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Locating mathematical activity a classroom study

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Locating mathematical activity: a classroom study

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Mathematics Education

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Abstract

This thesis recounts my research into value and purpose in the classroom construction of mathematical activity. The research took place in a boys' independent school where high examination expectations sustain. I am a teacher at the school; this thesis also narrates the development of a teacher-researcher identity through research.

Grounded methods and cultural-historical activity theory (CHAT) were used to explore the activity of the mathematics classroom in relation to pupils' developing mathematical capability. Pupil participants were interviewed from year 7 to year 10 and observed in lessons in order to reveal values associated with mathematical action through productive participation. Teachers were interviewed to explore the purposes they tried to convey in their lessons. By examining the actions engendered by participative, institutional and personal values I draw a picture of the object of mathematics classroom activity within established norms of order and work. Exploring the co-construction of classroom activity with CHAT revealed the persistence of systemic tensions within the stable activity. Neither pupils' developing authority nor teachers' awareness of these tensions impacted substantially on the roles that sustained. This stasis accompanied limits placed upon pupils' expectations of personal transformation, and inhibited scope for the teacher to introduce aims other than examination success.

I conclude that engagement with the generative potential of tensions was hampered by a restrictive focus on the high-stakes end point of compulsory mathematics education. Responsibility for this is attributed to the combination of cultural and institutional pressures which results in opportunities being placed aside and mathematical capability undeveloped. In developing a dual identity, I placed the aims and effectiveness of mathematics teaching in a wider socio-historical context, concluding that the development of mathematical authority as defined by examination curricula results in the classroom as a tool-and-result methodology, producing pupils in an enduring relationship with knowledge and knowledge construction.

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1 Overture and structure of this thesis

This thesis tells the story of my research into classroom mathematical activity. The research was borne out of a state of dissatisfaction: I perceived a mismatch between policy claims about the importance of mathematics education and the frequently encountered popular claim that one rarely uses mathematics outside school. This led me to question how the qualities and purposes of mathematical activity are understood, and how this understanding develops out of classroom activity. I was keen to learn more about the relationship between mathematical activity and mathematics teaching, by investigating classroom practice. As a graduate with a passionate attachment to mathematics, I wanted to understand the nature of mathematics as it is made manifest in classroom practice. As a teacher, I wanted better to understand how successfully the potential purposes of mathematical activity were made visible for pupils. The resulting story, a “confessional tale” (van Maanen, 1988), is about my intellectual and professional development through the experience of doing research (Flick, 2002, p. 240). This will be seen in the questions which emerged, the process by which I came to answer them and the form of conclusions reached.

In this chapter I trace the emergence of the problematic which led to my embarking upon further study (§1.1). This forms the basis for identifying the initial research problem, outlined in §1.2. In §1.3 I detail the aims which have guided the research and informed this writing, before outlining the narrative form of this thesis (§1.4). Finally, an outline of the thesis structure is given (§1.5).

1.1 The problematic: contradictory attitudes toward mathematics

Having nurtured a keen personal interest in mathematics from an early age I feel the ability to act mathematically has served me well in social and practical engagement with the world. However, I often hear the complaint that “I’ve never used any of the maths I learned at school”: many people do not see a connection between their experience of mathematics education and their adult lives. As a teacher I endeavour strongly to respond to the questions “What is mathematics for?” and “Why are we learning this?” whether

posed by adults or pupils, inside or outside the classroom. The need to combat negative attitudes towards mathematics propelled me to consider their possible sources.

In contrast to this disaffection, mathematics has long been privileged as a core subject for pupils up to the age of 16; parents, schools and employers place an emphasis on examination achievement in mathematics. Mathematics acts as a gatekeeper subject for entry into further study and employment and as a service subject for many undergraduate disciplines. The research which has revealed the 'earnings premium' on A-level mathematics in the UK (Dolton and Vignoles, 2002), indicates the desirability of a mathematics qualification and numeracy skills are seen as a key attribute in the workplace (CBI, 2008, 2009, 2010a, 2011, 2012).

This apparent contradiction in regard to the status of mathematics education raised some key questions for me: to what extent can mathematical activity be demonstrated to be important in people's lives? Is success in mathematics education anything more than just a token of academic capability? I saw these questions as pertaining to the ethical responsibilities of teachers to pupils, schools and to mathematics itself. There have been various curriculum changes and recommendations for mathematics education in England in recent years. These changes have been accompanied by claims that they will make courses more accessible and increase participation, improve attainment or serve better the aims of preparing pupils for their lives after school. Consequently, I started to ask how curricular aims were communicated through the teaching of mathematics, and how this might change. I was keen to understand how attitudes towards mathematics were fostered in the classroom.

1.2 Investigating the origins of attitudes: classroom mathematical activity

Pupil participation in the activity of the mathematics classroom is expected on the basis that adopting the aims of the curriculum will serve them well for their future lives. However, personal experience and research (Goodchild, 2001) have shown that whilst pupils might participate to a teacher's satisfaction, they do not automatically adopt the aims of the curriculum; they have their own attitudes towards mathematics (Picker and

Berry, 2000). Teachers also have their own implicit understanding of mathematics which influence their teaching (Thom, 1973). Informal observation had shown me that the delivery of any curriculum is mediated by the skills and interests of a teacher, and subject to institutional constraints. I had observed interplay between how pupils are instructed and the implicit messages taken from engagement in tasks in the classroom: explicit aims might be refracted through the form of activity and what is ostensibly achieved. The outcomes of classroom activity also appeared to be shaped by the attitudes of the pupils engaging in the activity. Thus, I resolved that if I were to investigate mathematical activity in the classroom it would be necessary to incorporate the pupils' point of view and understand how it influenced their participation.

My interest was in how pupils' intentions interact with teachers' constructions of classroom activity to lead to the emergence of actions identified as 'mathematics'. Acting mathematically involves not only identifying when mathematical action is appropriate and certain resources can be co-opted into action, but also recognising when mathematical resources are being used and being able to operate with them. I use ambiguity of the term 'locate' to indicate both placing and finding mathematical action.

In telling the story of my research, it will be seen that my focal question developed throughout the process. Hence it will be periodically refined throughout the thesis. The rationale behind this approach will be explained further in chapter 5. As the result of my reasoning into the problematic, my initial research question was:

How is mathematical activity located in the practice of the mathematics classroom?

1.3 Aims of this research

The purpose of this research was better to understand the construction of mathematical action in the classroom, and was explored through the following focused aims. These aims relate to pupils, teachers, researchers and myself as a teacher-researcher, and became substantiated over the course of the research.

- To form a description of how pupils locate mathematics in the classroom

- To better understand how this information can inform teachers' practice
- To identify implications for theory-building of the first two aims.

In relation to my primary aim, I hoped to address issues of how pupils' engagement in classroom activity related to their personal aims, how these aims related to those of the classroom, and the characteristics of mathematical action that were sustained there. These queries also related to pupils exercising their subjectivity in the structured mathematics classroom.

In response to my second aim, I hoped to contribute understanding of how teachers can work with, or with regard to, pupils' intentions; how these can be harnessed to unite with the broader purposes of mathematics education. I was interested in understanding how teachers can convey these purposes in productive ways. I see the question of understanding how mathematical activity is constructed as part of my role as a reflective practitioner. The intention that this thesis contributes to a conversation with other teachers is reflected in the language used throughout and the frame of reference in which I draw conclusions. I am a practising teacher who has undertaken a research project, and my focus has been the activity of the classroom, with the aim of contributing to understanding of what can be achieved there.

In undertaking this research I identified cultural-historical activity theory (CHAT) as a theoretical resource which would equip me to formulate a description of action in the mathematics classroom which appealed to my aims. Working with CHAT has shed light upon its strengths and limitations, complementing theoretical debates. I aimed to make a contribution to the ongoing theoretical discussion with observations grounded in empirical data. Framing my research and the production of this thesis as an activity in itself has facilitated reflexivity (Ball, 1990), providing a means of rigorously scrutinising the theory itself.

1.4 Narratives in this research: identity, questions and theoretical engagement

In writing in a narrative frame, I present my research experience as a means of reorganising my personal practical knowledge (Carter, 1993). Writing a narrative enabled me to chart my intellectual, pedagogical and personal development throughout the transformative process of research. In order to provide the rigour expected of educational research, my approach incorporates reflexive writing, sharing the challenges faced and processes by which I determined how to overcome them. Conle (2000a) notes that Bruner (1996) points to “Trouble” as the necessary *raison d’être* of narrative, which applied to my research: “What drives the story, what makes it worth Trouble: some misfit between Agents, Acts, Goals, Settings and Means” (Bruner, 1996, p. 94, cited in; Conle, 2000a, p. 190). This “Trouble” will be seen at the macro-level of the research process and writing this thesis and the micro-level of decisions made throughout. This narrative presentation of my enquiry includes exploration of contexts and social interactions, reflecting the fact that my research was embedded in school life.

In writing this thesis, adopting techniques from narrative enquiry (Conle, 2001; Polkinghorne, 2007; Hendry, 2010) offered a means of conceptualising my aims. Conle (2000a) describes the aim of narrative enquiry as *telos*, a “tacit end in view that drives the enquiry” (p. 193). However, understanding the process in reaching that end will be seen to be crucial: “It is in the course of the quest and only through encountering and coping with the various particular harms, dangers, temptations and distractions which provide any quest with its episodes and incidents that the goal of the quest is finally to be understood” (MacIntyre, 1988, p. 219). In the research, a dialectic relation between ends and means sustained, resonating with the theoretical approach at its centre. Throughout this thesis, as in the research process, the “tacit end in view” takes form to become a meaningful description of the mathematics classroom and mathematical activity. Through this retelling, I attempt to present a picture that will resonate with the experiences of other teachers, revealing a general case within this particular story.

Consequently this thesis is structured around a set of three related narratives. The first of these presents my personal and professional development as a teacher-researcher, through the orienting decisions I made in the course of my investigation. I refer to this as the *dual identity narrative*. As a reflective practitioner, it is part of my work to evaluate the effectiveness of teaching methods, and I saw academic research as a logical continuation of this, with exploration of the aims and mechanisms of teaching itself. However, the process has both complemented and conflicted with teaching practice, a situation which I explore in this thesis. The dual identity narrative constitutes chapter 2, and continues throughout the thesis to the end, where I detail the importance of my methods and findings to my practice. I aim to show that the process has been a powerful one in developing a dual identity as a researcher and teacher.

The second narrative (the *developing question narrative*) tells of the development of my research question in relation to my rationale. Following the initial statement of my enquiry, review of relevant literature enabled me to come to a more substantive formulation of the questions. My focus shifted from pupils' attitudes to the purposes and values operating in the classroom, and how these influenced the construction of mathematical activity. This enabled me to identify the features of classroom activity that would form my response to my questions. As my research progressed, the research questions became honed and increasingly concrete. This development is tracked throughout the thesis until I draw the threads of enquiry together in the final chapter.

The third narrative (the *theoretical investigation narrative*) concerns the adoption and exploration of a suitable theoretical framework with which to conduct the investigation. This develops out of and contributes to the second narrative, and complements my pursuit of a concrete response to the research questions, embedded in classroom activity. I show how my focus on participation, context, purpose and development led to the use of CHAT. Adopting the constructs of CHAT shaped my intellectual development, alongside the development of this research. I tell the story of my developing concretisation of ideas, as I begin to see the classroom as a tool-and-result methodology (Vygotsky, 1978) which continually justifies and recreates itself. The process by which I came to understand my

data cast light on applications of CHAT in other research settings and the various emphases placed on elementary concepts. In the pursuit of a coherent and purposeful description of classroom activity, I came to interrogate the concepts of activity theory, and determined those that could contribute to my aim of informing teaching practice.

Exploration of my data with the structuring apparatus of activity theory raised questions of the relationship between subjectivity and the systems in which one operates. In drawing together the narratives at the end of the thesis, I consider subjectivity as it pertains to pupils and teachers and also as a means of reflecting upon my research process and findings. I return to my position as a person working at the boundary of teaching and research, considering the constraints and affordances of that position. Reflection upon the choices I made in this research contributed to my understanding of both aspects, and to my conclusions regarding the construction of mathematical activity.

1.5 Thesis structure

In telling the story of my research into mathematical activity in the classroom, I begin (chapter 2) by exploring the situation that led to my initial queries. Initial reading into the pressures that influence mathematics in schools and lead to the competing stances on the uses and meaning of mathematics gave shape to the questions I wanted to answer, resulting in a shift in my focus from attitudes to values. In this chapter, I lay out the rationale which shaped my enquiry, and illustrate how making this rationale explicit enabled me to identify the key issues my investigation should address, providing a research framework. In chapter 3 I review the literature which formed a backdrop to my research and substantiated my rationale. This exploration enabled me to orient myself in relation to my aims, and led to the emergence of research questions which connected classroom practice with questions of purpose. In this chapter I explain it became clear that my description of classroom practice would have to have certain characteristics in order to answer my questions:

- My approach would have to relate in a concrete way to pupils' mathematical activity in the classroom.

- It would have to integrate the perspectives of both pupils and teachers, but also have the capacity to open up to broader fields of enquiry.
- It would have to incorporate my professional experience and outlook in this area: I should not subtract myself or my concrete experience from the exploration.

In chapter 4 I offer my justification for using CHAT in this research. With the aim in mind of communicating to other teachers, I made a choice that could offer tangible descriptions of the classroom which I felt would productively relate to the aims of mathematics education, and would serve well in ongoing dialogue with practice.

Chapter 5 details my reading in the history and development of CHAT, which opened possibilities of theoretical exploration. My need to consider pupils' views in the light of concrete action meant that my data had to have a suitably concrete treatment (recounted in chapter 6). This recognition led to the use of grounded theory (Glaser and Strauss, 1967; Strauss and Corbin, 1994; Charmaz, 1995) in my data collection and analysis. This approach spurred on the research, leading to the emergence of codes which I found powerful in constructing a description of classroom action. Chapter 7 details the results of this shift, which led to a structured understanding of pupils' experience and how it reflected aims of mathematics education. This description conveyed the tensions and complementarities within classroom action, and the mechanisms which sustained these. In chapter 8 I explore that description, bringing together CHAT and grounded theory. I conduct my formal analysis of the data, and develop a multi-voiced description, incorporating data from the teachers and classroom observation. These chapters also chart my reflexive investigation of activity theory itself, which led to the emergence of a rich description of classroom action from the pupils' vantage point.

With this description I explored the relationship between pupils' actions in the classroom activity system and the aims of those actions, in relation to their developing mathematical capability: characteristics of mathematical action and the activity are discussed in chapter 9. In this chapter I present my interpretation of the data and use it to address issues arising from the theoretical debates, specifically with reference to the aims of classroom

activity. In doing so I aim to bring an empirical perspective on the object of activity, a problem embedded within the 3rd generation of activity theory. By positing a model for the resultant shared object of teachers' and pupils' activity, I aim to contribute to the understanding of the relation between mathematical action inside and outside the classroom. This thesis ends in chapter 10 with a review of the research and further questions generated by my understanding of the emergent object. My discussion returns to address my original three aims, and is oriented towards informing teachers' practice.

2 Rationales for this study: policy and practice

In this chapter I lay out the complementary rationales for the research. These emerged from my initial exploration of the conflict outlined in chapter 1. I begin this chapter (§2.1) by detailing my initial consultation of literature into images of and attitudes towards mathematics; this shifted my attention to the values at play in mathematics classrooms. In §2.2 I consider recent policy debates into the mathematical needs of school-leavers and adults, and how these might best be served in the curriculum. I then turn to research into mathematical needs and “street mathematics” (§2.3). My exploration of the literature raised questions about the characteristics of mathematical activity and the classrooms in which it is learned, which are considered in §2.4. In light of this discussion, in §2.5 I justify my orientation towards teachers’ practice and pupils’ aims, and reformulate my research question.

2.1 Shifting my attention from attitudes to values

My initial exploration into attitudes towards mathematics revealed that negative attitudes have long been documented (see, for example, Henderson, 1981; Ernest, 1996; Rock and Shaw, 2000; Noyes, 2007). In this country, the Cockcroft Report (1982) reported research in which roughly 50% of potential interviewees refused to answer questions about mathematics. This observation was repeated 15 years later when an international survey by the Basic Skills Agency (1997; cited in Lim, 1999) about numeracy skills of adults reported double the rate of refusal to answer in the UK than in other countries (13%, as opposed to a peak of 6%). Lim (1999) states that “many adults of Anglo-American countries are not embarrassed to proclaim their ignorance or poor performance in mathematics” (p.14), and connects this with beliefs about the qualities of successful learners of mathematics. Negative attitudes to mathematical action are identified by Hoyles *et al.*, who assert that many adults are unaware of the mathematical activity they undertake, and the suggestion that they are doing something mathematical engenders “hostility, fear and even guilt” (1999, p.48).

Widespread negative “myths” about mathematics are documented by Ernest (1996, p. 785), who locates their roots in the “stereotyped experience of school mathematics shared by many.” These experiences stem, he proposes, from a “Modernist, axiomatic” understanding of mathematical activity, sustaining a conception of mathematics as an “objective, absolute, certain and incorrigible body of knowledge” (ibid, p. 807). This absolutist conception can emerge as a dualist approach to mathematical activity, in which “mathematical knowledge is in final form, and its foundations are permanent and secure, and if a human error is found uncovered it means that the questioned parts were not knowledge after all” (ibid, p. 807). Similarly, Boaler (1998; 2000) relates negative attitudes towards mathematics to pedagogic practices. I began to see that the relative positions of mathematical knowledge and persons acting might be at the root of negative attitudes.

The relationship between images of mathematical knowledge and views of mathematical activity formed the basis of Lim’s (1999) research. In a cross-cultural comparison of Malaysian and UK teachers and students, Lim found that views of mathematics informed by utilitarian values or as a system of symbolic expression (as opposed to a mode of engagement with the world) were more commonly held by those who claimed to dislike mathematics. She also found that images of mathematics were not differentiated from images of learning mathematics, suggesting mathematics teachers’ teaching styles, and motivation given by teachers and parents, may lead to differences in images and beliefs about mathematics. She posits that the views held by teachers influenced what they considered permissible action in the classroom, thus “that mathematics teachers might be one of the major influences on the respondent’s images of mathematics” (ibid, p. 340). Lim relates absolutist views of mathematics with the belief that hard work will lead to success and *fallibilist* (Ernest, 1991) views with conceptions of inherited ability. Fallibilist approaches to mathematics represent it as the outcome of social processes, and therefore contingent and open to revision, with no single fixed hierarchical structure. Lim found that UK teachers were more likely to hold fallibilist views, and proposes this might have had a subsequent effect on the image of mathematics transmitted through their teaching, and the qualities valued in pupils’ participation. The posited relationship between these

characterisations of mathematical action and the people who do it contributed to my need to focus on mathematics as a feature of human action, and those actions which would be valued in the mathematics classroom.

Lim's (1999) research suggests that teachers' fallibilist notions related to their conceptions of the personal qualities required for pupils to succeed at school mathematics. She restates the prevalence of "myths" about mathematical activity that reflect unfavourably on the personal qualities of learners, and highlights the pervasiveness of the notion that inherited ability forms a substantial part of mathematical competence (McLeod, 1992; FitzSimons et al., 1996). The myths cited include: mathematics is a difficult subject; mathematics is only for the clever ones; mathematics is a male domain. Lim posits that these myths can infiltrate teaching practice and expectations; negative recursive cycles develop in which lower teacher expectations lead to particular pupils having lower self-confidence and performing less well, thus reinforcing the myths (Gutbezahl, 1995). These observations seem to run counter to Ernest's (1991; 1996; 1998) proposal that recognising the power of fallibilist descriptions of school mathematics should help to develop and improve teaching practice. Rather they suggested that conveying a philosophy of mathematics through teaching is not straightforward and that any philosophy has a two-way formative relationship with events in the classroom.

Haladyna *et al.* proposed a causal link from qualities of teaching in the "social-psychological climate of the class" (1983, p. 19) to pupils' attitudes toward mathematics. In their US-based study, they found that this link was stronger in grades 7 and 9 than grade 4. Mcleod (1992; 1994) also notes stronger attitudes in older pupils, concluding that attitudes become more negative as pupils move from primary to secondary school. Swan (2004) draws attention to classroom practices that might contribute to negative attitudes towards mathematics. He highlights a need to attend to the role of the teacher, the qualities of learning that are inculcated (Skemp, 1971) and the levels of stress or interest that tasks can generate. By placing their focus at the level of the classroom, rather than the individual pupil, Haladyna *et al.* sought to incorporate social context into their analysis, explaining that "most classrooms are organised for producing [class] effects"

(1983, p. 19), and that teacher quality and the learning environment have to be understood at this level. Hannula (2002) investigates the intricate relationships between attitude, expectation, participation and achievement, tentatively suggesting a causal effect from attitude to achievement.

Berry and Picker (2000; Picker and Berry, 2000) explored images of mathematics and mathematicians held by 12-13 year-old pupils, an age which “seems to be a critical point in the determination of attitudes towards mathematics” (Aiken, 1970, p. 556). Pupils’ images (in the US and the UK) were found to perpetuate common stereotypes of gender, and to betray a limited understanding of the uses of mathematics outside school. The mathematics represented in the images could be described as ‘hard sums’; complex calculations built from successive arithmetic operations or simple calculations with large or decimal numbers. This sustained even though pupils in the study had encountered algebra which was more sophisticated. Berry and Picker attribute these largely negative images to cultural and societal stereotypes and memes (Dawkins, 1989) and the possible reinforcement of those stereotypes through non-specialist teaching in primary school. They suggest that mathematics teaching which creates a perception of the subject as a rigid system of externally dictated rules continues to contribute to negative images. They proposed a self-perpetuating cycle of stereotypical images of mathematicians and mathematics that remain unchallenged by teachers, complementing the recursive cycle of stereotypical images of learners identified by Lim. Placed alongside Lim’s research, this indicated a complex relationship between teachers’ conceptions of mathematics and that which might be perceived by their pupils, suggesting that the values sustained by institutional priorities and curricula need also to be considered.

Evans (2000a) notes that after leaving school, adults’ attitudes towards mathematics are related to the social contexts in which they might encounter mathematical activity and the value placed upon it. The extent and frequency to which adults needed to use mathematics in their daily lives influenced their attitudes, in relation to the purposes for which it was used. Cultural practices in which they acted were seen to influence how well

they could call up mathematical methods, in relation to how familiar those methods are, and how well they fit the context of action.

The literature offered some suggestion that cultural stereotypes (such as those of gender) are currently being destabilised: Mendick et al. (2007) note a recent increase in diverse representations of mathematics and mathematicians in popular culture. However, this research suggests that while diverse representations are available, they are not necessarily pervasive in popular culture. McDermott (2013) questions whether these representations necessarily challenge stereotypes or actually reinforce them, as the image of mathematics as a closed, technical and specialised discipline sustains. The incorrigibility of mathematics is often opposed to the fallibility of human actors, placing value on only those actions seen to be correct or effective.

The need to consider the confluence of pedagogical practices and pupil participation became clear to me through considering recent patterns in voluntary uptake of mathematics, post-16. During the 1990s there was a drop of nearly 10% in the numbers taking A-level mathematics in England (Smith, 2004). Although numbers are recovering, with the number of pupils who took A level mathematics in 2011 being the highest for 19 years, there is still a shortfall in the numbers who will require mathematics in their work or further study (ACME, 2011)¹. This ongoing increase coincides with a reduction in the 'applied maths' content of the A-level (Statistics and Mechanics), which took place in 2005, and raises the question of how mathematical needs can be served whilst also attracting students to the subject. Currently, roughly 85% of pupils in England end their mathematics education at 16 (Nuffield Foundation, 2010). In 2009, approximately 85,000 students studied mathematics post-16, when over 300,000 students per year are needed (ACME, 2011). Boaler (2002a; 2002b) demonstrated that pupils can be "turned away from mathematics because of pedagogical practices that are unrelated to the nature of

¹ The number of students completing A-level Mathematics continued to rise in 2012 and 2013, with 85 714 and 88 060 students (respectively), greater than 2011's 82 995 students. Data from the Joint Council for Qualifications: <http://www.jcq.org.uk/examination-results/a-levels>

mathematics”, echoing Burton (1999). Pupils who were successful at mathematics had decided to terminate their mathematics education at age 16, as they sought opportunities for “expression, interpretation and human agency” (Boaler, 2002b, p. 44), which would be valued in other disciplines.

My preliminary reading placed the relationship between attitudes and experiences of school mathematics in the context of the values imparted and sustained in mathematics education. Lim’s (1999) focus on the effect of mathematics teachers also recognised the influence of cultural and familial values. However, attitudinal research I encountered (such as Lim, 1999; Evans, 2000a) was undertaken with people who had completed their compulsory mathematics education, and had reflected upon their more or less distant experience. In this vein, Esmonde *et al.* (2013) explored the communicative resources available to people in talking about mathematics. They found that whilst people’s lives contained a deal of mathematical activity, this was dissociated from school experience in which ideas had had value only in terms of ability, authority and competitive pride. Mathematics was often seen as a set of procedures disconnected from purposes other than school achievement and consequently adults had little means to articulate the purpose of mathematical action in their lives. Lim’s research suggested there are potential effects of the pupils’ aims and values on their participation in school: for some learners, difficulty in mathematics might be seen as an enjoyable challenge, or if accompanied by pre-existing conceptions of inherited ability, it could lead to frustration, boredom and giving up on problems. This raised the question for me of the relationship between pupils’ values and images of mathematics as they progressed through secondary school.

My preliminary reading drew attention to the importance of values attributed to and within mathematics education, as a central component of pupils’ achievement (with its social and economic consequences) and their ongoing notions of mathematical activity. I saw these as key elements in a culture in which mathematics is valued to a greater or lesser extent, and in relation to the classroom where these values are instantiated.

2.2 Curricular aims: purposes of mathematics

In this section I briefly review the recent history of the role of mathematics in English education, considering policy recommendations made in relation to the needs served by curricula. Recent decades have seen repeated shifts in the English qualification structure, but throughout I found a sustained political emphasis on the societal value of mathematical skills, with a largely utilitarian focus (Noyes, 2008). However, it was not clear that this emphasis was accompanied by a coherent understanding of what those skills are or how they might be supported by school curricula.

Mathematics has long been considered part of the core curriculum in schools; all pupils in England are expected to gain the basic qualification in mathematics (commonly, the GCSE) at age 16. This qualification acts as a “gatekeeper” for future employment, as does the A-level examination. The introduction during the 1990s of SATs for 7-, 11- and 14-year olds reinforced the status of mathematics as a gatekeeper subject: achievement in mathematics, English and Science at these ages was used as a predictor of likely later achievement across all subjects, with all pupils targeted to achieve a baseline ‘level’ at age 11. The National Numeracy Strategy was introduced into primary schools in 1996, and extended into the mathematics strand of the Key Stage 3 Strategy in maintained secondary schools in 2001. These strategies were characterised by a focus on whole-class teaching and central prescription of acceptable mathematical methods (see Brown et al., 1998; 2000; 2003 for a discussion of the implementation of these strategies). The SATs for 7- and 14-year olds have since been dropped and replaced by teacher assessments across all subjects.

In 2002 the Smith inquiry convened to examine post-14 mathematics education provision and recommend changes to curriculum, qualifications and pedagogy. The rationale for this inquiry lay in the need to address “skills shortages that will adversely affect the Government’s productivity and innovation strategy” and “to enable those students to acquire the mathematical knowledge and skills necessary to meet the requirements of employers and of further and higher education” (Smith, 2004, p.1). The report thus

focused on meeting the needs of learners in order to become productive members of society. It represented mathematics as

...a major intellectual discipline in its own right, as well as providing the underpinning language for the rest of science and engineering and, increasingly, for other disciplines in the social and medical sciences. It underpins major sectors of modern business and industry, in particular, financial services and ICT. It also provides the individual citizen with empowering skills for the conduct of private and social life and with key skills required at virtually all levels of employment.

(Smith, 2004, p. 2)

Various recommendations were made in relation to compulsory mathematics education, some of which were adopted promptly. Change from a three-tier to a two-tier GCSE examination happened in 2006; the lowest tier of entry had previously rendered recognised 'pass' grades unobtainable to some pupils. This change happened alongside the reduction of the content of the A-level curriculum which saw the recent increase in uptake.

During the early 2000s there was an increase in the uptake of the International GCSE (IGCSE) award offered by examination boards to independent and international schools. The IGCSE stands as an alternative to the GCSE, in that it does not contain assessed investigative coursework, unlike the GCSE at the time (coursework was removed from the GCSE from 2008) and has syllabi explicitly designed to give pupils a foundation for further study in the subject (Edexcel, 2003), with topics such as set theory, functions and calculus. In 2012 the Education Secretary authorised the use of the IGCSE from one examination board (Cambridge International Examinations) for use in the maintained sector in England.

The report by Tomlinson (2004) into 14-19 pathways in education focused upon improving uptake and attainment in post-16 education in the maintained sector and strengthening vocational routes for students. The report recommended that all pupils study "Functional Mathematics" as part of their core learning in a Diploma programme. Functional

Mathematics was also proposed to comprise between 50% and 80% of the GCSE specification, and there was a recommendation that the Qualifications and Curriculum Authority “works with all stakeholders... to develop core components in functional mathematics” (ibid, pp. 31-2). Diplomas in health, media, information communications technology, engineering and construction were launched in 2008, each containing Functional Mathematics elements. The current programme of study for GCSE in England emphasises that pupils should learn to apply mathematical methods in “relevant real-life situations” (QCA, 2007)². Currently, pupils in the maintained sector in England study for a “Functional Maths” GCSE, introduced for first examination in 2012. This qualification is intended to have up to 40% of its assessment addressing functional elements of mathematics use, reinforcing the contestable utilitarian approach to mathematics education (Noyes, 2008).

At the time of writing, a pilot study of a double award in Mathematics is in its third year; this dual qualification would offer GCSEs known as “Mathematical Methods” and “Mathematical Applications”. The recent proposal of an English Baccalaureate qualification comprising a suite of essential subjects including mathematics was proposed by the Education Secretary, but rejected by the teaching profession. Proposals that, from 2015, mathematics should be studied until a student is 18 (Vorderman et al., 2011; Hodgen and Marks, 2013) have been partially implemented: as of September 2013 students who did not achieve a grade C at GCSE remain in compulsory mathematics education until they do or until they are 18.

The privileged position of mathematics in the curriculum would appear to be secure, but will continue to come under scrutiny during further reviews, as will notions of applicability

² At the time of writing, the programmes of study are under review, with new programmes to be followed from September 2014. The draft new programme of study states the pupils should “use mathematical knowledge to solve problems within and outside mathematics” and “model realistic situations mathematically” (DfE, 2013, p.4)

and “functional mathematics”. However, the extent to which proposed changes will impact upon teaching practice is far from clear. Recent proposals (DfE, 2010; Gove, 2011) have contained contradictory rhetoric about liberating teachers from a prescriptive curriculum, whilst increasing the coverage of topics such as calculus and statistics.

This recent history takes place against a backdrop of GCSE grade attainment which has increased substantially in recent decades (ACME, 2011), and the increasing extent of an “audit culture” (Williams et al., 2008) in English Education. However, Ofsted (2012) note that GCSE and A-level results continue to rise, “without corresponding evidence of pupils’ better understanding of mathematics to equip them for the next stages of their education and future lives ” (p. 6-7). Ofsted attribute this to “too much teaching concentrated on the acquisition of disparate skills that enabled pupils to pass tests and examinations but did not equip them for the next stage of education, work and life” (p. 9). Responsibility for grade inflation in recent years has been attributed “squarely with the regulatory system” (Vorderman et al., 2011, p. 8). Reports from the CBI (2008 - 2012) consistently call for an improvement in new and prospective employees’ numeracy and mathematics skills. The 2008 report stresses employers’ concerns over “functional numeracy” which they define as use of multiplication tables and mental arithmetic; percentages, ratios, fractions and decimals; different measures and conversion between them; spotting errors and rogue figures; odds and probabilities. School-leavers’ proficiency in these areas is perceived to be wanting.

2.3 Meaning and purpose with mathematics in everyday life and work

In order to make some more sense of this situation I conducted some preliminary reading into the mathematics that people do outside school. I saw the need to complement the overview of curriculum change with insights from research into everyday mathematics practices.

Research by Abreu *et al.* (1997) showed children aged between 7 and 13 using mathematics outside school in tasks involving money and measure, and in drawing geometric figures. However, the identification of actions *as mathematical* was dependent

upon their role in specific practices. Children in the research upheld a separation of out-of-school mathematics from school mathematics that originated in the classroom. Teachers of children in this study would exclude informal or out-of-school methods from classroom practice, and failed to explore pupils' idiosyncratic reasoning when it had led to incorrect answers. Recognition of out-of-school actions as mathematical depended upon making connection with authorised versions of mathematics, as defined by school practices. Abreu *et al.* posit that children inherited a conception that school mathematics was "superior" (ibid, p. 252), and that an asymmetrical relationship sustained in which out-of-school practices might incorporate school practices, but the converse would not occur. Children would refrain from classifying their actions as mathematical, if they had been excluded from "proper knowledge" by their schooling (p. 250).

Nunes *et al.* (1987) offered a means of understanding how people work with mathematical relationships in everyday life ("street mathematics"). This involves "understanding mathematical relationships that are embedded in particular activities, technologies and situations. *In order to function well in these cultural contexts, subjects must understand the mathematical invariants as well as the particulars of the situations*" (p. 139, original emphasis). Nunes *et al.* showed that in working mathematically outside school people adopted pragmatic approaches in response to specific circumstances, rather than pursuing general results or theories. The specific meanings in lived problems allowed for control of problem scenarios, but these did not restrict the potential for application: evidence was found for flexibility and generalizability of methods used. They suggest that successful learning and problem-solving in everyday life might be explained by the preservation of meaning during problem-solving activities, and raise the question of how well school practices recognised this. Masingila (1993, p. 18) also explains that problem-solving outside school is dilemma driven and goal directed, contrasting with school mathematics, which is dominated by a lack of context, relevance or a specific goal. In order to bridge this separation, Masingila makes recommendations that teachers should build upon pupils' knowledge from out-of-school situations, introduce concepts

through problem-solving scenarios and establish master-apprentice relationships in order to initiate pupils into the mathematics community (ibid, pp.19-20).

Whilst recognising the power of these recommendations, my experience as a teacher has made me sensitive to the difficulties and complications of incorporating out-of-school knowledge and practices into classroom action. This has been (in part) articulated as the *planning paradox* (Ainley et al., 2006): planning from objectives can be unrewarding for pupils, but planning from engaging tasks can result in unfocused activity that fosters learning which is difficult to assess. This situation was recognised by Walkerdine (1988): highlighting school mathematics as a specific discursive practice reveals the difficulty and artificiality of inserting contexts to problem solving. She draws attention to the tension between serving the needs of the curriculum and offering pupils appropriate challenges within tasks whilst preserving meaning in the contexts used. Recognising the bounded nature of school mathematics brought in to question whether it was appropriate to describe school mathematics as lacking context or specific goals. These readings highlighted the need to recognise school mathematics as inhabiting a particular context, rather than decontextualised (Noss and Hoyles, 1996), and to question how this related to the purposes pupils saw for mathematics.

2.4 Orienting my notions about mathematical activity

My preliminary readings initiated a change in how I located mathematical activity. In this section I detail that change, in order to make clear what a focus on practice would mean for me in the research, and to reveal the power of the classroom as a community in the formation of mathematical activity. This change lay at the heart of my development as a teacher-researcher, and informs my discussion throughout this thesis.

By the end of my formal mathematics education (a master's degree) I had completed a great deal of study in pure mathematics. My favoured approach to mathematics was formal and algebraic, connected with understanding in a highly specialised language and distanced from everyday applications. However, this was complemented by a strong sense that mathematics could be located within everyday behaviour and without requiring

formal language. Preliminary reading on ethnomathematics (D'Ambrosio, 1984; Nunes et al., 1992) alongside the comparisons of school and street mathematics (Nunes et al., 1987; Masingila et al., 1996; Abreu et al., 1997; Civil, 2005) helped me to come to a more tangible sense of mathematical activity in various forms, and a stronger understanding of how mathematical activity can be located in the world. At the outset of my investigations I conceptualised mathematics as a constellation of shared systems of behaviour, appropriating culturally-specific resources relating to the use and exploration of structure (most commonly numerical and spatial structure). Hence I saw learning mathematics as appreciating and adopting models of thought and action established by others and preserved as cultural habits or artefacts. I came to articulate mathematics as constituted in the actions of those who do things mathematically, rather than as an incorrigible body of knowledge. I found it constructive to abjure the use of the noun 'mathematics' in descriptions, in favour of the adjectival form 'mathematical.' This has enabled considering actions as comparable in terms of being more, or less, mathematical, rather than corresponding to an essentialising definition of mathematics. With this in mind exploring pupils' understanding *of* mathematics became a key adjunct to exploring pupils' understanding *in* mathematics. The development of a "proper meta-concept of mathematics... makes teaching in a school system possible" (Howson and Mellin-Olsen, 1986, p. 30). My research would aim to explore such development.

In my development as a teacher-researcher, theoretical exploration played an important part in clarifying the notions that would underpin my analysis, but also sharpened my understanding of the results of participation in classroom activity. Key texts which had formed my early understanding of the learning of mathematics were Skemp (1971) and the CSMS research (Hart, 1981), which led me to adopt a position hovering somewhere between the social constructivism propounded by Ernest (1991; 1998; 2004) and the more ascetic radical constructivism of von Glasersfeld (1990; 1991). The constructivist ideas I held accounted adequately enough at the time for my own experience as an independent learner. Through developing my own practice I found that viewpoint helpful in talking about learning the concepts of mathematics, but it offered little to say about classrooms,

the social aspects of how mathematics is taught, or how pupils fail to learn. I found I could not articulate a meaningful formulation of the social world in which ideas develop. With experience and further reflection and research I adopted a Vygotskian (1962; 1978) appreciation of learning, considering knowledge as inherently social and cultural, developed as the result of the internalisation of social processes. The basis for this research is the disjunction of classroom and the world outside, recognised from a conception of learning and capability as socially and situationally specific. The creation of mathematical capability happens within specific practices and gains its meaning from those practices. This stance offered some insight into the problem I had observed, but also indicated that the positions of pupils in relation to practices required investigation.

McDermott and Webber (1998) helped me to reorient my thinking about learning mathematics, by focusing on “thinking practices in social contexts to understand learning as historically arranged and institutionally consequential” (ibid, p. 321). Through the device of asking “When is Math or Science?” McDermott and Webber place a focus on people reorganising their relations to each other and the world around them, in order to avoid an idealised description, which would reflect badly on learners and those who have not achieved “success” in an educational system. They pursue instead a specification of social arrangements which identify moments in activity as being mathematical, with these key questions:

By what order of persons in relation to what organization of things are moments put aside as mathematical or scientific? By whom, with what consequences, and by what means of accountability?

When, under what circumstances and by what order of persons and behaviour, are there opportunities available to people in classrooms for making such math and science moments overlap systematically with the lives of the children?

(McDermott and Webber, 1998, p. 312)

I found in these questions a call to understand the activity of the mathematics classroom as a space in which development and empowerment are supposed to occur. In response to issues raised in the previous section, I pick up the assertion that lessons are everyday

situated productions, in that they are in the world of the children and subject to multiple uses through their participation. Through those lessons, pupils develop ideas as to what it means to be successful with mathematics, and when it is fitting to behave mathematically. In Schoenfeld (1998) I found an articulation of the need to understand how pupils come to be in a position to make these judgements, and on what basis. Schoenfeld calls for a reconceptualisation of what it means to do mathematics, along the lines of how one recognises competence, operates in the context of problem-solving and understands how mathematical knowledge can be organised (ibid, p. 312).

Such aspirations could be seen to influence discussion papers on school mathematical provision (such as AAAS, 1989). This report conveyed a sense of what pupils should know about mathematics as an endeavour, regarding mathematical action as the exploration of patterns and relationships, and related to roles of inquiry and the place of mathematics in science and technology. Cuoco *et al.* (1996) offer inspiration in reorienting priorities, to mimic the communal habits of academic mathematicians in the classroom. They propose considering the habits of the community as desired characteristics of classroom action, with the aim of empowering pupils. Aspirational documents such as these demonstrated the connection between the characteristics of mathematical action and of the acting person.

2.5 Developing question narrative

The UK Advisory Committee on Mathematics Education reports that it is “...impossible to talk about learner’s needs without talking about mathematically-appropriate pedagogy” (ACME, 2011, p. 4), and that applicability of mathematical knowledge should result from a continual process of applying knowledge in new situations and conscious building upon knowledge in order to progress. This focus on teaching resonated with my own professional experience, and raised the question of how national and governmental priorities become manifest in classroom practice. ACME also notes that institutions have become more accountable for results than mathematical understanding, and recommend that this situation be addressed by bringing a focus on to teaching practices. I saw

recommendations such as this as an appeal for research to contribute directly to classroom practice by communicating with teachers. In response to the political contingency of curriculum development and detachment of policy from research, I felt it necessary to make a contribution to teaching practice that could sustain across curriculum and examination changes. My research would aim to make a direct connection between classroom practice and the purposes of mathematical activity.

The need to speak directly to teachers in the face of curricular change and ministerial ideology is supported by Vorderman et al. (2011), who advocate a return to a situation in which teachers had much greater influence over school mathematics. Current curricular changes and the process of their introduction are under contestation (ASCL, 2013), with policy recommendations developing without reference to research. The development of meaningful mathematical capability should stem from the modes of engagement in the classroom, rather than relying on curricular specifications.

By undertaking this research I sought to understand the construction of mathematical action in the classroom. My preliminary reading indicated this would involve considering not only the actions that were meaningful in the classroom but also the processes by which meaning was validated and the capacity the pupils developed for establishing mathematical meaning outside the classroom.

My initial research question was:

How is mathematical activity located in the practice of the mathematics classroom?

In light of the reading detailed here it was reformulated to address my needs:

What are the processes that shape pupils' understanding of the purposes of mathematics education and the means of locating mathematical action in the world?

In order to respond to the situation from which my problem originated, and to speak about the construction of mathematical action, I had to:

- Explore the “social-psychological climate” of the classroom

- Explore how the purpose of mathematical activity was communicated through classroom action
- Inquire into the preservation of meaning in mathematics and the values placed upon pupils' actions
- Explore the extent to which subjectivity can be exerted by pupils
- Explore the extent to which engagement in mathematics lessons is purposeful and the means by which classroom action connects with the world

By responding to these priorities I hoped to be able to:

- Address the practice of the teacher in guiding pupils through school mathematics, whilst acknowledging that pupils have an obligation to learn mathematics
- Focus on school as a practice that claims to prepare pupils for their later lives, and look for evidence of development and empowerment
- Answer questions about the sources of meaning which are admitted within the mathematics classroom, and how teachers' supra-curricular aims might be recognised by pupils and could be realised in the activity.

3 Literature review

Chapter 2 is formed around the dual identity narrative, telling how I began to orient myself as a teacher-researcher. In this chapter I chart my exploration of the literature, which further shaped my research question. This narrative continues here, initiating the developing question and theoretical engagement narratives. Having detailed the elements of the problematic (Brown and Dowling, 1998) in chapter 2, I first review research into values attributed to mathematical action (§3.1) and show how these are related to classroom and cultural purposes. This raised questions of the expectations and capabilities created by classroom action and the values imbued in them, in relation to the purposes served by mathematics education.

I then consider in more depth recent policy responses into the workplace mathematical needs of school-leavers (§3.2), and the extent to which classroom practice can relate to the workplace, alongside different conceptions of knowledge construction. I then open up considerations of the material which would substantiate my questions and responses (§3.2.2). In §3.3 I consider research into the use of mathematics outside school as a means of reflecting upon the particularities of school mathematics practice. Different conceptions of mathematics in use offer alternative means of describing action, which contributes to the developing question narrative, and to a deeper exploration of the theoretical requirements of my research. In §3.4 I show how my concerns led to the adoption of cultural historical activity theory. In §3.5 I begin to recast my research questions, in light of the theory.

3.1 Research into classroom values and purposes

As a result of the shift in my attention from attitudes to values and purposes I became interested in seeing how notions of purpose informed pupils' participation and teachers' work, and how these related to the values imparted to mathematical action. In this section I review literature from the point of view of an experienced teacher and novice researcher, and share the formative effect this had on my research question.

3.1.1 Values and purposes in the classroom

From Evans' (2000a) demonstration that adults' attitudes to mathematics develop and change in relation to ongoing notions of purpose and value, I turned my attention to classroom research, where I found similar observations of the dynamism of attitudes in relation to tasks and contexts. Nardi and Steward (2003) illustrate the effect that patterns of classroom action can have on attitudes toward mathematics, in relation to pupils' images of effective teaching. They show that engagement based upon school or parental pressure can result in a profile of "quiet disaffection" with school mathematics. This dynamism was echoed by Hannula (2002; 2004), who constructed a framework for exploring attitudes which included cognitive evaluations of goals and an emotional disposition towards them. Cognitive interpretations of tasks and associated affects together yielded attitudes dependent upon the extent of an individual's engagement in a task. Hannula suggests a reciprocal relationship between attitudes towards and understanding of mathematics, and posits that a negative attitude could be a successful defence strategy of a positive self-concept: attitudes continually develop in relation to personal goals and meaning. Mellin-Olsen (1981) notes that differing notions of purpose can give rise to differing qualities of engagement in classroom work. He identifies motivation to engage in work as part of the development of a self-image as the *S-rationale* and motivation to work in relation to success in school as an instrument for future success as the *I-rationale*. These papers brought my attention to the need to consider the personal meaningfulness of the tasks in which pupils were engaged. Meaningfulness might be intrinsic to the task, or could be related to extrinsic factors and values, related to the purposes envisaged for learning mathematics and participating in school.

Perceptions of the purposes of education, aligned with respondents' wider cultural conceptions of mathematics and education were also related to the images of mathematics, learning mathematics and mathematicians reported by Lim (1999) and Berry and Picker (Berry and Picker, 2000; Picker and Berry, 2000). These conceptions incorporated notions of value and use. Lim cites Elliott *et al.*: "perhaps more important are children's familial, peer and cultural perceptions about what constitutes real and

meaningful educational achievement and the extent to which this is seen to be of such intrinsic or extrinsic values as to evoke significant effort” (Elliott et al., 1999, p. 91). In a cross-cultural study (UK, US and Russia) Elliott *et al.* found complex local relationships between attributions of value to what was studied, the immediate purposes and values of educational attainment and enjoyment of school. This observation highlighted for me the importance of understanding which values sustained within classroom activity, the purposes to which they related, and the influence of cultural values.

The research by Berry and Picker (2000; Picker and Berry, 2000) revealed that values relating to classroom practice become related to images of mathematics. They found that lower-secondary school pupils associated doing mathematics with working under coercion, at a remove from their activity out of school and at the weaker end of a power relationship with authority. This connection of values casts a light on the observation by Evans (2000a) that whilst pupils might be successful within school, notions of proficiency and the value of school achievement become strained outside the classroom. This could in turn lead to a devaluation of mathematical action.

As a practising teacher I am conscious of classroom practices that can contribute to pupils’ dislike of mathematics and those which are designed to foster positive engagement. However the relation of values and participation is not straightforward. Swan (2004) cites the ease with which pupils can become successful in mathematics through rote learning, but also that for some, rote learning can be a source of anxiety (Skemp, 1971). Teaching literature abounds with ideas for engaging children’s interest in mathematics, such as “Mathemagic” tricks and the use of trivia as the basis of mathematical investigation (Swan, 2004). However, research suggests that the practice of creating games and rewards as a motivating factor for pupils could implicitly devalue the content of tasks (Lepper et al., 1973; Bates, 1979; Lepper et al., 1996; Deci et al., 1999), and thus contribute to negative images of mathematics.

In considering the communication of values through mathematics teaching, writings such as those by Ernest (1991; 1994; 1998) suggested that philosophies of mathematics

education are conveyed through the tasks set and the form that activity takes in the classroom. I began to see that this viewpoint derived from the assertion by Thom (1973) that all mathematics teaching rests upon a philosophy of mathematics, however inchoate. However, I questioned the extent to which an individual's philosophy can be upheld whilst still being part of the "institutional voice" (Williams et al., 2008) of a school, with its particular targets, resources, priorities and ethos. Complementing these viewpoints, Evans (2000a) brings in to question the affordances for thought and action created by different positions in classroom practice. The ideas the teacher intends to convey might not be the same as those the pupils infer from their engagement in a task. Together these writings indicated to me the importance of appreciating the pupils' viewpoint, in coming to understand the formation of mathematical action from classroom experience and expectations.

Boaler (2002b) described the nature of pupils' agency in the mathematics classroom using an analytic frame from Pickering (1995) which describes the interplay between individual agency and "the agency of the discipline" (ibid, p. 116) in the work of professional mathematicians. Pickering identified periods of creative and exploratory work, in which initial thoughts are explored or established ideas are extended, but also periods when mathematicians need to follow standard procedures and subject their methods to conventional methods of verification. Boaler demonstrated the empowerment of pupils who were able to work in both modes, engaging in "the dance of agency" (Boaler, 2002b, p. 46). In contrast, Nardi and Steward (2003) concluded that the mathematical potential of learners may remain defunct through their "quiet disaffection" when submitting to the primacy of the classroom and the curriculum. Goodchild (2001) supplements Mellin-Olsen's (1981) pupil rationales for participating in classroom practice with the *P-rationale*: a pupil undertakes tasks because it is seen as their role to do as the teacher requests; they accept and comply with the expectations of the practice. Goodchild found an association of the *P-rationale* with "blind activity", unmotivated toward a particular goal and unrelated to developing insight into mathematical structures.

3.1.2 Developing question narrative

The rationale I had laid out for this research entailed that I consider how values were imparted within the classroom to mathematics, learners and learning mathematics. The literature suggested bringing attention to the “social-psychological climate” (Haladyna et al., 1983) of the classroom, focusing on the activities which are valued and promoted. This valorisation would come from both the pupils and the teachers, in relation to their purposes; my research would have to explore the purposes perceived for mathematical activity and classroom participation. I would have to find a means of talking about the values imparted, and the means by which cultural values were imported. I would need to observe how teachers’ views and values of mathematical activity might be (explicitly and implicitly) conveyed, whilst also exploring the effect of pupils’ purposes on anticipating and engaging in classroom tasks (Hannula, 2002). Purposes form part of the classroom context in which pupils work, and I saw that my research interest in locating mathematical activity should explore the effect of pupils’ purposes.

My attempt to connect observations from research conducted with adults (Evans, 2000a; Lim, 2002) with the setting of the classroom brought into question what could be said about the correspondences between the classroom and the world outside. Specifically, the need to investigate the relations between mathematical activity inside and outside the classroom became clear. It was observed that individuals’ mathematical activity outside the classroom can be difficult to anticipate (FitzSimons et al., 1996; Baker, 2005) or to characterise adequately (Kent et al., 2007). My research would have to examine the extent to which the future contexts of pupils’ activity could be anticipated in classroom activity. Personal experience told me that teachers’ practice often does incorporate engaging tasks and contexts that try to make appeal to pupils’ interests and the world outside the classroom. The effectiveness of such approaches on the construction of mathematical action would have to be investigated. Conversely the increasing prevalence of iconoclastic images of mathematics (Mendick et al., 2007) might be observed in popular culture, but this provides no assurance that they infiltrate or are accepted as part of classroom activity. My research would have to incorporate these constraints and

inhibitions as features of classroom activity. Consequently my queries turned to the actual, rather than anticipated effects of such efforts, and extended beyond the value of a successfully completed task.

Suggestions made for enlivening the curriculum did not seem to address the issue of the classroom as a structured space in which activity is constrained by institutional obligations and co-ordination. A question which emerged for me was how classrooms strike a balance between disciplinary agency and the agency of the pupils (Boaler, 2002b), and the extent to which teachers are aware of this. Evans' (2000b) observations regarding mathematical proficiency drew my attention to the notion of mathematical authority in the classroom. I was interested in exploring the relationship between mathematical capability and authority and the means by which authority could be asserted.

3.1.3 Reflection: Dual identity narrative

My responsibilities as a teacher and as a researcher complemented each other in that both require attention to the mathematical development of pupils. My work in both practices is motivated by a belief in the power of flexible mathematical capability as a means of making sense in and of the world: my interest extends to how teaching practice and classroom habits permit such 'knowing'. As a critical practitioner I review lessons for the features that promote academic success, but I saw undertaking research as an opportunity to evaluate how these features empower pupils to make claims to knowledge. I aimed to investigate the bases from which claims are allowed, the evidence pupils are expected to provide and the authorities they have to satisfy for the claim to be sustained. In this, my pedagogical knowledge would undergo a rigorous process of development, as my focus turned to practices and subjectivity.

To satisfy my aim of informing teachers' practice, my research focus would remain at the level of the functioning classroom rather than the individual, and I would keep my research observations in the context of busy school life, recognising the influential pressures on teachers and pupils. I felt that my dual position conferred the advantage of insight into classroom events and sympathy with its protagonists. As a teacher coming to

research, I am predisposed to be sympathetic to teachers working under the constraints and pressures of school. Curricular and institutional expectations mean that one cannot avoid “practices unrelated to mathematics” (Boaler, 2002b), and as a teacher I know that my work incorporates such practices. The need to work with pupils’ interests can make it difficult to sustain teaching practice which is consistent with one’s epistemology. As a teacher, I have experienced the necessary compromises that can be the result of the need for short-term efficacy. I was aware that much of the research cited above focused on individuals, yet work as a teacher is structured around dealing with classes (Haladyna et al., 1983), in interactions which function through mutual recognition of relative social positions. I aimed to use this research work actively to explore these constraints and inform teaching practice with a sense of guiding principle – how to manage these intrusions upon working practice, convey epistemological consistency and a strong sense of purpose for learning mathematics.

However, this sympathy does not entail becoming uncritical of teaching practice. The mathematics teaching literature is populated with entertaining suggestions, games and diversions with which to engage learners’ interests. I have always found such suggestions problematic, for the tension they reinforce between intrinsic and extrinsic motivations. Discovering research which suggests the motivational effect might be counter-productive in the long term fuelled my intention to interrogate classroom action. Similarly, teaching practice readily incorporates illustrating mathematical points with reference to the world outside the classroom, and teaching materials are populated with ‘contextualised’ problems. However, I would have to explore whether these function as more than illustrations, with meaningful connection between classroom and everyday action. This raises the question of the extent to which teachers are able to make such meaningful connections, thus destabilises a common classroom practice.

As a researcher I wanted to step back to examine the constraints in practice for the formative effect they have on the development of mathematical action. The notion of “practices unrelated to mathematics” raised the question of how I would identify mathematics in the context of this research. Having taken the stance that mathematics is

constructed in specific social practices, observing the constraints on classroom practice would enable me to describe the constitutive effect they have on mathematical activity. This stance might also enable me to explore the nature of “traditional” classrooms (Boaler, 1999), in the context of institutional practices and priorities.

The connection made between images of mathematics and images of learning mathematics resonated with the links I saw between my two positions in the research process. As a teacher, my focus is primarily on aiding pupils in developing capability in mathematics, but as a researcher I was turning my attention to the processes and practice in which this learning would happen. As a teacher I would naturally focus on those features of activity which related to pupils’ progress towards curricular aims. As a researcher, I wanted to examine those aims, their relation to the needs and perceptions of the pupils and the means by which they were instantiated in the classroom. The meaningfulness of classroom action is thus different for the two practices, but related through the intersection of my research and professional interests.

However, these links could become conflicting, as my research would query the process and outcomes of teaching, asking on what basis aims are established and by what means they are maintained. My research interest questions the formulation of mathematical capability and extends to include the processes of valorisation of actions within the classroom. Introducing notions of purpose raised the question of the extent to which meta-mathematical awareness (Howson and Mellin-Olsen, 1986) is a desirable goal for mathematics teaching, and whether this should be seen as part of the curriculum. What counts as mathematical action is defined by mathematics lessons, which are superficially controlled by the teacher. As a researcher I positioned myself to examine this *status quo*, to ask whether pupils had the opportunity to query what counts as mathematical action (and when it is appropriate) and whether they should have these opportunities. In exploring the constitution of mathematical activity I would query the notion of progress, by asking what it is that pupils progress towards. These questions relate to the ethical case for my research, deriving from broader educational purposes. Through education we equip pupils to construct realities (Wiliam, 1998), but I could discover that classroom

practice might inhibit the knowing application of mathematical skills. As a researcher my responsibility would be to articulate and explore this situation, but as a teacher my first priority had to be the academic progress of pupils, in the terms of schools and syllabuses.

My research sits in the context of broader educational practice, as does the mathematical activity of the pupils. My description of the classroom would reflect this embeddedness, not only from a practical standpoint, but also in terms of the values inherited by classroom mathematical action from its institutional context. The influence of the institution and its expectations cannot be ignored, nor can I exclude the aims embedded within the examination curriculum. This research recognises these as the backdrop to pupils' activity, but also queries how these aims are manifest in practice. I would look for evidence of broader educational purposes emerging in classroom action, and ask the extent to which pupils are aware of these, and the effect they have on their participation and the resultant mathematical capabilities.

A working teacher is continually reminded of their curricular responsibilities, but these can become reductive and bureaucratic, and draw focus away from mathematical ways of being (William, 1998). In this sense, my research identity becomes broader than my teaching identity, as political and social concerns are embedded in my questioning. As a practising teacher, I was aware of the institutional and curricular constraints that shape the activities teachers prepare for their pupils: the communication of teachers' values might be compromised by these constraints.

My position as teacher would have to remain separate from the research practice, as I would have to guard against making assumptions that closed opportunities for developing my understanding. I would have to find ways of balancing this dual identity when switching between modes in the school, and in respecting the different standards of rigour which adhere in making judgements from these points of view. Ethical concerns will be discussed more fully in §4.4, but here I note that seeing the classroom as a problematic space created a tension for me: how could I, as a teacher, continue with classroom practice whilst querying it through this research? My aim was for this research to be

purposeful for me as a practising teacher, but I also had an obligation to make a rigorous contribution to the research literature. I also had to be sensitive to the potential difficulties of critically exploring my colleagues' work. Research methods which undermined their practice would be damaging to their classroom authority and consequently to pupils they taught. I had to plan my research carefully so as to respect their practice and to be seen to respect their practice. The resolution to this tension came through taking a critical approach to my own tentative findings. This enabled me to confine any interim judgements to the research whilst critical reflection should give substance to this thesis.

3.2 Research into needs – curricular responses

In §2.2 I began to explore curricular responses to pupils' needs. My exploration continues here, with a focus on the notion of "functional mathematics". In this section I begin to substantiate the research questions that I am positioned to answer as a teacher-researcher, and identify the features of classroom action that will provide form to my response. I also begin to anticipate the theoretical requirements of my research.

3.2.1 Functioning mathematically

As an historical precursor to functional mathematics, Noss (1997; 1998) traces the development of notions of numeracy, which he identifies as part of an educational culture of utility. He notes that in an increasingly technological age the uses of mathematics are growing, but also that as mathematics in working practices becomes less visible, the mathematical needs of adult life appear increasingly insignificant in quantity and quality. "The utilitarian perspective gives rise to a recursive cycle, in which what is taught at school becomes less and less relevant to working practices, as working practices show less and less evidence of making use of what is taught at school" (1998, p. 3). This paradox is accompanied by a second, in which, through efforts to address a lack of confidence and alienation on the part of many, school mathematics has risked becoming decoupled from its roots in science and technology. "In trying to connect mathematics with what is learnable, we have disconnected school mathematics from what is genuinely useful" (ibid,

p.3). He advocated addressing these paradoxes by looking afresh at what mathematical activity people do in their working lives. He showed that people try to make sense out of complex situations by building models and making structural features visible, which requires “tools which bring the model to life (like graphs, variables and parameters) and the means to express its structure (like numerical, algebraic or geometrical tools)” (p. 5). He also shows how professional expertise and intuitions are mobilised to make sense of situations, to fine tune and modify models. Noss concludes that the presence of technologies in the workplace has direct implications for teaching:

...more and more people will need to modify and rebuild systems with their own variables and parameters, not just plug in values to someone else's. It will mean that the distinction between domain specific knowledge of mathematical facts and generalisable skills will become increasingly obsolete. And, for our teaching of numeracies, it will involve constructing new educational cultures in which individuals have the means to make sense of the models, and the means to express them algebraically, geometrically, and computationally. New cultures of work are redefining the boundaries of what needs to be understood as a whole, rather than as isolated skills.

(Noss, 1997)

Hoyles *et al.* identified that “clarification of how mathematics is used [in the workplace] is urgently needed to plan curricula for the new millennium” (Hoyles et al., 1999, p. 61). The recent introduction of “functional mathematics” to the curriculum for England could be seen as a governmental response to this situation, with a move towards demonstrating the value of mathematical action in everyday life.

However I felt it was far from clear that this would adequately address the problems of equipping pupils with the transformable skills they need. Curricular change alone is also inhibited as a generator of changes in practice, as teaching aims are shaped by examination expectations. Pope (2011) notes that regulatory structures inhibit innovation and development in examination content, and consequently have little impact on classroom practice, citing the removal of coursework from the GCSE course as an illustrative example. The skills articulated in the curriculum as “using and applying mathematics” were previously assessed through the coursework. However, “[d]espite incorporating UAM into GCSE and NC [National Curriculum] tests there is little evidence

that this has actually impacted on classroom practice. Many teachers think that UAM is an add-on to the curriculum rather than something to be integrated” (Ofsted, 2008, in; Pope, 2011). Similarly, a change in the statutory curriculum is not necessarily accompanied by changes in teaching methods. Brown *et al.* (2000) discovered multiple interpretations of centrally-determined policy in teachers’ practice, resulting in the recontextualisation of recommendations to fit local conditions, despite attempts at tight prescription and control of practice. Adopting new curricular initiatives and developing classroom practice instigates an interplay between histories of practice and beliefs about mathematics teaching in which practitioners’ goals might become aligned with policy (Venkatakrisnan, 2004), resulting in an “enacted” rather than “intended” curriculum (Remillard, 2005).

Research into out-of-school mathematics revealed the capacity of knowledge to transform and be transformed by everyday activity. In exploring the mathematical actions undertaken in the workplace, FitzSimons (2005) investigated fundraising practices, post office work, modular shed construction, rental business, graphic design, playgroup, hairdressing and warehouse management, working in a care hostel and chemical spraying and handling, concluding that individuals’ mathematical knowledge mediates their activity, just as the constraints and affordances of the workplace mediate the relevant mathematical knowledge as it is co-opted into use. Through such action the actor and the world are changed and the mathematical knowledge is transformed through its connections with action. Similar conclusions were reached by Hoyles *et al.* (1999, see also; Noss et al., 2000; Kent et al., 2007) in investigating the decision-making processes of nurses, bank employees and customer enquiry workers in a pensions company. Coben (1997) notes that co-opting mathematical action into tasks requires judging the appropriateness of that action. This appropriateness depends upon the particularities of the job that needs to be completed, and the values of accuracy and efficiency associated with the work. I found that this mediation was often overlooked in policy discussions of numeracy or mathematics in the workplace (see, for example, CBI, 2008 and successive reports).

Bakker *et al.* (2006) develop the term “techno-mathematical literacies” (TmL) in favour of “numeracy” to describe the use of mathematical knowledge in the workplace. The Organization for Economic Co-operation and Development’s Programme of International Student Assessment defined “mathematical literacy” as:

An individual’s capacity to identify and understand the role that mathematics plays in the world, to make well-founded judgements and to use and engage with mathematics in ways that meet the needs of that individual’s life as a constructive, concerned and reflective citizen.

(OECD, 2003, p. 24)

Kent *et al.* (2007) found more relevance in this term than in “numeracy”, following critiques by Noss (1998) and Coben (2003), and included the prefix “techno” to emphasise the mediating effect on mathematical knowledge by technology in the workplace. The inclusion of the plural form was intended to stress the importance of an appreciation “of how the same symbols are constitutive of different meanings across different contexts” (Kent *et al.*, 2007, p. 66). For my research I found it constructive to use the notion of TmL as a characterisation of the future mathematical needs of pupils, for its emphasis on making contextualised meaning with technologies. However, my review encompasses literature which discusses “numeracy” in workplaces. I understood the more recent term as a development of “numeracy”, extending its connotations whilst preserving its concerns.

This articulation of mathematical activity raised the question for me of how assessment in functional mathematics might emphasise transformation and mediation, in the midst of a system of teacher accountability and tightly defined expectations. Pope explains that in the process of developing assessment materials, “Novel items which expect learners to make decisions about what mathematics, information and strategy to use are trialled... As responses are not as statistically robust as more traditional items, few survive... Where they do survive items are likely to have become shorter and more structured” (Pope, 2011, p. 64). This standardisation of test responses is a result of the market structure in educational qualifications in England: examining bodies preserve their market share by

maintaining quality of the product that they offer. This entails producing tests with minimal variation, in order to satisfy the responsibility of the regulator (currently Ofqual, until 2012 the Qualifications and Curriculum Development Agency) to ensure consistency across time and organisations. The processes in place that ensure consistency of standards stifle innovation and variety (Wolf, 2009), but enable teachers to work with well-defined goals. Pope (2011) explained that the extent to which assessment materials for the functional mathematics GCSE would differ from the previous model had yet to be seen, and that awarding organisations would be expected to maintain consistency of standards whilst working to significantly different criteria. Drake *et al.* (2012) analysed assessment materials for 16 year-olds in England with the aim of understanding the relation between 'functionality' and the extent to which mathematics was contextualised in human activity. They concluded that a superficial appearance of functionality can serve to disguise routine calculations and connections between contextualised assessment materials. Their judgement that a model of functional mathematics has yet to be established offered an indication of the persistence of the assumption on the part of policy-makers that mathematics education can deliver a "general intellectual resource which can be transferred from the classroom to the workplace" (Noss *et al.*, 2000). Noss *et al.* note that "attempting crude behavioural classifications based only on the mathematics of school curricula fails to evoke the authentic details of real work practices" (*ibid*, p. 18). It would appear that current forms of assessment inevitably result in crude classifications of mathematical behaviour being tested, which will undermine any progress made in curricular development.

Dowling (1991) posits that in making the assumption that workplace mathematics is the embodiment of school-learned mathematics, policy approaches in the lineage of the Cockcroft report (Cockcroft, 1982) make a fundamental error, identified later by Greeno as the "thinking with the basics" approach. According to this approach, "the job of classroom learning is to provide basic scientific or mathematical knowledge that students can then use in thinking mathematically or scientifically after they have learned enough

and if they are sufficiently talented and motivated” (Greeno, 1992, p. 39), and can result in the acquisition of knowledge without transformative capacity.

Since I began this research, the validity and reliability of national assessments at age 16 have also come in to question. Studies such as the Increasing Competence and Confidence in Algebra and Multiplicative Structures project (Hodgen et al., 2012) suggest that whilst increasing examination attainment would suggest improvements, there has been no significant change in pupils’ understanding in the last 30 years. Similarly Ofsted’s observation that result statistics continue to rise comes with the caveat that this is seen as “a consequence of the high priority accorded to them by teachers and leaders in secondary schools, but without corresponding evidence of pupils’ better understanding of mathematics to equip them for the next stages of their education and future lives” (Ofsted, 2012, pp. 6-7). ACME (2011) note that whilst more pupils than ever before are attaining pass grades at GCSE, many of those are not capable of working with percentages, fractions or interpreting data.

Complementing the utilitarian focus, the case for paying attention to the abstract structures underpinning mathematical concepts persists: “without this, young people are neither able to apply their knowledge in new situations nor have the confidence to function mathematically” (ACME, 2011). A narrow utilitarian focus will produce “students who may learn one or two recipes but who will not be able to transfer this knowledge to progress in mathematics or apply it unfamiliar ways” (ibid, p. 3). The need for undergraduate students of STEM subjects (Roberts, 2002; Smith, 2004) demands that pupils acquire strong technical fluency with abstract structures and the formal language of mathematics. Attention to technical and algebraic detail (by teachers and pupils) and connection between mathematical topics are required to equip pupils for the next stages of education (Ofsted, 2012). The need for teachers to attend to the elements of mathematics practice at a remove from pupils’ everyday experience lies at the root of many of the tensions detailed above. However, within this research, I saw the question of how pupils become equipped to make claims to knowledge in abstract mathematics (and

how they will be able to sustain those claims in successive educational or technological contexts) as an aspect of my exploration of mathematical authority across practices.

Evans (2000b) calls for “points of articulation” between practices, and suggests that analysing discourses as systems of signs might provide a means of doing this. This complements the focus placed on ‘boundary objects’ by (Hoyles et al., 2007; Kent et al., 2007), who suggest that connecting multiple interpretations of objects can facilitate communication across practices. Similarly, the use of breakdown situations (Hoyles et al., 1999; Noss et al., 2000) could create opportunities for making implicit mathematical models explicit and for adapting standardised models of action in situation-specific ways. However, the observations from research that pupils’ future workplace activities are broadly unpredictable suggest that such points of articulation could be difficult to identify and convey purposefully in the classroom. The literature and my classroom experience together suggested that epistemologies of transformation and mediation are not anticipated in school curricula.

Boaler (2002a; 2002b) demonstrates the benefit of seeing knowledge, practice and identity as co-constitutive. Moving away from a conception of knowledge as a property solely of the individual enables the development of generative descriptions of persons acting mathematically. Adopting situated perspectives, in which knowledge is seen as distributed between people, activities and systems of their environment (Lave, 1988; Boaler, 2000), allows the researcher to recognise knowledge being used differently in different situations, emerging as a co-construction between individuals and activities. Noss *et al.* (2000) bring an epistemological focus in considering how mathematical meanings are generated from different lived-in cultures. They show how mathematical meaning develops from interleaving mathematical objects with context-specific nuances, illustrating the benefit in taking a generative approach to descriptions of mathematical meaning.

In connecting practice with epistemology I found that the question of abstraction came to the fore. I began to question whether, if I regarded learning mathematics as a socially

constituted productive practice, I would be able to identify abstract knowledge. Hoyles *et al.* (1999) raise the question of the nature of abstract and concrete knowledge in the classroom, and how this relates to knowledge in the workplace. They posit that the assumptions underlying school-based learning are that the trajectory of learning moves from concrete to abstract, and that “the litmus test of expertise is the stripping away of contextual, concrete and experiential knowledge in favour of abstract, decontextualised knowledge” (p. 59). In contrast, workplace learning is seen in the development of concrete experiences and meanings around initially ‘abstract’ rules, algorithms and principles. For me this raised the question of whether school learning could be constructively explored, querying whether it should be considered so fundamentally different from workplace learning. The notion of expertise as a web of interconnected knowledge (Hoyles *et al.*, 1999) rests upon an interplay between abstract principles and response to specific conditions, akin to Boaler’s “dance of agency”. Noss and Hoyles (1996) suggest that mental models of situations and problems acquire meanings through use, and that meaningfulness results from “identification with particularities of the setting” (*ibid*, p. 60). They argue that general rules and concepts are shaped by practice, and gain further meaning by connection with mathematical structures.

3.2.2 Developing question narrative: investigating tensions in the construction of mathematics

My reading suggested that current curricular and assessment practices support the “thinking with the basics” approach. The reading detailed in this section also brought to the fore tensions between focusing on mathematics as a body of knowledge and mathematical capability as a property of the acting person. In considering the particularities of the classroom setting I should take in to account the working patterns that guide pupils’ actions, alongside the mathematical structures they encounter.

It became clear that in this research I would be following Boaler (1998; 1999; 2002b) in investigating the relationship between teaching and mathematical activity, and how mathematical knowledge is shaped in the classroom. Boaler concluded that mathematical knowledge is constituted by pedagogical practices, echoed by the suggestions made by

Lim (1999). I take this as the start point of my enquiries into the values that emerge through that constitution. The pedagogy I might observe would not only influence the amount of mathematics knowledge pupils would develop, but also the practices in which that knowledge was located, and the means by which mathematical authority could be asserted. I intended to explore classroom empowerment in anticipation of pupils' future needs.

My concern was what this approach to knowledge could reveal about learning in the mathematics classroom. Hoyles et al. (1999) query the notion of "abstract" as "decontextualised", which undermines the potential for mathematical meanings to emerge. I became concerned with how adopting an approach in which abstraction is understood as making multiple connections, rather than stripping away context, can be informative in pupils' development of mathematical activity.

3.2.3 Reflection: Dual identity narrative

At this stage in my research I was aware of the potential for tentative conclusions to influence my ongoing practice. I would have to monitor this tendency through reflexive consideration of the basis of those conclusions and clear articulation of their relevance. As a reflective practitioner, I frequently learn from evaluation of my own practice, observation of others' and discussion with pupils. However, drawing conclusions for my own practice should be separate from those I intended to explore from the basis of theory in this thesis. Observations I would wish to share had to be explored for the degree to which they were specific to my work situation or might be considered representative of wider concerns. I also had to frame my questions with regard to those I would be capable of answering: researching in my own workplace placed practical and ethical constraints upon my research methods. These are considered in §4.4.

The recognition that pupils' values and purposes played a part in shaping classroom activity brought into consideration the ways in which teachers manage pupils' expectations, choosing to work with or to counter them. The tension between intrinsic and extrinsic motivations cannot be avoided in a classroom in which the teacher works to

harness the interests and expectations of a large group of learners. I was interested in how the adaptation of tasks to pupils' interests and motivations influenced the resultant mathematical activity, in terms of the values attributed to it. I wanted to investigate the interplay of subjectivity and classroom activity, being interested in how teachers make connection between pupils' predispositions to act and the capabilities it is desired they develop. Reappraising the role of the pupil in this way was a fundamental part of stepping aside from my usual practice as a teacher.

My exploration of the literature raised questions as to the ways in which school and examination success are equivalent to pupil empowerment, an assumption that has to underlie effective teaching practice. In looking for connections made between the classroom and the world, I resolved to describe the structure and purpose of classroom practice and query whether this relates to other practices. I would explore how pupils are positioned to use their own knowledge in transformative action and the extent to which they are encouraged to reflect upon this; this might take place in relation to breakdown situations or in using boundary objects. In the work they undertake generalities are represented: I should explore the extent to which these are interpreted as intentional and the meanings that are both conveyed by and imparted to them. I would consider whether pupils are encouraged to reflect upon the practice, and how their methods acquire the characteristics of mathematics. I would ask whether pupils have the opportunity to query the mathematical models and methods they are expected to use, as part of their learning, and in relation to their goals.

3.3 Street and school mathematics – separation of practices

The literature detailed above illustrated the need to consider the potential relationships between school and everyday mathematics, indicating that a separation of practices resides in part in the different values applied to action. In order better to understand these values, I explored the actions in which these distinctions are sustained.

3.3.1 The literature: values in action

Investigations into the uses of mathematics in everyday practices, such as Martin et al. (2009) reveal the mutual influence of mathematical and cultural processes, in the service of personal goals and values. In everyday practice, mathematical rules and structures often become subordinate to the aims of the practice. As McDermott (2013) notes, in making connections between school and everyday practices, mathematical capability should be seen as “a resource for people’s activities”, rather than defining them. Noss *et al.* (2000) emphasise that different notions of efficiency predominate in school mathematics and workplace mathematics. For the mathematician, efficiency relates to determining general methods that can be applied to classes of similar problems, whereas in the workplace efficiency is determined by the specificities of a given problem and the circumstances in which it arises: “orientations such as generalisability and abstraction away from the workplace are not part of the mathematics with which practitioners work” (ibid, p. 32). Strategies and resources are used as and when necessary and often rendered unavailable for scrutiny. Routines are rarely interpreted as instantiations of general mathematical concepts or relationships, and are not investigated as such. The salient generalities of workplace settings might be limited to circumstances that work *in situ* but nowhere else (ibid, p. 33). The research by Evans (2000a) showed that outside school, different notions of competence held and mistakes had different consequences and values. Evans found that distinctions and comparisons between school mathematics and practical mathematics should be operationalised cautiously, with sensitivity to the mutual influence of context and attitudes.

Considering what people actually do in the workplace offered a more detailed perspective of the skills they need. One first distinction emerges in the work of Noss *et al.* (2000), who report that adults’ behaviour in the workplace indicates mathematics being used to make sense of situations in ways which “differ quite radically from those of mathematicians” (p. 17). They review research into professions spanning a range of explicit requirements in formal mathematical training: dairy workers, carpenters, carpet-layers, seamstresses, automotive industry workers and civil engineers. Noss *et al.* conclude that problem solving

at work is not governed by consistency or generality, but by pragmatic considerations focusing on specific problems. FitzSimons (2005) echoes these conclusions in her review of workplace practices. Occupational and professional concerns predominate over mathematical ones, with strategies emerging from professional expertise in relation to the features and regularities of the working environment.

The evidence that people neither use only syntactic, domain-independent rules of logic in their reasoning, nor develop only narrow, context-specific rules from experience had earlier led Cheng and Holyoak (1985) to propose the development of *pragmatic reasoning schemas*: abstract knowledge structures induced from life experiences consisting of sets of “generalized, context-sensitive rules which, unlike purely syntactic rules, are defined in terms of classes of goals (such as taking desirable actions or making predictions about possible future events) and relationships to the goals (such as cause and effect or precondition and allowable action)” (ibid, p. 395). In order to articulate the relationship between practical reasoning and mathematical representations, seen to be problematic in their research of street mathematics, Nunes *et al.* (1987) supplement this theory with Vergnaud’s (1985) theory of concepts. In this theory, a concept involves a set of *situations* which give the concept meaning, a set of *invariants* that are constituted by the relationships essential to the concept and a set of *symbols* used in the representation of the concept. Thus the way symbols are used in relation to problematic situations influence what can be achieved within them.

Nunes *et al.*’s research indicated that it was necessary to incorporate considerations of representation in order to understand the differences in children’s success in numerical problems in and out of school. They illustrate that in school mathematics the meaning of arithmetical situations is rarely represented, and plays no part in the algorithmic work with symbolic systems of number or algebra. (This context-independence is what gives arithmetic its power in applicability.) In contrast, oral arithmetic depends less on syntactic rules, preserving the meanings inherent in problem situations. The retention of meaning seemed to help children in using pragmatic reasoning schemas, whereas the use of formal

systems of representation introduced complexities and errors relating to the rules of the systems.

Nunes *et al.* summarise that in street mathematics people are involved in particular activities, engaging available technologies and situational particulars. Helpful representations contain elements of the situation, but are also amenable to transformation for more general situations. The rules of the representation are under the user's determination and control. By solving everyday problems in these situations, people develop pragmatic reasoning schemas for numeracy. The retention of situation-specific meaning in the representations in these schemas is not problematic, however, as it allows for control and for answers to be checked for reasonableness.

A second distinction between practices lies in the relative positions of authority that sustain in different settings (Abreu *et al.*, 1997). The endorsement of school methods as "superior" and the prohibition of out-of-school practices together inhibit connections being made between the two forms of knowledge, whilst also impinging upon pupils' capacity to assert mathematical authority outside the classroom. Abreu *et al.* (1997) found that school-taught algorithms could be applied uncritically and hence produce incorrect answers to problems in out-of-school contexts. Pupils had had little opportunity to examine the structures that could have united their flexible and accurate out-of-school methods with their algorithmic school-endorsed methods. Dowling (1991) notes that home and workplace mathematical practices are pathologised by being negatively defined with respect to school mathematics, and claims that efforts to democratise mathematics by redirecting it away from abstract mathematical action actually devalue social practices. This happens as a result of defining social practices on the basis of their mathematical content, which is only recognised when articulated in the terms of the universal signifiers used in the official discourse. Dowling notes the contribution made from anthropological research in understanding knowledge in action, and the contribution made by Marx to comprehension of the reproduction of working practices. If productive practices are recognised and defined only in school mathematical terms, this imports the values of school mathematics and these practices are required to demonstrate their value on terms

alien to the practices themselves. Highlighting the reproductive nature of engagement in practices brought in to focus the productive effect of school practice on the pupils themselves. Abreu *et al.*'s research suggested that alongside knowledge of mathematics, the pupils adopted valorisations of action in classroom-mathematical terms.

In an effort to draw practices together, Baker (2005) proposes working with learners' "knowledge, experience, histories, identities and images of themselves" (p. 16) in order to capitalise on their interests and social and cultural relations. By doing so, he claims, education can empower learners within the classroom and foster engagement. Similarly, Engle and Conant (2002) suggest that giving pupils authority over problems regarding subject matter can foster productive disciplinary engagement. Valuing pupils' authority should contribute to developing capability in switching between formal and informal practices. However, such moves can invoke the *planning paradox* (Ainley et al., 2006), referred to above, which presents an ongoing tension for teachers working to a prescribed curriculum. The appeal to meaningfulness of classroom tasks can be subverted with the introduction of the pupils' out-of-school priorities, and mathematical development can become sidelined as pupils' knowledge disrupts task structure. Research by Jurow (2005) likewise showed that teaching mathematics through a project-based curriculum creates a challenge for the teacher to capture the pupils' interest in the theme of a project and engagement in the intended mathematical purposes: pupils who engaged with the theme treated mathematical objects pragmatically, in service of the goals of the project, as would be done in a real scenario, rather than as structures to be explored.

The introduction of out-of-school contexts and 'realistic' problems to classroom tasks also raises questions about the meaningfulness of mathematical action. These strategies convey messages which Dowling (1996) refers to as *the myth of participation* and *the myth of reference*. The myth of participation suggests that formal mathematics deals with the public domain and mathematical solutions to problems replicate the structure of practical ones. The myth of reference suggests that mathematical behaviour would retain meaning in practical contexts and would be in a person's interest; the cultural arbitrariness of purportedly relevant mathematical structures is concealed. Recognising the presence of

these “myths” in classroom practice reinforced the need for me to speak directly to teachers through this research.

Despite these myths, Boaler’s research into forms of mathematical practice concluded that pupils learning mathematics through open, group-based projects developed “more flexible forms of knowledge that were useful in a range of different situations, including conceptual examination questions and authentic assessments” (Boaler, 2002b, p. 43) than those who had worked through textbook exercises. She notes that the two groups had developed different skills, with the former being more amenable to transformation and application. The pupils who had learned through projects also had a higher pass rate on the GCSE examination than the traditionally-taught pupils: 88%, in comparison with 71%.

This would appear to suggest that the epistemological complications of open, project-based work could be overlooked. However, at the time of Boaler’s research, the ‘pass’ grades on the GCSE ranged from A* to G, with grades C and above being desired by employers and further education. Both schools achieved only 11% A* to C grades (Boaler, 1996, pp.170-1), which might indicate that the project approach did not confer any advantage in meeting the academic demands of the full curriculum³. At a time when competition for high examination grades is intense, the implications of this research might now be revisited.

In exploring the potential for making connections between school and work mathematical practices, Noss *et al.* (2000) identify the use of *visible mathematics*, defined as derived from school mathematics; “conventional mathematical symbolism and representations... but also the use of concepts, strategies and methods of the mathematics classroom” (p. 23). Visible mathematics was largely concentrated in two types of activity: finding solutions to specific problems in procedural ways using algorithms; carrying out routine data gathering and representation. These forms of activity represented problems and

³ This percentage is also significantly below the national rate at the time (46%); Boaler suggests that “both sets of students were probably disadvantaged in some ways in the GCSE examination.”

scenarios which were understood well and could be encapsulated within pragmatic rules-of-thumb and efficient use of available resources. However, when conflicts or novel problems arose, judgements were justified in terms of professional expertise or 'intuition', rather than mathematical knowledge. This research showed that "intertwined with those judgements were mathematical elements – *but not necessarily those of visible mathematics*" (p. 28, original emphasis). When required to justify their judgements, practitioners revealed their dependence upon mental models which were not necessarily 'visible' beforehand. These observations illustrated processes by which implicit mathematical ideas could be exposed and made amenable to transformation.

In response to Noss (1998), Kanes (2002b) identifies three complementary characterisations of numeracy. *Visible numeracy* is characterised by using commonly accepted language and symbols to formulate mathematical relationships and communicate these. This numeracy is explicit and undisguised, reproducing canonical numerical knowledge and methods, as seen in curriculum documents and textbooks. *Useable numeracy* is concerned with knowledge as it is engaged in problem-solving, defined by its use in everyday tasks, and involves the transformation of numerical concepts in use (Nunes et al., 1987; Lave, 1988; Scribner in Tobach et al., 1997). Useable numeracy is adaptable, being deeply responsive to its context of use. In contrast to these approaches, *constructible numeracy* refers to that which can be purposefully produced by individual or social constructive processes, usually in a learning situation. Following the constructivist line of development in mathematics education, constructible numeracy relates the meaning of numeracy to meanings which sustain in the learning context. This approach involves "a valuing of educational context and the conditions of learning mathematics as intrinsic to the learning process" (Kanes, 2002b, p. 5). In this view, social, historical, cultural and economic contexts are integral to learning processes.

Examining instances of everyday numeracy, Kanes shows that these attributions of numeracy lend themselves to complimentary descriptions, but also that they help to highlight the tensions within approaches to numeracy. The judgement of the Cockcroft report (Cockcroft, 1982) that "it is possible to summarise a very large part of the needs of

employment as a feeling for measurement” can be reformulated as an emphasis on useable mathematics which reduces the range of visible mathematics. As Noss (1998) argued, this would then result in less mathematics being useable. Studies into working and everyday practices also indicate that people make use of mathematics without wishing or needing to make it visible. Kanes also reformulates the paradoxes identified by Noss in these terms. Firstly, concentration on visible numeracy oversimplifies issues relating to useable numeracy, and thus numeracy becomes less useable than would otherwise be the case. The second paradox is rephrased in terms of constructible numeracy: that which is easily constructible lies in tension with that which is genuinely useable. Kanes proposes this articulation not merely to rephrase long-standing problems, but to emphasise that they represent intrinsic tensional features of numeracy, which should be used as keys to understanding the cultural bases of numerical activity.

The paradoxes highlighted by Noss derive from the progressive mathematisation of society and concomitant demathematisation of individual activity: the increasing mathematisation of intellectual and social life (Hoyles et al., 1999) has the potential to result in the demathematisation of individuals’ skills and valorisations of mathematics. Jablonka and Gellert (2007, p. 8) support this suggestion: as processes and abstractions are realised in our technologies, “the existence of materialised mathematics in the form of black boxes reduces the importance of mathematical skills and knowledge for the individual’s professional and social life”. They raise the concern that this development also results in a diminishing shift of values, suggested by Fischer (1993): the “value of meaning” of mathematical knowledge becomes replaced by the “value of utilization” as meanings are embedded within technology and rendered invisible. However the value of utilisation diminishes as the mathematics becomes invisible within the technologies.

Jablonka and Gellert indicate that the process of mathematisation is not only a societal process, but can be traced within the actions of individuals. Noss *et al.* (2000) draw attention to the potential of focusing on the invariant mathematical structures indicated by situational specifics, citing Nunes *et al.* (1992, p. 558):

In this view ‘mathematising’ reality is representing reality in such a way that (a) more knowledge about the represented reality can be generated through inferences using mental representations, and (b) there is no need to manipulate reality further in order to verify this new knowledge. Invariant logical structures are embedded in mathematical knowledge, regardless of whether mathematical knowledge is developed in or out of school. It is the ability to make inferences using these structures – not the content of knowledge – that distinguishes mathematical from other kinds of knowledge.

The process of forming a mathematical description of a scenario can be described as *horizontal mathematisation*, whereas the development of a systematically principled and hierarchically organised structure can be described as *vertical mathematisation* (Treffers, 1987), echoing the notion of a *vertical discourse* (Bernstein, 1996, 1999). The activity of the classroom could be seen to encourage both forms of mathematisation, but Adler (2001) highlights tensions in both processes. If horizontal mathematisation does not involve changing the language of description from the public domain, then mathematical knowledge remains constrained to that domain. However, if classroom talk is mainly esoteric, the individual construction of meaning is inhibited. In the classroom, both forms of mathematisation represent mathematics as the exploration of mathematical structures, rather than establishing understanding of the world (Jablonka and Gellert, 2007, p. 4).

I saw in the descriptions of pragmatic reasoning schemas, mathematisation and visible mathematics an indication of the transformative effect of engaging mathematically with the world in everyday action. This effect is seen in the users’ transformation of mathematical knowledge and the transformative effect of the mathematical understanding on the user, making sense in action of mathematical ideas and making ideas mathematical. Bakker *et al.*’s (2006) identification of techno-mathematical literacies illustrates the need for education to equip learners with accessible mathematical skills and an outlook which does not enclose those skills within school practice. These needs present a challenge to the predominant “thinking with the basics” (Greeno, 1992) approach to knowledge in action, suggesting that “knowing cannot be separated from the activity in which it takes place” (Bakker *et al.*, 2006, p. 344). FitzSimons (2005) traces a middle path between these positions, noting that, aside from specialised computer training, most mathematical learning in the workplace seems to be “on the job” (p. 36), constituting the

adaptation of mathematical ideas to the idiosyncrasies of the workplace. She posits that this adaptation is dependent upon formal education, with a strong “general mathematics”. In workplace practices, the values of efficiency and personal meaning (safety, satisfying obligations and performing one’s job well) reposition those of mathematical correctness or precision. Adopting an activity theory perspective, FitzSimons demonstrates the importance of contingent and contextual factors for mathematical action in the workplace. Mathematical competence was supported by the use of tools or artefacts which reduced cognitive loads, and the division of labour which supported individuals through social structures (Wake and Williams, 2001).

3.3.2 Developing question narrative

As a teacher focusing on the functioning of the classroom, I recognised the above observations about workplace mathematics, in which situational contingencies have as much importance as logical mathematical structures, as having direct bearing upon the classroom *as the pupils’ place of work*. Whilst engaging with mathematical tasks, they are simultaneously in the process of carrying out their day-to-day “work”, in which they play a role in the classroom, and produce evidence of their efforts for the teacher. This connection suggested the potential for understanding classroom participation analogously to participation in work. Mathematical actions will have intrinsic meanings, in terms of solving the problems with which they are faced, but also meanings in the practice of the classroom. Consequently, my exploration would have to grasp the relation between the activity of the classroom and the conceptualisations of mathematics this generated, a need identified for research by Vergnaud (2000). Recognising the classroom as the pupils’ place of work, with its own priorities and aims, should enable me to describe learning mathematics as a social practice or system of activity, which is socially, culturally and historically situated (FitzSimons, 2005). This would entail describing the system of obligations in which pupils work, and the expectations held of teachers.

I understood Noss’ calls for a broader relation between school and work (Noss, 1997, 1998) to refer to more than the curriculum through which pupils work, and encompassing the modes of mathematical action which are encouraged in the classroom. I was

conscious that McDermott's (2013) call for mathematics to be "a resource for people's activities", rather than defining them presented an obvious point of departure from mathematics in the classroom, which defines pupils' activities. In informing my research, I saw the need to consider how pupils were positioned to exercise their subjectivity in the classroom; his need represents a challenge to common classroom practice. I also recognised that the mathematics classroom has not been immune to the intrusion of technology and thus it became important to consider the extent to which technological devices influenced the construction of mathematical activity.

In order to consider the roles played by pupils and teachers in the formation of mathematical capability, my description of the mathematics classroom would have to detail the patterns of teachers' and pupils' actions, those they choose to focus on and the meanings and values ascribed. In this way, I should be able to describe how 'proper' mathematical action is identified and encouraged, and the grounds on which action is permitted to be known as 'mathematical'. Answering these questions would relate to the values and purposes which underlie the teachers' and pupils' activity. In doing so, I should come to understand what constitutes meaningfulness for pupils, and how they are predisposed for the internalisation of classroom processes as their own mathematical capability. In doing so, I should explore the relations of authority and subjectivity, touching upon classroom interpersonal relations in relation to mathematical authority. Assembling a picture out of these elements would enable me to describe the relationship between classroom practices and pupils' knowledge, from their viewpoint. In light of the observation that rationales for pupils' engagement do not necessarily relate to their goals (Mellin-Olsen, 1981; Goodchild, 2005), I should seek to explore which aspects of motivation and practice do relate to each other.

When looking for "personal identification with the particularities of the setting" (Noss and Hoyles, 1996) I should ask what pupils choose to focus on and why. In doing so I should discover whether personal transformation is a salient aspect of pupils' engagement and motivation. I need to detail the social consequences of mathematical activity, and relate these to pupils' aims, asking whether they achieve what they expect to, or whether there

are unforeseen consequences. This will stem from and influence opinions and conceptions of mathematical activity.

The literature offered a set of analytical descriptors with which I might be able to constructively articulate relations between pupils' actions and the circumstances in which they act:

- Pragmatic reasoning schemas (Cheng and Holyoak, 1985) and concepts (Vergnaud, 1985)
- Visible mathematics (Noss et al., 2000)
- Mathematisation and demathematisation (Jablonka and Gellert, 2007)

I also saw the literature as offering a language for describing the qualities of pupils' mathematical action in the classroom:

- Visible, useable and constructible mathematics (Kanes, 2002b)
- Horizontal and vertical mathematisation (Treffers, 1987).

In order to accommodate these concerns, I turned to the literature for suitable theoretical support.

3.4 Theoretical engagement narrative: adopting a theory

The theory I would choose had to offer a framework for describing what happens in the classroom that leads to the construction of mathematical activity as a value-laden and purposeful feature of human action. I would need a theory that could describe relations between contexts and goals and the outcomes of action. Considerations of the context would entail actions encouraged and permitted, and those frequently engaged in. The wider context of school life and institutional goals would have to be considered, as influencing participation in mathematics lessons. The theory would have to admit a description of the effect on the "social-psychological climate" (Haladyna et al., 1983) of personal and cultural values, and the actions of pupils. I wanted to describe the effect on pupil participation of their expectations of themselves and of the activity's purpose.

Conversely, I aimed to be in a position to consider how engagement in the practice relates to pupils' and teachers' aims. The theory should also offer a means of describing the relationship between the pupil and the school system: the relationships between subjectivity and compulsion would have to be articulated. This should lead to an understanding of the role of the pupil, and what counts as authority in the mathematics classroom. It also brings in to play the role of the teacher, with its responsibilities and affordances.

My rationale, aims and reading indicated I would have to form a dynamic description of the classroom, from which a formulation of transformative mathematical action could emerge. The theory that would enable this description would have to offer means of dealing with school mathematics as a discrete endeavour that is supposed to presage the use of mathematics outside the classroom. In order to communicate with teachers, I needed a theory that could offer a recognisable description of the mathematics classroom, whilst admitting the influence of pupils' goals. I hoped that this would contribute to an understanding of how "the dance of agency" leads to the construction of mathematical activity.

The theory I would use should frame but not presuppose the outcome of classroom action. The research suggested that the valorisation of events in the classroom creates and defines mathematical capability. This valorisation takes place at the intersection of teachers' and pupils' aims, in relation to the instantiation of the curriculum in classroom action. I would need a theory that allowed for the description of mathematical capability as a result of pupils and teachers acting together with multiple purposes.

Research I had encountered in my explorations had indicated that cultural-historical activity theory (CHAT) (Engeström, 1987) could meet my priorities. My reading had shown the theory had provided a constructive framework for describing the place of mathematics in work practices (Williams et al., 2001; FitzSimons, 2005; Martin et al., 2009) and for examining classroom activity (Roth et al., 2002; Wells, 2002a; Hardman, 2005a; Ho, 2006; Hardman, 2007; Roth and Lee, 2007).

My choice to use activity theory lay in four primary characteristics:

- The theory was oriented towards purpose in its description of activities
- The theory offered a formal description of activity within communities and regarding the use of specific tools
- The theory describes individuals and their purposes as mutually constitutive, and developing over time
- The theory facilitates a natural process of reflection, enabling my own role in the research to be articulated

The theory has clear definitions of the terms *activity* and *action* (amongst others, see chapter 5); from this point in the thesis those definitions are observed, whereas previously they have been used interchangeable. The term *practice* has been used in the everyday sense of 'habitual action' and continues to be used in this way.

3.5 Developing question narrative

As a result of this review of the literature my research question developed from:

What are the processes that shape pupils' understanding of the purposes of mathematics education and the means of locating mathematical action in the world?

To:

What are the aims and values that shape and are served by participation in the mathematics classroom, and how does the resulting activity provide pupils with the means of locating mathematical action in the world?

This new question incorporates the formal understanding of activity, to be explained in the following chapter. The adoption of CHAT should offer the articulation of the relationships between practice and knowledge that I sought, and would continue to have a formation effect upon my question as I better understood the theory. This continued development is traced throughout chapters 4 and 5.

4 Adopting cultural-historical activity theory as a research framework

Cultural-historical activity theory (CHAT⁴), as developed by Engeström (1987), inhabits the Vygotskian tradition of research into participation in social practices, and developed from preceding versions of activity theory (Leont'ev, 1978; Lektorsky, 1999), inheriting a focus on outcomes resulting from engagement in practice directed toward a goal, the *object* of activity. The historical development and differences between the theories are discussed in chapter 5; in this chapter I share the basis for my choice in the first instance.

I found that activity theories offered a means of discussing stable practices in which people exercised decision-making capacity, and how resulting behaviour related to their purposes and values. These features resonated with my aim of describing the construction of mathematical activity in the classroom, and I could see the benefit of using the framework of CHAT as a basis of communication with other teachers. I saw that CHAT would offer a means of describing the “social-psychological climate” of the classroom, incorporating individuals’ values and the collective focus on an object, as that object was being constructed. The theory should afford a description of how that construction related to pupils’ and teachers’ sense of purpose and the connection made with anticipated uses of mathematics.

4.1 Adopting a Vygotskian approach: foregrounding practice and participation

My reading had reinforced an understanding of mathematical activity as a cultural phenomenon. Studies of everyday mathematical cognition (Nunes et al., 1987; Lave, 1988) had demonstrated that mathematical activity develops and is sustained in the specific cultural history of certain groups of people at certain times. For individuals this emerges as a result of interaction with their environment, occurring in different ways at different times and places in their lives. Differences manifest themselves in the values placed upon

⁴ The literature contains a great deal of slippage between the terms “activity theory” and “cultural-historical activity theory”. In this thesis, I use the former term for work deriving from Vygotsky’s and Leont’ev’s theories of activity, and the latter strictly when discussing work employing Engeström’s (1987) framework. A fuller discussion of language issues in this thesis can be found in §5.1.

systems of representation, means of communication and applicability of ideas, as well as varying requirements in terms of precision, efficiency and practicality (Walkerdine, 1988). My focus on the classroom as a formative space which establishes particular values regarding mathematical activity would entail examining the classroom for its own specific cultural history.

The Vygotskian (1962; 1978) approach had already contributed to my understanding of learning, with insights into the interrelation of the acting person and their surroundings. Lerman (2000) identifies four key elements of Vygotsky's theory of learning: the priority of intersubjective action; internalisation; mediation; and the zone of proximal development (ZPD). I was familiar with some of these elements from my work and training as a teacher, but wanted to deepen my understanding of the implications of these concepts and broaden my perspective from the level of individual to the level of the classroom. CHAT offered such a scalable perspective (Beswick et al., 2007), and in doing so would admit awareness of the cultural and historical background to education, and the influence of values which may be more or less proximal to the classroom action itself (Coupland and Crawford, 2002).

Abreu (2000) highlights two strands in Vygotskian empirical research into learning mathematics. In the first strand specific tools are seen as repositories of cultural knowledge, bringing a macro-context into the micro-context of the classroom; to understand a classroom culture requires an understanding of the cultural tools used by a group to represent their mathematical ideas. These tools can be material artefacts such as rulers, writing equipment and calculators, but can also be "symbolic sign systems [used] to represent mathematical ideas, such as counting systems or measuring systems" (ibid, p. 4). Within the activity, social practices constrain and afford problem-solving and the use of tools: "Features of the macro-context invoked in the micro-context are more complex than the tool itself and include an institutional definition of the tools and knowledge that is valued in each activity" (ibid, p. 8). This observation appealed to my aim of understanding the purposes and values invoked in engaging in mathematics in the classroom. It also offered a point of articulation in understanding how mathematics

practice inside the classroom could anticipate that outside: “data suggest that unfamiliar situations are approached on the basis of tools acquired in other social contexts, and that situated strategies are constructed through progressive specialisation within the practice” (ibid, p. 8). Using CHAT, Kuutti (1996) refers to tools as “artefacts”; persistent structures across time and space which can be co-opted into use. Artefacts are created by people to direct their own behaviour, and convey a particular culture and history, thus by co-opting an artefact into use a person’s actions and relations to an object are mediated by that artefact. I saw in the theory the capacity for regarding representations created in the classroom as tools which had the potential to become boundary objects (Kent et al., 2007) between practices.

The second strand identified by Abreu represents learning as structured by the immediate context of social interaction. This emphasis on social context in learning is conveyed by the notion of the ZPD (Vygotsky, 1978). The ZPD is “the distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem-solving under adult guidance or in collaboration with more capable peers” (ibid, p. 86); this concept describes learning as the development of capacities within social relations and points to learning processes as developing participation in a social practice. Working through the ZPD by engaging in tasks that have been set out for them is “a process by which children grow into the intellectual life of those around them” (ibid, p. 88). To adopt the concept is to see learning as “primarily a process of enculturation, and to emphasise the crucial role played by both children’s interactions with more knowledgeable others and their mastery of tools that are specific to the culture” (Cobb, 1995, p. 123).

Abreu suggests that the two strands of research provide complementary ways of describing the same features of activity, as tool use is contained within social context. This is reinforced by Rogoff and Lave: “Information regarding tools and practices is transmitted to children and other novices through interaction with more experienced members of society. In practical situations the context provides information and resources that facilitate the appropriate solution of the problem at hand” (Rogoff and Lave, 1984, p. 4).

However, research such as Saljö and Wyndhamn (1993), which found children's arithmetic strategies in solving postage problems varied according to the context in which the problems were presented, indicated that whilst behaviour varied according to context, it was not completely determined, as individual decision-making plays a part. I saw in the emphasis placed by CHAT on social structure and subjectivity a means of exploring the development of the self as emergent from pupils' participation, their aims and values.

I found activity theory and CHAT offered the capacity to describe the interplay between the enduring practice of the classroom and the instability created by individuals' behaviour (Beswick et al., 2007), and creativity (Lektorsky, 1999). CHAT gives a theoretical emphasis to the developmental effect of this interplay on both the acting subject and the activity itself. Under the theory, development results from conflicts arising between individuals' capacity for transformation and the constraining nature of tool use and social practices. This emphasis substantiated my questions about the relationship between pupils' learning, mathematical capability and participation in classroom practice. I became keen to explore whether the activity of the mathematics classroom could be seen to develop alongside pupils' development. CHAT would provide a means of constructively analysing such conflicts in activity, and thus serve my aim of informing teaching practice. With the aid of the concepts of CHAT, I could explore the conditions which promoted or inhibited development in classroom activity, in relation to pupils' development.

Under activity theory and CHAT, the activity provides the meaningful context for behaviour and analysis. Described by Leont'ev (1977) as "the molar unit of life", an activity comprises the object, the acting subject, their actions and the tools they use. Barab *et al.* (2002, p. 78) refer to the object as "the thing at which activity is directed and which is molded or transformed into outcomes with the help of physical or symbolic, internal or external tools." The activity and acting person are described in a mutually constitutive relationship: the person generates the activity by acting towards the object, but the activity shapes the person through the availability of artefacts and the specific conditions in which action takes place. The object may exist both as a material thing, and be recognised through its relations with the elements of the activity, and as a mental

representation guiding the subject's actions (Leont'ev, 1977). Consequently the context is both external to the acting person and internal as a mental representation.

In sustaining a focus on the development of mathematical action through participation in practice it was important to delineate the place of knowledge in my research. My Vygotskian stance predisposed me to consider knowledge as existing “relationally, between people and settings” (Lerman, 2000, p. 26); evidenced in and constituted by the things that people can do in any given situation. I saw this as fitting with my aim to describe how pupils were empowered to locate mathematical action in their lives. This emphasis entailed shifting away from metaphors of acquisition when discussing knowledge, and toward descriptions of participation. I saw this not only as a description of ability, to be compared on a scale of greater or less, but also as relating to the constraints and affordances experienced by the acting person. This stance was particularly important in the context of school practices of assessment and reporting, but also meant that I could not ignore those practices for their formative effect on conceptions of capability. The pupils' knowledge will be seen in the choices they make for action, and will reveal the aims they hold, the means by which they think they can attain those aims and the expectations they hold of others in the classroom.

This approach to knowledge would form my understanding of mathematical authority in the classroom. As a complement to describing the construction of their mathematical capability, a focus on practice would entail understanding how pupils' capability could be instantiated as mathematical authority, conforming to curricular, institutional and classroom expectations. This authority also corresponds to their specific cultural history with mathematics, informed by their experience in previous schools and outside school. I would use the theory to explore how pupils' mathematical authority is framed by their fluency in the culture of the classroom.

4.2 Cultural-historical activity theory – CHAT

In this section I offer a brief introduction to CHAT, and justification for its use as the theory guiding my research.

4.2.1 Key notions of CHAT

Under CHAT the interaction of the individual with the practice is contained within the unit of analysis: the activity. The activity is defined in relation to an object (in the sense of “objective”⁵) which is held by the subject and which motivates and guides their actions. According to activity theory, the features of activity derive from this object (Leont'ev, 1977, 1978). However CHAT admits a more complex relationship between these features, with the community, its rules and division of labour invoked as mediating influences; these are discussed in Chapter 5. The framework for the unit of analysis in CHAT is often presented diagrammatically as a structure of mutually influential features (Figure 1).

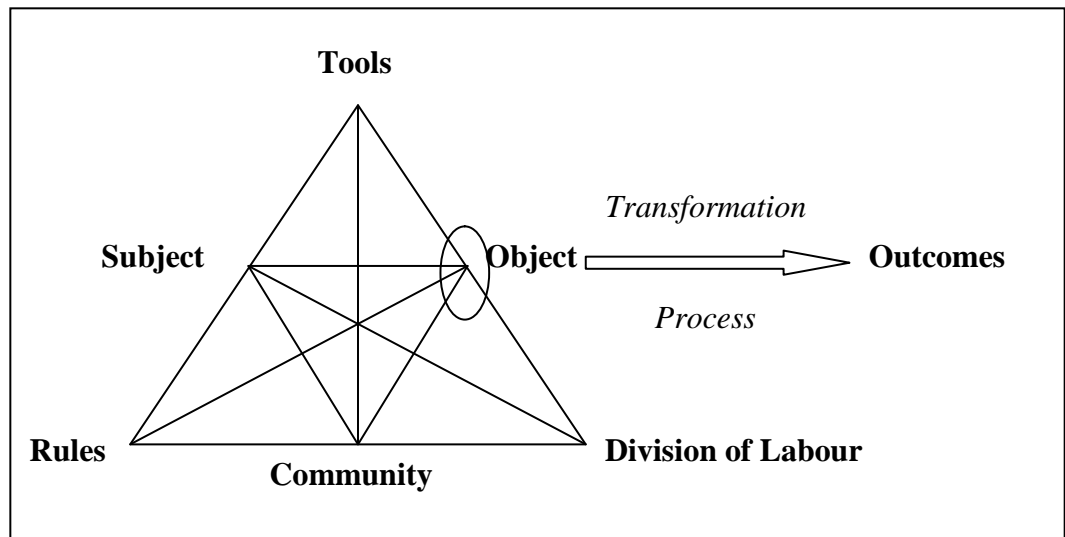


Figure 1. The Activity Triangle Model or Activity System (Engeström, 1987)

Barab *et al.* (2004) reviewed the literature in CHAT, concluding that the six components of the activity system were used as “buckets for arranging data collected from needs and

⁵ There are various issues regarding vocabulary when working with activity theory in English. These are explained in more detail in § 5.1.

task analysis, evaluations and research” (p. 207). The implication that this would facilitate the management of data suggested that my analysis could quickly progress from an assembled picture of the mathematics classroom. Beswick *et al.* (2007) describe the activity as “totally structured” by this representation. I could also see the potential power of this representation (referred to as the “expanded mediational triangle” (EMT) by Goodchild and Jaworski (2005; Jaworski and Goodchild, 2006)) in communicating my research to other teachers. A theoretical framework which could be understood intuitively and provided a structure for conveying insights should facilitate professional conversation and reflection. This framework is explored further in §5.1.

4.2.2 Theoretical engagement narrative – comparing theories

My review of the literature had revealed the uses of various theories in socio-cultural explorations of learning. Here I detail some pertinent comparisons for my research. A more systematic review of CHAT in comparison with other socio-cultural models can be found in (Nardi, 1996d).

Goodchild (2001) used activity theory alongside social constructivism and cognition in practice, in order to provide a multi-levelled analysis of pupils’ goals in classroom activity. He explored the effect of these goals at the personal and public social levels, and at the interaction of social and individual. Goodchild posits that the distinction between activity theory and cognition in practice is not strong, lying in the prominence given to individuals’ goals in guiding their actions and the extent to which internalisation functions as a description of learning (p. 41). The similarity of the theories is seen in the exchange of concepts between activity theory and cognition in practice in his analysis, and the overlapping contribution the theories make to understanding learning at both the public social level and the interaction of social and individual. However, under activity theory the structure of activity derives from the object (Leont'ev, 1978, 1981), which acts as a goal identified by the acting subject and around which a community coheres. This contrasts with situated action approaches, which assert goals are “retrospective and reflexive” explanations as to why something is done, with activity and its values being created simultaneously (Lave, 1988). My experience as a teacher indicated that I had to recognise

the intentionality of the classroom as a social system, alongside the interplay between personal, curricular and institutional aims. The framework of CHAT appeared to offer a means of describing that intentionality.

The potential for the constituents of an activity to change in response to the object is an integral part of CHAT. However, I soon found that within CHAT and activity theory the role of the object had various treatments. Some approaches treated objects as transformable in the course of activity (Kuutti, 1996), whereas others treated them as fixed points around which the activity developed (Nardi, 1996b; Jaworski and Goodchild, 2006). The relationship between the object of activity and the motive for pursuing it was also treated differently across applications. The formative relationship between the object and the structure of activity was also not treated consistently, and whether change should best be seen as continuous or episodic was unclear (c.f. Engeström, 1999d; Hyysalo, 2005).

I saw the potential for describing the dual transformation of acting subject and motivating object as offering a means of answering my questions regarding the construction of mathematical capability. In the classroom, the pupils work towards a named but as yet unknown goal, which gives meaning to their actions as it is created. I felt that an approach such as the four parameter model (Saxe, 1991) used to describe mathematical understanding would not offer me the means to describe this mutually constructive relationship.

My aim to explore the construction of mathematical activity through pupils' participation in broad and institutionally determined patterns of activity would have to be served by a theory that could accommodate differing conceptions of the object. Engeström (1987) notes that multiple subjects in an activity might have varying conceptions of the object which nonetheless provides a common focus and motivation for their actions. Coupland and Crawford (2002) reiterate the capacity for CHAT to accommodate multiple perspectives on activity, in relation to the histories of those involved, whilst all contribute to a communal aim. This assertion is supported by Beswick *et al.* (2007), who investigated the actions of members of a mathematics department in relation to the object of their

activity, the desired qualities of mathematics learning of their pupils. They demonstrate that the shared object becomes instantiated in different ways in relation to the individual teachers' working habits, yet retains its coherence as a focus for action. Roth (2010) identifies this variety as the "difference-in-itself" of the object, which acts as a source of change.

Responsiveness to individuals' perception of the context is a feature that CHAT shares with the cognition in practice approach (Lave, 1988), which constructs the *arena* as the stable institutional framework in which a person acts and the *setting* as the "personally ordered, edited version" of that framework. Both approaches place an emphasis on the ongoing flux of activity as it unfolds, emphasising the responsive qualities of the environment and the improvisatory nature of human activity. The concepts of setting and arena indicated the explanatory power in considering the classroom as a space saturated with personal meaningfulness. However, cognition in practice draws emphasis away from the stability of institutional phenomena, focusing more on momentary actions and improvisations in activity (Nardi, 1996d, pp. 71-2).

In my research I would look for the artefacts of the classroom that were co-opted as tools in the development of mathematical capability. In common with theories of distributed cognition, CHAT recognises the formative effect of the tools available to the acting person. However, distributed cognition places people and things as conceptually equivalent (Nardi, 1996d), with the implication that things are cognising and communicating entities, rather than items which respond programmatically to inputs. CHAT treats artefacts, co-opted into action as tools, as media for knowledge in action, thus offering the potential for self-initiated and unpredictable responses from the acting subject, through the novel appropriation of cultural tools (Abreu, 2000, p. 18). The assertion that tools need not be only material artefacts also opened the possibility of regarding pupils' established mathematical capability as a useable tool (following Kaner's (2002b) characterisation of useable and constructible numeracy). Emphasis on the use of material and intellectual tools placed pupils' mathematical capability in the relationship between them and the classroom, thus positioning the object of their learning as a property of their interactions.

I saw this as a means to investigate relationships between the micro- and macro-contexts of the classroom as the pupils saw them, and the effect of the values imparted to tools, capability and social interaction.

My position as a practising teacher obliged me to recognise the asymmetric positions of teachers and pupils in the classroom community. My theoretical approach would have to afford a description of the hegemony of the classroom and how this was experienced by pupils. The four parameter model (Saxe, 1991) offered a means of describing the community through *social interactions* and *conventions*, but I felt this was not powerful enough to adequately recount the pupils' response to the position of the teacher. I perceived that CHAT offered a means of describing pupils' position not only in the community, but also in relation to the tools of the classroom and the object of study.

My aim of placing the pupils at the centre of my analysis entailed employing a theoretical frame that would identify and privilege their perception of those properties and artefacts that were significant in forming their activity. The centrality of the subject in CHAT offered me the means to move beyond personal descriptions of activity, and relate pupils' intentional behaviour to the pursuit of mathematical capability, as recognised in the classroom. Placing the pupils' action in an inter-subjective space would afford a description which incorporated individuals' aims and purposes with the desired qualities of mathematical action. Research using CHAT also demonstrated the capacity of the theory to accommodate local concepts in describing specific activities (see, for example, Monaghan, 2004; Jaworski and Potari, 2009), which I saw held potential for conveying pupils' idiosyncratic conceptions of the mathematics classroom.

4.3 Reflection: Dual identity narrative

As a novice researcher in education I quickly discovered the need for a deeper engagement with theory than had been necessary for effective teaching practice. In doing so, it became clear that in order to pursue theoretical consistency I should see my research as constituting an activity in its own right. This would make it amenable to reflection and criticism on the terms of CHAT, and also bestow certain intellectual and

philosophical responsibilities. CHAT requires attending to the historicity of activities and objects, and I saw this requirement applying to my own application of CHAT. Consequently, in chapter 5 I recount my historical survey of the development of activity theory and CHAT, and explore how they have been used in research. I found that the philosophical roots of activity theory challenged some of my conceptions about learning mathematics, but provided a more rigorous basis from which I could explore the application of CHAT. The focus on practice also cast light on the presence of philosophies of mathematics education (Thom, 1973; Ernest, 1989, 1991), and the ethical imperative to articulate ideas regarding epistemology in mathematics. However, my attention remained on the means by which they might be conveyed, rather than querying a specific philosophy.

This engagement with theory continued through my data collection and analysis, which presented challenges to some of the assertions I found in the literature. As will be seen, I found using the concepts of CHAT as categorical “buckets” into which to place data problematic. My further reading also highlighted the contested nature of the “object” in CHAT, which related directly to my use of the theory. At the outset of any period of learning pupils cannot conceptualise or instantiate the things they are about to learn. Though engaging in tasks in lessons titled “Mathematics” the pupils come to know what mathematics is. I saw the activity of the mathematics classroom as motivated by the need to learn “Mathematics” but as an enculturation process in which the idea of “Mathematics” takes on meaning. It became clear that this thesis would have to convey parallel lines of construction: that of the object of pupils’ study and of the activity formed by my research. I hoped to explore the substantiation of the object both as the focus of my research, and as means of engaging with the idea of the object as the source of activity. My empirical engagement with these problems runs through my data analysis chapters (7 to 9).

4.4 Developing question narrative

Preliminary understanding of CHAT led to development of my research question as I questioned the relationship between the structure of activity and pupils' development. CHAT predicts that an activity changes as the subject and object change. This would suggest that classroom activity should develop alongside the pupils' developing mathematical proficiency and understanding of mathematics as a discipline. I became keen to see the extent to which this happened, and was recognised by the pupils. I was also keen to use my empirical research to make concrete sense of the concepts and methods of CHAT. Consequently, my research question took on an additional aspect, developing from

What are the aims and values that shape and are served by participation in the mathematics classroom, and how does the resulting activity provide pupils with the means of locating mathematical action in the world?

to the pairing of

What are the aims and values that shape and are served by social interaction and tool use in the mathematics classroom, and how does the resulting activity provide pupils with the means of locating meaningful mathematical action in the world?

How can CHAT equip me, as a teacher-researcher, to explore this question with reference to the subject developing in relation to an object of study?

5 Understanding Cultural-Historical Activity Theory

In my development as a researcher, it was important for me to understand the philosophical and historical origins of CHAT. I discovered that in the heritage of activity theory the relation between the acting person and the world has been conceived radically differently from the prevailing Western dualistic view. Re-evaluating my own conceptions of mind-world separation has been a central part of this research process, as I understood better the concept of dialectic mediation which lies at the centre of Vygotskian theory.

In §5.1 I give a thematic overview of the development of CHAT, to clarify how I drew upon the theory in this research. As a theory of activity, the potential for CHAT to be applied reflexively enabled me to articulate a key concept, the object, in my use of the theory (§5.2). I reflect upon my dual identity in light of CHAT (§5.3), then consider the uses of activity theory and CHAT in education, as a theory of learning and research tool (§5.4). I continue the *developing question narrative* in §5.5.

5.1 Thematic introduction to CHAT

In this section I trace a path through the development of ideas which have led to my understanding of CHAT. I consider its philosophical basis, its roots in twentieth-century psychology and the emergence of activity theory as a field, leading to an overview of the central concepts of CHAT as I understood them. My mode of presentation is not intended to imply a strict delineation between these fields of human thought. As will be seen, CHAT emphasises the connectedness of concepts and human action. The developments of philosophical, psychological and activity-theoretical ideas have been subject to continual interaction and mutual influence. I have chosen to present the separate strands in this way in order to emphasise the points at which the approaches coincide and have helped to formulate my approach to the research questions. More strictly chronological and critical overviews can be found elsewhere (Engeström, 1987; Roth and Lee, 2007; Bakhurst, 2009).

5.1.1 Philosophical heritage

In activity theory, human activity is seen as inherently object-oriented (Leont'ev, 1977). The concept of the *object* has developed from its use in the dialectics of Hegel (Hegel, 1802/1979) through its interpretation by Marx (1845/1969; 1909; 1973) and successive theorists. An understanding of the notion of the object will be seen to be crucial for this research and a fuller discussion follows in §5.5.1. Here I briefly survey the origin of the concept as it developed in a dialectical category with the acting and cognising *subject*.

In Hegel's philosophy, understanding of the human condition should be derived from observing the activity and interactions of humans within and with their culture (Blunden, 2008). Hegel proposed that human consciousness comes into being through the struggle to satisfy physical and psychological needs. In any society with systems of labour and exchange there will be delayed gratification of needs, mediated by a labour process. Thus the activity of the society is divided into consumption and production; in the gap between need and satisfaction consciousness emerges. Hegel took two categorical sources of knowledge from Kant: practical activity and experience (*Intuition*) and understanding of the world, including institutions, words and tools (*Concept*) (Blunden, 2007, 2008). In *The System of Ethical Life* Hegel (1802/1979) reconciled these two distinct categories by arguing that when acting in the world, the human continually subsumes *Intuition* under *Concept* and vice versa. When using a tool, a human employs a concept according to reason, yet reason simultaneously operates according to norms determined by the sensuously comprehended world. Hegel saw the continual subsumption of one category under the other as a generative contradiction at the heart of consciousness and in the world of human affairs, thus becoming a driver for personal and social history (Blunden, 2008).

The human subject is defined by Hegel as the identity of three aspects, each of which mediates the other two: the finite, mortal and self-conscious *Individual* psyche; the *Particular* continuing activity of individuals in definite forms of social practice and relations; and the *Universal* material products of culture (including language) and means of production, mobilised in activity and mediating the activity of individuals (Blunden,

2007, pp. 258-9). The subject is thus constructed as a relation between these aspects, each continually mediating the others. Consequently, in human consciousness there is ongoing dynamism, resulting from the contradiction between the world as it is imagined and the world as it is. The world exists as things which have their own force and resistance (in German, *Gegenstand*) and as reifications in the world of concepts held by humans. Through action in the world and use of things, there is continual dialectic interplay between cultural products and the psyche, characterised by conflict between the brute *Gegenstände* and the idealised images of them.

Understanding the relation between humans and the world as dialectic in nature entails seeing objects as the media through which the social and material reality of human lives is constituted. Humans relate to objects in the world through inherited cultural categories and customs; meanings of human conduct and social organisation are imbued in the use of things. However, the resistance of *Gegenstände* has an influence on these meanings, transforming them when things are used in pursuit of needs. As a result, when humans act to fulfil needs through transforming nature, qualitatively new and different ways of relating to the world can emerge (Hyysalo, 2005). Thus, in engagement with the world, objects and subjects both undergo transformation. This mutual constitution and simultaneous development of subject and object has been a central idea in the heritage of activity theory. To describe the simultaneous development of subject and object in activity, Hegel introduced the idea of sublation⁶, a verb which has the contradictory meanings of overcoming (or destroying) and preserving the distinction between subject and object (Roth, 2007). In activity, an object becomes internalised as part of the subject's understanding of the world and capacity to act in it, yet also remains distinct as a *Gegenstand* whose properties are better understood and which plays a greater part in shaping future action.

⁶ Translated from the German verb *aufheben*, meaning "to cancel", "to keep" or "to pick up".

Hegel's theory of dialectics countered Kant's assertion that humans possess inherent categories through which they comprehend the world. In contrast, he proposed that humans have constantly evolving relationships with the world, in which categories of comprehension develop throughout a person's life history and thus in the course of the history of mankind. This dialectic understanding was proposed as a means of transcending and resolving some of the difficulties posed by Kant's categories and the opposing metaphysical stances of naïve empiricism (according to which we perceive nature directly) and idealism (which holds that we do not truly possess knowledge of material reality). As such the theory challenges the persistent Cartesian separation of mind and world underlying both approaches.

In Hegelian descriptions, the term *object* refers merely to things, which have both a material and ideal existence. Under Leont'ev (1978) this notion develops to become the *object of activity*, incorporating needs, motivations and outcomes. Thus objects in the world are those things towards which human action is directed: a given object defines a certain type of activity, so we can speak of that activity as being object-oriented. Object-orientation creates discontinuities in behaviour, determining acts as directed towards a target from which those acts derive their purpose. Object-oriented activity is a structured, discrete and finite part of life, distinguished from the continuous, free-flowing operation of human consciousness and behaviour (Leont'ev, 1977); this practical activity is the means by which consciousness develops and the subject learns. The discontinuous nature of object-oriented activity is contained within the term *Tätigkeit* (in Russian, *deyatelnost*) which does not translate easily into English⁷. The term stresses the nature of *an* activity,

⁷ The history of CHAT sees its origins in German philosophy and Russian psychology. Consequently, it has inherited concepts and distinctions best expressed in those languages, and which do not translate easily into English. The nuances between objects as things in the world and objects of activity can be conveyed with the Russian terms *objekt* and *predmet*, and the notion of objects as having resistant force is contained within the German term *Gegenstand*. Each of these translates as "object" in English, and the relations between terms in the two languages has not been consistently maintained (Kaptelinin, 2005; Blunden, 2008). Leont'ev maintained a consistent distinction between *objekt* and *predmet* only in later activity theory work (Leont'ev, 1978). To emphasise the distinction in English however, is problematic. The term *predmet*, referring to the

which disrupts the continuity of consciousness and action and is driven by a subject co-opting specific tools and language. Tätigkeit is an emergent product of continuous psychic processes in response to specific conditions (Schurig, 1988), resulting in the production of material outcomes.

The theme of human development in relation to material conditions was developed by Marx as his theory of dialectical materialism (McLellan, 1975). However, Marx emphasised the role of consumption in the formation of the self, subordinated to three aspects of human activity, viz. production, distribution and exchange:

Production creates the object which correspond to the given needs; distribution divides them up according to social laws; exchange further parcels out the already divided shares in accord with individual need; and finally, in consumption, the product steps outside of this social movement and becomes a direct object and servant of individual need, and satisfies it in being consumed.

(Marx, 1973, p. 89, in; Engeström, 1987)

In Marx's analysis, the subject produces not only the material means by which he achieves his object and satisfies his needs, but also consumes his abilities and those means in production. Distribution (of goods and people) is both the consequence and prerequisite of production and, through interaction and communication, exchange is contained within production. Thus the subject produces both himself and the object in practical activity, and the four categories form "the members of a totality, distinctions within a unity" (Marx, 1973, p. 99) in which production dominates. In developing a description of purposeful human behaviour in this manner, self-society dualism begins to dissolve. Individual agency and socially distributed or cultural aspects of action and thought

focus of an activity, could translate to *subject*, as in 'the subject of dispute', but this is likely to be confusing, for obvious reasons. The term subject is also problematic when applying CHAT to research in education. We refer to school subjects, which are arguably the objects of the pupils' activity. In this thesis, to avoid confusion, I refer to school subjects as 'disciplines'. For similar reasons, the items explored by mathematical action and on which mathematical action takes place will be referred to as 'structures'.

become incorporated in a dialectical relationship, centred upon production. Hegel argued that subjects in a society develop the capacity to interact with each other through mechanisms such as the division of labour and the creation of surplus product (Hegel, 1802/1979). In Marx's analysis these features of activity result in the emergence of the *exchange value* and *use value* of the results of production. The use value of a product resides in its capacity to satisfy a "definite social want, and thus hold its place as part and parcel of the collective labour of all", whereas the exchange value derives from the capacity to "satisfy the manifold wants of the individual producer himself" through social relations of exchange for other products (Marx, 1909, p. 44).

In coming to understand the mutual development of subject and object in the context of learning mathematics I draw on the theory of ascent from the abstract to the concrete (Hegel, in Ilyenkov, 1960). The abstract concept is that which is defined in terms, failing to express the "sensually contemplated reality in its entirety", whereas the concreteness of a concept lies in the unification of its diverse definitions and their meaningful cohesion. An abstract concept becomes concrete when embedded and understood within its context of a scientific theory or lived experience:

The genuine sense, genuine content of each abstract definition taken separately is revealed through its links with other definitions of the same kind, through a concrete unity of abstract definitions. The concrete essence of a problem is therefore always expressed through unfolding all the necessary definitions of the object in their mutual connections rather than through an abstract 'definition'.

(Ilyenkov, 1960)

Ilyenkov explains that a concrete definition exists only when embedded in a sensually given image or a well-developed system of theoretical definitions; it does not exist as a separate word, term or symbol. Concrete concepts are imbued with their interrelations and transformative capabilities. Thus a truly developed concept "includes in it a conception of the dialectics of the transformation of the individual and the particular into the universal" (Ilyenkov, 1960, cited in Engeström, 1987).

It is with this frame of philosophical concepts that I came to understand activity as the “non-additive, molar unit of life” (Leont'ev, 1981, p. 46), in which the transformative relationships between subjects and objects could be understood.

5.1.2 Psychological developments

As has been pointed out by Blunden (2008), the work of Hegel can be seen as early work in cultural psychology, hence some of the ideas covered above immediately inform a psychological understanding of the human acting in the world. I first review these points before considering the contribution from twentieth-century psychology:

- Object-orientation creates discontinuities in behaviour, isolating acts as directed towards a certain target. *Tätigkeit* disrupts psychological continuity, where the psyche is seen as a continuous processing of information and action, being live, plastic and flexible.
- Through exploring new understandings, possibilities and actions in the world, the active subject goes beyond himself and is transformed. Through engaging with the world, individuals develop a richer appreciation of the reality which they inhabit, in both a material and a social sense (Ilyenkov, 1960).
- The dialectic relation between humans and the world means that objects are not only items of focus for human action, but offer a means by which the social and material reality of the world is constructed, through the material production of outcomes mediated by practical tools.

Through his work in early twentieth-century psychology, Vygotsky sought to overcome the subject/object, intellect/affect and mind/environment dualisms prevalent in the predominant paradigms of psychology at that time (introspection and behaviourism) (Vygotsky, 1978). Stressing the importance of analysis of units, rather than analysis of elements (Vygotsky, 1962), he situated the development of higher cognitive functions in the context of human social activity. To do this, he formulated a triadic model of stimulus-response (S – R) relations mediated by signs brought into operation by the actively engaged individual. The sign (X) has a double effect of initiating or facilitating the

transformation of something in the world whilst also changing the individual who acts (Figure 2).

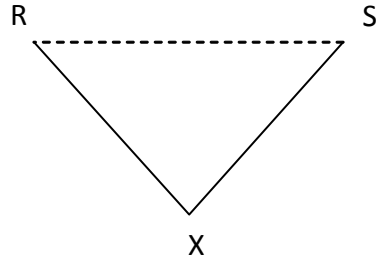


Figure 2. The mediated act (Vygotsky, 1978).

Vygotsky claimed that this mediated connection with the world was “basic to all higher psychological processes” (Vygotsky, 1978, pp. 39-40) and thus the structures of behaviour are determined not only by biological development, but also by culture. Vygotsky noted that tools and signs have analogous roles in mediating activity, but with an important distinction in the ways they orient human behaviour. Vygotsky saw the tool as being externally oriented, leading to changes in the world, whereas the sign is internally oriented and as “a means of internal activity aimed at mastering oneself” (ibid, p. 55). Using practical productive human labour as an explanatory category thus extends the stimulus-response model to a broader subject-object mediated connection.

A.R. Luria followed the work of Vygotsky in exploring the effect of tool use on human development (Luria, 1928). In his model, consciousness develops through engaging in transformative activity, mediated by those tools (Kozulin, 1986). As Cole and Engeström (1993) note, ‘tools’ in this context did not mean only material devices for shaping the physical world. Language was considered an integral part of the overall process of cultural mediation. Later work closed the distinction between these orientations, arguing that both happen simultaneously, with tools recognised as signs of their potential transformative effects and signs seen as tools in human intercourse. Mediation by tools was said to be more outwardly oriented, mediation by signs was more inwardly oriented,

but both aspects occurred at all times alongside each other (Cole and Engeström, 1993, p. 6).

Vygotsky's notion of the ZPD brought the use of signs and tools in social interaction into conceptions of learning and capability to learn: "human learning presupposes a specific social nature and a process by which children grow into the intellectual life of those around them" (Vygotsky, 1978, p. 88). The ZPD identifies not only the child's current capabilities, but also the capacity he has for further learning, in that his learning "creates the zone of proximal development; that is, learning awakens a variety of internal developmental processes that are able to operate only when the child is interacting with people in his environment and in co-operation with his peers" (ibid, p. 90). Under this theory, learning precedes development: learning new meanings or mastering new operations becomes development only when those capabilities are internalised. This model of learning challenges dualistic representations of mind and the world and introduces a notion of learning as distributed, with subject development mediated by practical and cultural factors. The mediating influence of semiotic and material tools indicates how cultural meanings become part of individuals' psychological development through internalisation of social acts (Vygotsky, 1962). In Vygotsky's analysis, the focus was placed on signs as 'psychological' tools as these "imply and require reflective mediation; consciousness of one's... procedures" (Engeström, 1987, p. 38).

5.1.3 Activity theory and CHAT

The psychological insights established by Vygotsky open the possibility of understanding human behaviour from a basis of:

- viewing the mind as embodied, stretching across material environments
- dialectic philosophy, viewing people as shaping and shaped by social contexts
- problematising notions of knowledge as isolated from action.

Leont'ev (1981) built on Vygotsky's original idea of mediated action to create a psychological theory of socially meaningful object-oriented activity. He refocused on practical activity and found justification for an individual's actions in the collective activity

to which they contribute. He saw human labour as inherently co-operative, but divided between individuals. In this theory, activity is structured into three qualitatively different levels of human functioning (Figure 3). An activity driven by a shared object-motive is divided into constituent actions which have specific goals. Individuals' goals might not coincide with each other or with the motive, but will contribute towards attaining it. Actions are carried out in concrete circumstances by means of operations: reactions to the constraints and affordances of the situation, which might not necessarily be consciously apprehended by the subject. It is possible for behaviour to 'slip' from one level to another, in response to unforeseen difficulties, or as a result of developing proficiency (Leont'ev, 1978).

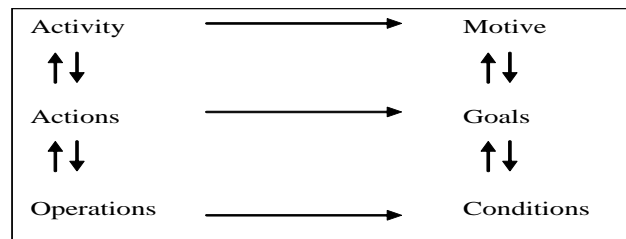


Figure 3. Hierarchical Model of Activity (Leont'ev, 1978).

Leont'ev offered the example of a primeval collective hunt, in which an individual employed as a beater undertakes actions whose result is to scare animals towards their predators, and his activity ends there. The processes of his activity do not coincide with what had stimulated them in the first instance (the need for food or clothing). "We can say, for example, that the beater's activity is the hunt and the frightening of game his action" (Leont'ev, 1981, p. 210). In stating this, Leont'ev posited that an undirected need state on the part of the individual is not sufficient to motivate human action. Needs become motives through being directed towards imaginary or material objects which are able to relieve those needs. In Leont'ev's theory, activity is based on material production of outcomes, mediated by practical and psychological tools. It also means that social, cultural and historical dimensions have to be taken into account if we are to form an understanding of psychological development. In this theory the unit of analysis became

the historically evolving object-oriented practical activity (Kozulin, 1986). However, with this psychological basis, it is difficult to apply the framework in fields other than psychology in order to deal with collective activities (Kaptelinin, 2005). The potential for behaviour to slip from one level to another under the pressure of circumstances demonstrates the situated nature of activity and the need for aspects of the environment and the person to be included in the analysis.

The emergence of activity theory in Anglo-Saxon academia was facilitated by the work of Michael Cole through the Laboratory for Comparative Human Cognition at the University of California, San Diego (Roth and Lee, 2007). The sociocultural approach was introduced by Cole as a means of understanding the process of change by which cultural content is transformed into cultural differences in cognitive processes (Cole, 1985). Cole traced parallels between developments in anthropological theory and developmental psychology, and posited activity as the site in which culture and cognition could be jointly examined. By acknowledging culture embedded in the tools whose use becomes internalised by the learner, research could take a cultural-historical approach to distributed cognition as a theory of learning and development (Cole and Engeström, 1993). Leont'ev's approach offers a means of structuring activity to understand how actions relate to objects, but does not offer the means of situating learning in a wider cultural context, accounting for the collective and evolving nature of activities (Engeström, 1987). Vygotsky's emphasis on semiotic mediation also leaves aside issues of social position, power and control by condensing them into the workings of language (Hardman, 2007) and placing less emphasis on practical activity. That model centres mediation of development on the individual's actions with signs and does not offer means to securely place cognitive change in a broad cultural context. Focus on the mediation of activity by society opened up a middle ground connecting psychology with sociology and anthropology (Cole et al., 1978).

Engeström (1987) put forward the case that the subject of activity needs to be seen more fully as a collective subject, operating in a community, rather than as a sole agent (Engeström and Cole, 1997). To this end, he extended Vygotsky's mediational triangle

model to represent the activity system, incorporating the community in which the subject is situated, the rules which govern that community and the division of labour which takes place (figure 1, p.73). This is a system of multiple mediations: just as tools (material and semiotic) are seen to mediate between the subject and the object, so rules mediate between the subject and the community, and the division of labour mediates between the community and the object. However, there are relations between all elements of the expanded mediational triangle, and relations of production, consumption, exchange and distribution can be located within this framework (Figure 4, below). Continuing the work of Marx in identifying these relations with their mutual interactions and co-constitution, he notes that “boundaries between the sub-triangles are blurred and eventually given up” (Engeström, 1987, p.30). CHAT is characterised by the use of the triangle as the unit of analysis when describing the active cognising subject interacting with their environment, allowing one to situate developmental processes in a communal context.

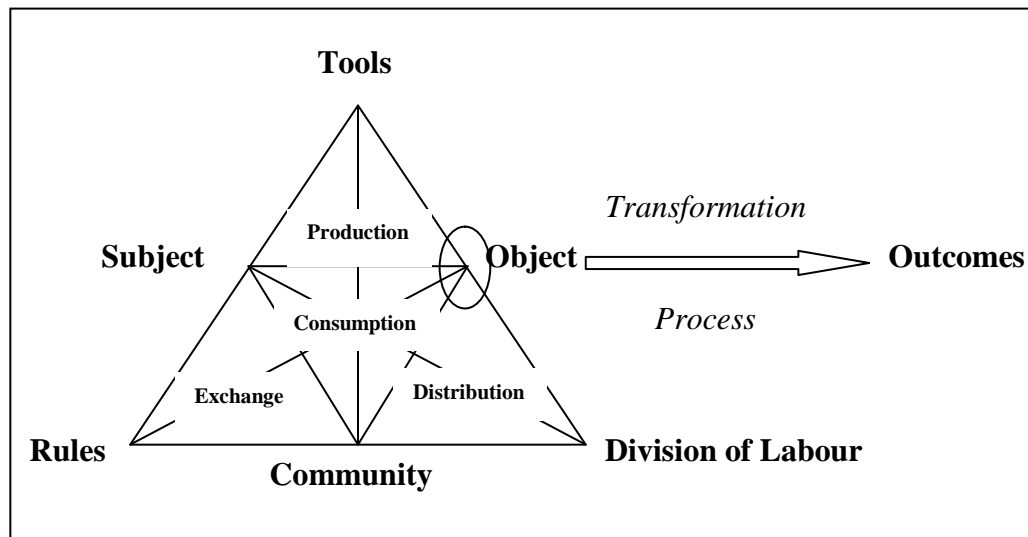


Figure 4. The structure of human activity (Engeström, 1987).

Kaptelinin (2005) notes that Leont’ev’s example of the hunt shows the dissociation between activities and actions which emerges as a result of the division of labour in

collective activities; the individual's actions are still social, as they are directed ultimately towards the shared object, although not directly collective. However, Leont'ev's theory does not offer a means of situating human activity in context, to demonstrate how individual actions are co-constitutive with the collective activity. Engeström's formulation thus comes into play as a framework for discussing the interactions of community members, the ensuing division of labour and the rules which govern the activity, using the framework as a heuristic for situating cognition in context, and as the basic unit of analysis. Hence the activity system becomes a means of describing the "person-environment interface" (Cole et al., 1978) or 'mind in society' (Vygotsky, 1978); the action of mind between and among the institutions that mediates relationships to ourselves, each other and the world (Newman and Holzman, 1993, p. 24). This enables discussion of how semiotic and material resources are transformed into outcomes such as artefacts, in collaborative work processes (Nardi, 1996a; Miettinen, 1997; Engeström, 2001). Hyysalo (2005) argues that the model rests on an assumption that societal needs motivate local activity systems that seek to fulfil societally recognised outcomes. These outcomes are then exchanged with other activities, bringing back resources for consumption. Thus the object of activity no longer directly relieves the original need, but provides outcomes which have exchange value. Engeström (1987) emphasises the point that there can be no activity without production: material production of an outcome also entails production of the tools co-opted into the activity, of the subject's abilities and of the relations in the community. It is through activity that these notions are concretised in the world, thus the activity system as a whole is a productive entity.

Engeström's description also emphasises the generative effect of structural contradictions in activity: "historically accumulating structural tensions within and between activity systems" (2001, p. 137)⁸, which become apparent as disturbances in actions. Contradictions are not just inevitable features of activity, but also provide the driving

⁸ The literature uses the vocabulary of both *contradictions* and *tensions*, with no formal distinction, e.g. (Engeström, 1999d; Engeström, 1999e)

force for the development of activities into qualitatively new forms: in resolving contradictions (through the creation of new tools, new ways of dividing labour, etc.) relations to the object or the object itself are transformed, thus a new activity emerges. Historical traces of the original activity remain and might be discovered by understanding features of a given activity system as the resolutions of previous contradictions. Through the development of individuals' actions within developing activities, the subject changes and establishes new relations. Inheriting the Marxist analysis of labour via Ilyenkov (1960; 1977), Engeström (1987) demonstrates that four levels of contradiction may be discerned in activity. The four levels of contradiction are briefly summarised in Table 1 (below).

Primary	Inner conflict between exchange value and use value within each constituent component of the EMT.
Secondary	Conflicts between the nodes, such as a clash between a division of labour and possibilities offered by the development of new tools.
Tertiary	Conflict which appears when representatives of a culture introduce the object/motive of a culturally more advanced form of the activity into the dominant form of activity.
Quaternary	Conflicts which occur between the central activity system and its "neighbour activities"; those which have produced the tools, subject and rules and consume the objects and outcomes.

Table 1: The four levels of contradiction in activity systems. Summarised from (Engeström, 1987)

CHAT describes the interplay between stable practices and instability within systems such that "change is manifested as a crisis that requires reorientation of parts of the system, renegotiation of roles and rules; introduction of new mediating tools and meanings; and redefinition of objects" (Beswick et al., 2007, p. 122). The continual effect of generative contradictions on the activity system entails a constant shaping and re-shaping of the activity system, through re-organisation and development. The object is represented by an oval in Engeström's model (Figure 4, p. 91) to indicate that object-oriented actions are always characterised by ambiguity, surprise, interpretation, sense making and potential for change (Engeström, 1999d). Change within activities is seen as "a fundamental part of the dynamics of human evolution" (Cole and Engeström, 1993, p. 8), so activity systems

should be viewed as complex bodies in which stasis is unusual and contradictions continually exert pressure towards change. Over time these can instigate significant qualitative transformation of the activity.

In an effort to articulate dialogicality and multivoicedness in joint activity within the collective system, Engeström (1998) introduces “3rd-generation activity theory” (Figure 5, below). In this, the basic model represents two subjects, each with their own initial conception of the object (Object 1), which emerges through productive action as a version of the shared object (Object 2). This is then influenced through the actions of and interchange with others, leading to the emergence of a third object (Object 3).

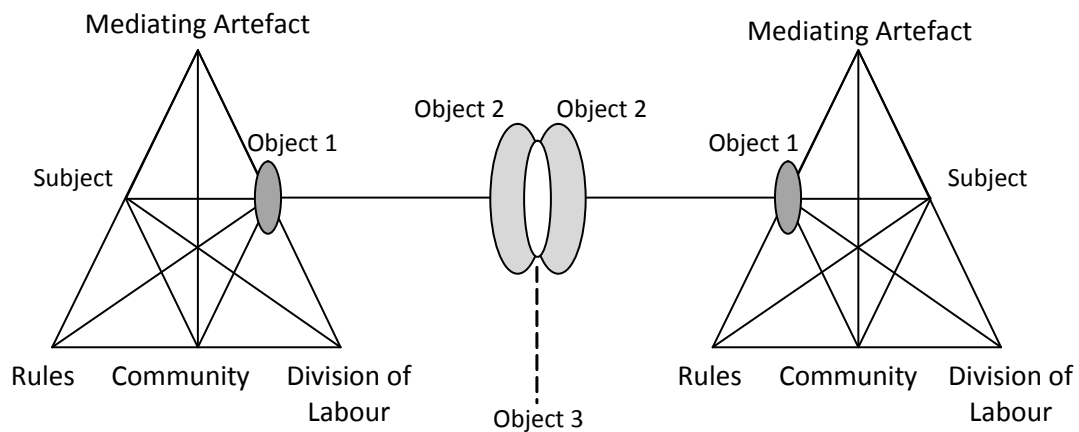


Figure 5. Two interacting activity systems as minimal model for third generation of activity theory (Engeström, 1998).

The different initial objects result from subjects' different positions in the division of labour. Subjects inhabiting different roles in a community carry diverse histories, which are embedded within their activity systems in rules, community norms and the use of artefacts. In common with the previous model, the 3rd-generation activity theory triangles should be seen as inherently dynamic, as the object of activity subordinates to itself through its different images in the transformative action. This transformation reverberates throughout each activity system, through actions of translation and negotiation. The multi-voiced description of activity which results from this model enables

the exploration of contradictions which generate disturbances and lead to innovative attempts to change the activity. This development can lead to new versions of the activity, with new norms and a new conceptualization of the object of activity, which can “embrace a radically wider horizon of possibilities than in the previous mode of the activity” (Engeström, 2001, p. 137). This model of activity allows for the description of processes of social transformation and includes the structure of the social world in analysis, taking into account conflicts in social practice. In CHAT, knowledge is understood as an emergent property between the acting subject and the world. Cause-and-effect explanations of knowledge creation are avoided and a central role is played by subjects’ interpretations of events.

Engeström (2001) summarises CHAT with five key principles:

- Collective, artefact-mediated and object-oriented activity systems, in relation to other activity systems, are the prime unit of analysis. Individual or group actions are subordinate moments, to be interpreted within the context of an activity system.
- Activity systems are multi-voiced, through the different positions, points of view, histories and interests of the members of the community.
- Activity systems take shape and are transformed over lengthy periods of time, and their history provides a means of understanding their potentials and problems.
- Contradictions in activity (historically accumulating structural tensions) are sources of change and development.
- Activity systems contain the possibility of expansive transformation: reconceptualisation of the object of activity which embraces wider horizons of possibility than the previous mode of the activity.

5.2 Theoretical engagement narrative

In my exploration of the location of mathematical action in the classroom, I needed a theory which could account for engagement *with*, as well as *within*, mathematics, and offer means of speaking about the activity in which this engagement is fostered. I found in

CHAT the means of exploring this engagement, related to meta-concepts of mathematical capability (Howson and Mellin-Olsen, 1986). The theoretical frame would enable me to identify and privilege those relations and artefacts that were most significant in forming learners' conceptualisations (Nardi, 1996d). The nature of my questions placed the learner at the centre of the analysis; third generation CHAT offered the means of placing pupils and teachers at the centres of their own related activity systems.

The above review is far from being a definitive or complete account of the development of CHAT. Many concepts are still contested, as are theoretical relations which sustain or diverge through the history of CHAT: the continuity between Vygotsky's work and Leont'ev's is debated (Hyysalo, 2005) and the theory itself allows for its own development, as concepts are further explored (Davydov, 1999). Bakhurst (2009, p. 197) argues that the development has been extensive enough that the current formulation of CHAT is "in tension with the concerns of the Russian founders of the tradition". The difficulty of translating some key terms from Russian into English hinders the formation of a definitive account of the development of the theory, as will be shown in §5.4. Relations within and between concepts can be seen to change in prominence and importance in Leont'ev's work (Hyysalo, 2005), and the notion of the object remains contestable (Kaptelinin, 2005). As the moment in the activity system from which the others derive their importance I found it important to look deeper into the concept of the object, with the aim of determining how to identify the object in my research, and the distinct object *of* my research.

5.2.1 The object

Developments in the notion of the object and object-orientedness have been traced in §5.1. Whilst prevalent, the representation of the object in current literature is not completely consistent. For some the object is understood to be a part of the world – that which provides the "energizing force" which drives activity forward (Jaworski and Goodchild, 2006), which is very close to Leont'ev's idea of the motive of activity. Bedny & Harris (2005) take the stance that the object has been confused with the idea of an objective, closely aligned with the motive of an activity (see Bellamy, 1996), but they place

the object separate from and subordinate to the goals of activity: "that which is modified and explored by a subject according to the goal of activity... [and] it is only the desired future final state of an object that corresponds to the notion of the goal of action or activity" (Bedny and Harris, 2005, p. 131).

In order to understand the history of the concept, it is necessary to introduce three terms which would all translate to 'object' in English. As stated in the discussion of the Hegelian and Marxist heritage of this work, the term *Gegenstand* (*Objekt* in Russian) refers to material items in the world, existing independently of the mind, offering resistance to the actions of humans and therefore impinging upon the possible cultural constructions that can be made of them. *Predmet* (Russian) on the other hand means the target of a thought or action, emphasising the subject's objective orientation within an activity. Kaptelinin (2005) notes that in his early work, Leont'ev conflated the two notions of *Objekt* and *Predmet* (Leont'ev, 1981) , but came to distinguish between them more systematically in his later work (Leont'ev, 1978)⁹:

...the object of an activity is its true motive. It is understood that the motive may be either material or ideal, either present in perception or exclusively in the imagination or in thought. The main thing is that behind activity there should always be a need, that it should always answer one need or another.

(Leont'ev, 1978)

Here, though, I saw the lack of determination in the object which had opened up opportunities for differing interpretations. For Leont'ev the object of activity is the "object of individual activity", with the activity being sited in a structure of social relations from which it cannot be isolated. All activities are social, but the focus of his theory is on "concrete individuals" (Leont'ev, 1981, p. 51) engaged in individual activity. This does not preclude the notion of collective activity, and Engeström (1987) notes that Leont'ev's theory is a move towards this idea, but his framework is designed for understanding

⁹ The reversed publication dates here reflect the order in which Leont'ev's work was published and available in English.

individuals' activity. The object for Leont'ev is the only thing which can relieve the needs behind the motives of individuals, thus connects the actions of various participants under the same motivating whole and creates an object-related horizon that is transformed in the course of the activity (Leont'ev, 1978, p. 62-65). To carry out a piece of educational research focusing on classroom activity in a wider social context, I needed to be able to situate the subject of activity in a cultural context at varying scales, ranging from the classroom to the world outside school. The formulation of three hierarchical levels of activity does not directly help in pinning down the nature of the object which drives activity. However, Leont'ev illustrates that the object need not be consciously held during the activity, by commenting that when the motive of an activity becomes conscious it should be considered a 'motive-goal', concomitant with specific actions within the activity.

At this stage the inability of English to express well the subtle distinctions in thought and action upon which the theory rests became apparent. The object of thought (*Predmet*) cannot be understood independently from the object of practical activity (*Gegenstand/Objekt*) but nor should it be identified with it (Marx and Engels, 1846/1964). Leont'ev's theory relates the object of activity to motive, whereas Engeström relates it more closely to the processes and outcome of practical activity. The connection to motive is not rejected, but we are brought closer to understanding the complexities involved in the motivational aspects of collective activity (Hyysalo, 2005). Kaptelinin (2005) discusses the differences between Leont'ev's and Engeström's notions of object, and concludes that a conceptual separation between the object of an activity and its motive is necessary. Leont'ev's unification of the two ideas does not allow for multiple motivations, the dynamic construction of the object in activity in response to situational constraints, the interrelation of parallel motives or the interaction between motives held by different subjects in a collective system. Separating the object from the motive and allowing the object to be "cooperatively defined by the whole set of motives that the subject strives to attain in their activity" (ibid, p. 16) permits it to be seen as flexible and emergent from the activity. In coming to an understanding of the object, then, it is not sufficient to merely proclaim its role in directing unfolding activity, but the researcher must explore how the

object is manifested in activity, how subjects relate to it and how it shapes specific actions. Davydov (1999) claims that “true activity” is always connected to the transformation of an object in reality, where transformation of an object means “changing it internally, making evident its essence and altering it” (p. 42). I interpreted this description to refer to socially established meanings of objects, discovered and created in activity.

In Engeström’s framework, the object is something other than the motive driving the activity. It is “the raw material or problem space at which activity is directed and which is moulded or transformed into outcomes” (Center for Activity Theory and Developmental Work Research, n.d.). Moreover, objects do not exist for us in themselves, directly and without mediation. We relate to objects by means of other moments of the activity system, and there is continual interchange between the nodes of the activity. What initially appears as object can be transformed into an outcome, tool, or rule (Engeström, 1996). This means that objects appear in fundamentally different roles: as *Gegenstand* and in mediating artefacts or tools. Consequently the material makeup of an item itself does not determine whether it is object or tool. The constellation of the activity determines the place and meaning of the item (Engeström and Escalante, 1996, pp. 361-362).

By highlighting the dual material/ideal nature of the object, I was in a stronger position to explore not only the relationship between object and motivation in activity, but also how that activity unfolds in practice. Cole and Engeström (1993) represent the object of thought and the produced object as two different things, by adding the dimension of time to the analysis. Over the course of time, aspects of the directly experienced and culturally mediated *Gegenstand/Objekt* are synthesised to produce a new subject with a new conception of the object. In order to grasp processes of change and the non-self-identity of the object, Roth (2010) urges the adoption of a diachronic perspective on activity. The notion of object-orientedness of activity can now be explored by focusing on what is acted upon, what results from the action and how the structuring of activity derives from the ideal and material aspects of the object. Given that the object is a qualitatively rich and

meaningful part of the world, it is also not possible for the subject to know the future final state of the object. At the outset of activity, an object can only be known through abstract definitions. Specifically in terms of education, as made clear in discussions of the ZPD, the object occupies a horizon which can never be reached. However, the object defines the continuity or unity of activities through being known to others (teachers) who structure and lead activity. Hardman (2007) writes of uncovering an *emergent* object in classroom research, as the results of learning are produced in activity.

For this research I identified the object as “the focus of activity, the issue or thing that is being acted upon” (Daniels, 2004). Similarly, Barab *et al.* (2002) refer to the object as “the thing at which activity is directed and which is molded or transformed into outcomes with the help of physical or symbolic, internal or external tools” (p. 78). To summarise, my conception of the object is as something which:

- provides a specific activity with a focus and directs the unfolding of the activity
- inhabits both an ideal and physical existence, through the interplay of use and meaning in action
- marks out divisions in time and activity, defining distinct activities
- pre-exists the individual subject’s engagement but is dependent on the engagement of the subject in activity for its development
- exists in a dialectical relationship with the circumstantial particulars and material conditions of activity
- the subject cannot claim to know in its entirety at the outset of activity, but can be understood well enough to make evident generative tensions and contradictions which drive development
- might still be ‘on the horizon’ at the ostensible completion of activity.

Formulations of the object as motive assume that the subject is motivated by (and towards) the object, and that their participation exists on that basis. As noted previously, this is not an assumption that holds in relation to pupils’ participation in the classroom. However, Hyysalo notes that the motive for participation has grown more complex as the

theory has developed, with dissociation between the motives spurring action (such as hunger, social recognition or personal growth) and the object/motive in a collective activity, participation in which brings relief to the original needs. Consequently, “the self-motivating power of participation in activities can become the key, sense forming motive in itself” (Hyysalo, 2005, p. 22).

5.2.2 The object in my research and the object of my research

I saw in CHAT the opportunity to articulate dialectic co-constitution of object, subject and means in the context of mathematics education. In this sense the notional object (*Predmet*) directs the activity, but the meaningful result of practical production (both material and ideal), is a result of the meeting of *Objekt*-determination with social, institutional and environmental constraints. I aimed to show that by investigating the practical-productive activity in mathematics classrooms I could develop an instructive sense of the actuality of the object as created.

Leont’ev’s implication that the motive may not be consciously held in the mind of the subject drew my attention to what the subject might actually know about the object. Kaptelinin states that the term indicates things that exist regardless of our feelings about them (1996, p. 123), embedded in a world which is meaningful independently of a specific subject. Thus the object is not necessarily determined by the subject’s own immediate priorities, but may be informed by meanings beyond the subject’s current understanding. The transformation of the object makes evident more of its social meanings, bringing them into the subject’s understanding. Kozulin (1986, p. 271) hints at this when he states: “When entering human activity an object... appears as an object of collective social experience”. This aspect of the object indicates something of the subject’s awareness of it: van Aalsvort (2004) refers to the object as that in which the subject can ‘see’ the product of their actions. In relation to the observations made by Barab *et al.* (2002) above, part of the transformative process in mathematics classrooms involves subjects making mathematics visible to themselves.

However, activity is not an additive phenomenon: it is realised in actions, but its overall social meaning cannot be derived from those actions alone. Bartolini Bussi notes that “no motive can be taught directly, as activity exists only by means of actions” (1996, p. 22); this assertion applies not only to those observing activity but also to the subject at the heart of it. These statements point to the necessary dialectical relationship of the object and the actions it dictates, and the need for the subject to engage in activity under the assumption that their initial notion of the object will become more rounded and stable, as their activity progresses. A specific object as presented *ab initio* needs to be accepted by the subject as a ‘shell’ which they will re-shape and fill through the developing activity. The object of their activity is both the prerequisite and result of that activity.

At this stage, I found it meaningful and helpful to describe the object (*Predmet*) of classroom activity as a transformable mode of systematic behaviour facilitated by access to the conceptual resources of number, algebra and spatial and statistical reasoning. This stood as an abstract definition of pupils’ mathematical capability, waiting to become concretised through the research. Features of the activity of the mathematics classroom would mediate internalised relations to mathematical structures, effectively constituting aspects of their concepts. This definition stood as the object *in* my research.

Applying understanding of the co-constitution of object and activity to my own research, I was aware that CHAT (used as a tool) would mediate my understanding of pupils’ mathematical activity. Consequently, in pursuing my aim of contributing to teaching practice, the transformative process of undertaking further study comprises the substance of this research as much as any conclusions that I draw. I saw this approach as an exercise in ascent from abstract to concrete, in which understanding of classroom mathematical activity would come about through “unfolding all the necessary definitions of the object in their mutual connections rather than through an abstract definition” (Ilyenkov, 1960). I came to see the construction of this thesis as an opportunity to engage in Vygotskian tool-and-result methodology, in which the method is “simultaneously prerequisite and product” (Vygotsky, 1978, p. 65). Newman and Holzman describe the tool-and-result methodology analogously:

The toolmaker's tool-and-result is that tool specifically created to assist in the development of something that we wish to create. Tools of this sort are paradigmatically 'prerequisite and product' in that... the tool... is a precondition for the product. [However] It is not linearly in advance of the product, either conceptually, or materially. Tool and product of tool are therefore, of necessity a produced unity.

(Newman and Holzman, 1993, p. 47)

My reflexive methodology thus results from the practice of research, rather than from pre-determined premises. Newman and Holzman (1993) discuss the centrality of a search for ecologically valid method in Vygotskian psychology; my search for an appropriate research method reflects a desire to work from the premise of humans “in their actual, empirically perceptible process of development under definite conditions” (Marx and Engels, 1846/1964). This rationale lies behind the narrative form of this thesis, in telling the story of the evolution of my research and presenting the development of my research questions.

The inclusion of Marxian philosophy to describe the classroom and the products of pupils' activity raised the question of surplus value, alienation and commodification in education. It could be argued that this research has been a surplus product of my own making: few teachers identify with the need to embark on extensive part-time study. However, my personal involvement and identification with the research and this thesis sustained a self-conscious balance between use and exchange value. The object *of* my research was the joint (3rd generation) activity systems of pupils and teachers (Figure 5, p. 94) connected through their joint object, defined at this stage as the pupils' mathematical capability. Focus on the object *of* my research constituted an extension to my practice as a teacher and a supplement to my personal interest in mathematical activity. In this, I was pursuing the extent to which the object *in* my research might be produced as an alienated category for the pupils.

5.3 Dual identity narrative

In reflecting upon the use and exploration of CHAT in this research, I was drawn to consider the role of theory throughout this thesis. My primary aim in this research was to uncover (for myself) the constituents of activity in the mathematics classroom, and to

present my findings with a view to informing ongoing teacher practice. As discussed above, CHAT offered a framework which I felt could facilitate such conversation with other teachers, through its ease of appreciation. However, as a researcher I was committed to producing an account of that would meet the standards of the genre in terms of rigour and clarity in argument and suitable placing within the context of pre-existing education research. This would entail making appeals to theory in order to connect and make suitable sense of emergent categories in analysis, and justify their use in argument. My empirical exploration would thus constitute an investigation of the use of CHAT as a framework for classroom investigation, for its sufficiency and deficiencies. My review of CHAT literature indicated the incorporation of additional theoretical tools, such as the analytical descriptors highlighted in §3.3, as need arose (see, for example, Goodchild, 2001; Potari and Jaworski, 2002; Ho, 2006).

Seeking to participate in two practices resulted in a negotiation between priorities: in order to communicate effectively with teachers I should emphasise empirical data and concrete conclusions, whereas in rigorous intellectual exploration I should incorporate theoretical references and justifications. I saw this interplay as an attempt to participate in both horizontal and vertical discourses (Bernstein, 1999), which at times came in to conflict. The site of this conflict was the writing of the thesis itself, which in turn influenced the research process. As a basis for making choices as to when and where to make appeals to theory or current practice in developing my argument, I focused on the principle that underlies the research, and my understanding of the two practices in which I was involved: *acting mathematically is a means of making sense in and of the world*. I intended that this principle should offer a common point of focus between teachers and researchers, offering intersubjective understanding around which conversation could be articulated. Theory which could not contribute to the understanding of the classroom description from this point of view would not be included.

As a teacher, my concern is primarily with the efficacy of ideas and methods in teaching mathematics, allied with an ethical concern as to how the results of my teaching will outlast the pupils' time in formal education. As a researcher, these concerns shift to a

focus on the constituent elements of the classroom, and how I can meaningfully describe them for analysis. Engaging with theory through the literature raised the issue of the extent to which concepts could be precisely defined before beginning empirical investigation. In relation to my aim of learning about the construction of mathematical activity and informing teaching practice, I had to be able to articulate coherently the notions which form my description. The potential for theoretical engagement with the concepts of CHAT was limitless. However, I had to close this exploration at a point from which I could begin to engage with the active classroom. In ascending from the abstract to the concrete in my research, I would develop meaning for these concepts through empirical experience and reflection. To this end I decided not to use an extensive *a priori* description of the mathematics classroom, such as can be seen in (Engeström, 1998); I wished to preserve the pupils' perspective.

The research and writing of the thesis represent a possible first step in an “expansive visibilization” cycle (Engeström, 1999e): making disturbances and innovations visible and analysable, including the identification and questioning of “myths that are typically invoked by practitioners to explain away and defend disturbing aspects of work practice” (ibid, p. 68). My development as a teacher has been to uncover those myths and begin to question them. The second step in the cycle is to undertake analyses that connect incidents with the contradictions they reveal. However, the cycle is intended to be a collective act, involving other teachers and researchers in stages of reflection, planning and transformation (see §5.4.1).

In undertaking this research, I had to negotiate a route through my own ZPD in relation to mathematical activity: “the Vygotskian enterprise... is to create zones of proximal development – environments where people can perform life – and in doing so transform alienated reality” (Newman and Holzman, 1993, p. 29). Revolutionary activity is expressed through making new meanings and tools with which to articulate them. The meaningfulness of this research resides in the context of its production and findings, and the production is the context of the findings. Consequently, the story of the development of my research is an integral part of the research itself. Through adopting CHAT I was able

to see this research as the beginning of an enterprise in which I could contribute to teachers making new meanings and tools in teaching practice.

5.4 CHAT in mathematics education

In this section I consider activity theory and CHAT in their use as theories of learning and in investigating mathematics education. Differing emphases on aspects of the theories have resulted in different structures of learning being articulated with various foci in research. CHAT has been used increasingly in education research in recent years (Roth and Lee, 2007). My review of activity theory research in education is not exhaustive; I present here those studies which have been informative in my development as a beginning researcher. These also informed my ongoing development of the research question, discussed in §5.5.

5.4.1 Activity theories as theories of learning

Leont'ev's hierarchical activity structure offers a means of describing learning as a cycle of developing capability in socially meaningful activities, prompted by motives. In the hierarchy classroom actions and activity co-constitute each other, being regulated and ordered by motives and goals. In action, results are produced upon which the learner reflects for their relevance to the learning situation. The consciousness borne of this reflection contains a changed mental image of the object and the process by which it is achieved, thus motivating further action (Van Aalsvoort, 2004).

The structure also enables a characterisation of the results of internalisation of social processes in the mathematics classroom. When a mathematical method is first encountered as part of classroom activity, it is enacted as an action in which the goal is to produce connections between mathematical structures. When a pupil develops fluency in that method it takes on the character of an operation, being applied in response to problems and structures as they are considered relevant. In this process the *Gegenstände* of mathematics are explored for their resistant properties, through social production and reflection.

Van Aalsvort (2004) uses Leont'ev's structure to investigate cycles of learning prompted by Chemistry curricula. He addresses a lack of relevance, perceived need and motivation by proposing that curricula and classroom practices pre-empt Chemistry-related social practices. In this formulation, participation in "school versions" of chemical preparation and production is engendered in the context of socially meaningful and evolving Chemistry practices. Reflection in these school versions takes place by means of scientific results, rather than school values, and is posited to engender further positive motivation.

CHAT posits learning to be a process whereby the learner internalizes socially mediated tools, in particular language and symbol systems. Third-generation CHAT claims to provide the tools to "locate and articulate internal contradictions and to design concrete collective activities to remove them" (Roth, 2004, p. 6). It thus presents an explicit theory of teaching, but also a means for describing the interrelation between the learner and the world around them, with cognition seen as inseparable from activity. Learning takes place in the subject's movement through the ZPD, but this is not to say that learning is solely embedded in action. Reflection plays an essential role in learning, in particular "when the flow of action is broken by events that were not anticipated" (Raeithel, 1990, p. 36), i.e. in response to disturbances revealing systemic contradictions. Thus, teaching should focus on the creation of ZPDs and work with pupils within these, presenting them with challenges which interrupt the flow of activity, leading to reflection and internalisation of tools. All external phenomena which become involved in the learning activity can be seen as socially mediated and semiotically mediating tools in the activity.

Exploring the mechanisms underlying internalisation and movement through the ZPD, Kaptelinin and Cole (1997) hypothesise learners as involved in two hierarchies of actions, related to the formation and pursuit of individual as well as collective goals. These hierarchies have to overlap "otherwise people would not participate in collective activities at all." However the incomplete nature of this overlap creates the potential for contradictions, from which originate new forms of activity. Wells (2002b, p. 8) identifies learning as ascent along a "spiral of knowing" in cycles of coming to understand through

participation. This leads to the presentation of a taxonomy of curricular activities in which participant structures facilitate reflection and construction through dialogic exchange.

In order to support the processes of internalisation and movement through the ZPD, Engeström (1987; 1999e; 2001) offers the structure of “expansive learning”. In this structure developmental cycles begin with questioning and historical analysis, leading to modelling and testing new models of practice before reflecting upon and consolidating changes as new stable forms of practice. Partnering the five tenets of the theory with questions about the subjects, motivations, content and methods of learning, Engeström (1987) proposes using expansive analyses of learning organisations in designing environments that “energise serious learning effort,” echoing Rizzo (2003). However, the cycle of expansive learning is applied to reflecting upon and designing activities, rather than as a description of how learning might happen within an activity.

Wells (2002a) emphasises the role of spoken discourse as a tool in classrooms, and explores dialogue in the differing forms of agency offered to learners. Focusing on co-construction of tasks in dialogue between peers, he creates a framework of two overlapping expanded mediational triangles, as part of a micro-analysis of task completion. In this framework the construction of meaning constitutes the action which leads to outcomes. This also enables him to represent discourse in the ZPD, concluding that with ‘expert’ and ‘novice’ interlocutors the dialogue itself is constitutive of the object, preserving relative positions of authority (Figure 6, overleaf).

In models of effective learning established from activity theories, learner autonomy and motivation toward the object are central. Gifford (1997) develops a mediated learning model in which individual motivation develops out of learners exercising their autonomy in working at their own pace in teacher-supported environments using ICT. In this model, learners’ exploration establishes ZPDs which ICT then guides them through, whilst teachers act as mediators and regulators of academic concepts. Lim and Barnes (2005) use this model to explore ICT in Economics courses, noting that differences in the resulting object of courses depended upon the learning environment, the role of the teacher and

the extent to which learner autonomy was fostered. Using second-generation activity theory, Lim and Chai (2004) similarly conclude that for opportunities for learner autonomy to be taken up, learners require “orienting activities” (sic) such as tuition in tools, instructional objectives and tools for post-instructional reflection. Time, curriculum and assessment constraints can impinge upon the role of the teacher in providing such orientation.

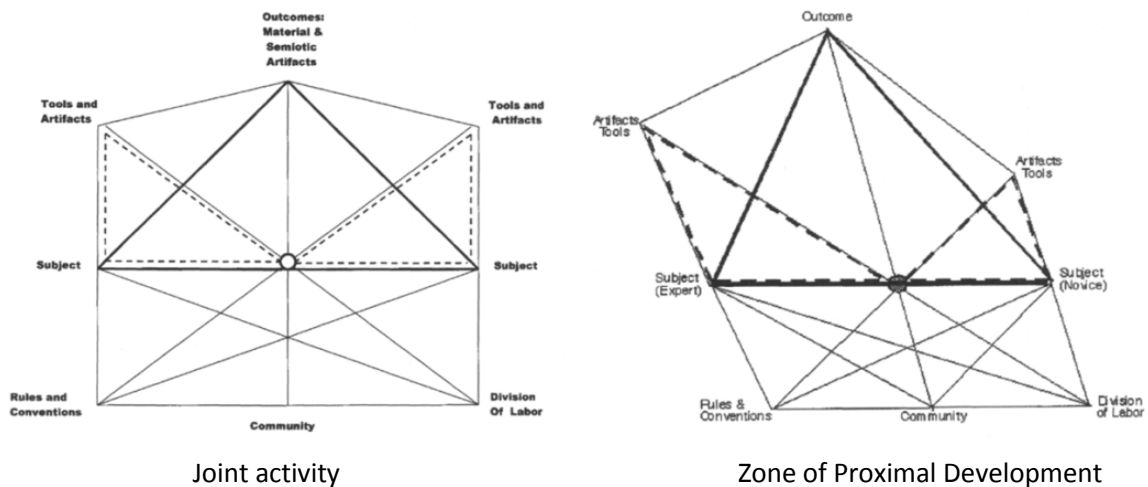


Figure 6. Discourse as a tool in joint activity, and in the zone of proximal development (Wells, 2002a). The object is represented as the central node.

5.4.2 CHAT research in mathematics education

In an example of “gap analysis” (Barab et al., 2004) between school and workplace practices, Jurdak and Shahin (2001) compare the spatial reasoning of plumbers and school pupils in problem-solving in their respective practices. They use activity theory to identify the differences between the methods adopted in terms of the motives, socio-cultural settings, tools and constraints under which tasks were executed. They found that the plumbers’ approaches were characterised by meaningfulness and idiosyncrasy related to materials, methods and situational particulars, whereas the pupils’ approaches conveyed the meanings and methods of the classroom, and were more generalisable to unusual constraints. In contrast to expectations, plumbers’ workplace methods were not error-

free. Their research points to the need for a path through mathematics in which the learner can establish the power and generalisability of formal mathematics whilst also engaging in tasks which offer meaning to quantitative and spatial relations. They posit that the paradox of mathematical language, both derived and detached from the situations which give it meaning, is not only a challenge but an opportunity. I saw that adopting CHAT as a framework would be a means of engaging with opportunities to move on from description of contradictions.

Gap analysis was also undertaken by Williams *et al.* (2001) considering workplace practices and college mathematics. They use CHAT to understand the position of students in relation to these practices, highlighting the limitations of the students' experience. Observing a student making sense of a graph from an industrial chemistry laboratory, they reveal contradictions between practices in ways of knowing, pertaining to generality and idiosyncrasies. They note that it is the student himself who brings the two activities into contact and contradiction, and who has to reconstitute his academic graphical knowledge in the graph as an effective tool in the workplace. They conclude that "the conventions upon which our mathematics curriculum is built should be problematised for students" (ibid, p. 80), in order that they can experience different conventional meanings in different situations.

Whilst noting that the concepts of CHAT can be used as "buckets" for arranging data, Barab *et al.* (2004) offer the "cautionary note" that this can result in treating the EMT as a static representation, and "compartmentalization also runs the risk of the ontological compartmentalization and static portrayal of reciprocally defining and transacting components" (ibid, p. 209). Nevertheless, Nunez (2009), conducting a review of the operationalisation of the concepts of CHAT in the mathematics education community, concludes that the buckets approach is often taken. Her review notes that there are no systematically agreed ways to apply the theory, but notes that the contradictions identified have tended to be between components (secondary) or in relation to the involvement of new tools or objects (tertiary). This restriction might be as a result of failing to accommodate the internal dynamics of the concepts of CHAT. Nunez observes

that use of the theory has largely been used to observe and describe learning situations, rather than as a basis for intervention (ibid, p. 16).

Following Nunez, I turned to the literature for exploration of the dialectic co-constitution of components of activity systems. Coupland and Crawford (2002) found evidence of an intricate relation between the tools of learning, qualities of learners and the actions which contributed to success. Introducing computer algebra systems to a tertiary mathematics course saw those with extensive computer experience become successful adopters of the algebra systems. However, those students who had a high level of engagement in mathematical learning and those who were socially adept at group work could overcome poor computing backgrounds. This change saw students move from being adept rule-followers to adopting self-direction and risk-taking approaches.

Hardman (2005a; 2005b; 2007) focuses on the introduction of computers into primary mathematics classrooms as a tool for shifting pedagogic practices. She tracks transformation and identifies shifts in the object of the classroom which leads to shifts in other elements of the activity. She undertook interviews and lesson observations to develop “thick descriptions” (Cohen et al., 2000) of the activity systems, with the teachers positioned as the subjects. Contradictions between nodes of the activity system were identified, as well as conflict between the “computer as a tool for creative student-centred learning and the computer as a tool for lower level drill and practice skills” (Hardman, 2005b, p. 12). In this research I found characterisation of the object as “emergent”, resulting from the subjects’ motivations, the problem spaces in which they act and the available tools. This terminology appealed to my understanding of sublation as a process through which the object becomes simultaneously adopted as part of the subject but also preserved as something distinct and identifiable.

Goodchild and Jaworski (2005) use contradictions within mathematics classrooms to facilitate the design of inquiry based classroom communities, in a development research project, Learning Communities in Mathematics. They identify contradictions as ‘perceived’ or ‘latent’ and work with teachers to resolve these. They also demonstrate the reflexive

use of CHAT, in evaluating the progress of the project. Applying CHAT to the LCM project itself revealed divergent goals of subjects and obstructions to change. With the aim of using inquiry as a tool to lead to inquiry as a way of being (Jaworski, 2006), the project encountered intransigencies in school activity and established ways of thinking about it (Jaworski and Goodchild, 2006). CHAT offered a means of analysing the complexity of relationships, interactions, organisational demands and established ways of thinking.

Jaworski and Potari (2009) involved teachers in a developmental research project investigating sociocultural complexity in mathematics teaching, reporting on work undertaken with “lower achievers” in a UK secondary school. They use the EMT to investigate questions of pupils’ *resistance to teaching* and *productivity* (terminology adopted from the teacher in the project). In their analysis they highlight tensions using the EMT and the *Teaching Triad* of *management of learning*, *sensitivity to student*, and *mathematical challenge* (Potari and Jaworski, 2002). Their analysis revealed the need for effective teaching to respond to the learning behaviour of pupils, and the complexity which surrounds it. Ho (2006) similarly accompanied the EMT with additional concepts in analysis of mathematics classroom practices, focusing on the prevalence of mathematical problem-solving. The framework of the Singapore mathematics curriculum is used to form an *a priori* description of the classroom as an activity system and categories of action that form the basis of a grounded coding scheme. Lessons were then segmented into mutually exclusive categories and analysed for the emphasis placed on problem-solving. This application of CHAT moved away from use of the nodes of the EMT as “buckets”, but in the mutually exclusive assignment of class events to categories obscured dialectic processes.

5.5 Developing question narrative

My exploration in the development of CHAT revealed that applying the theory bestowed a responsibility to respect a considerable philosophical heritage. This heritage endowed the theory with a substantial set of concepts with which to explore and explain the relation of the acting person to the world. Consequently my questioning came to incorporate

concerns for how pupils were positioned in relation to mathematical action. I was keen not to recreate the findings of previous research, representing school learning as “encapsulated” (Engeström, 1996) or as homogenised curriculum delivery in “traditional” classrooms (Miettinen, 1999), nor would I take these as assumptions. I saw research as a process by which I would make the familiar mathematics classroom strange to myself, in order to explore these characterisations.

However, for my research not to be “built on to sand or pinned onto thin air” (Engeström, 1999e, p. 66) I found it constructive to sketch out a representation of the classroom activity systems (Figure 7, below), based upon my experience as a teacher in the school. This provided a useful starting point, with the caveat that all descriptions were open to revision in light of the data.

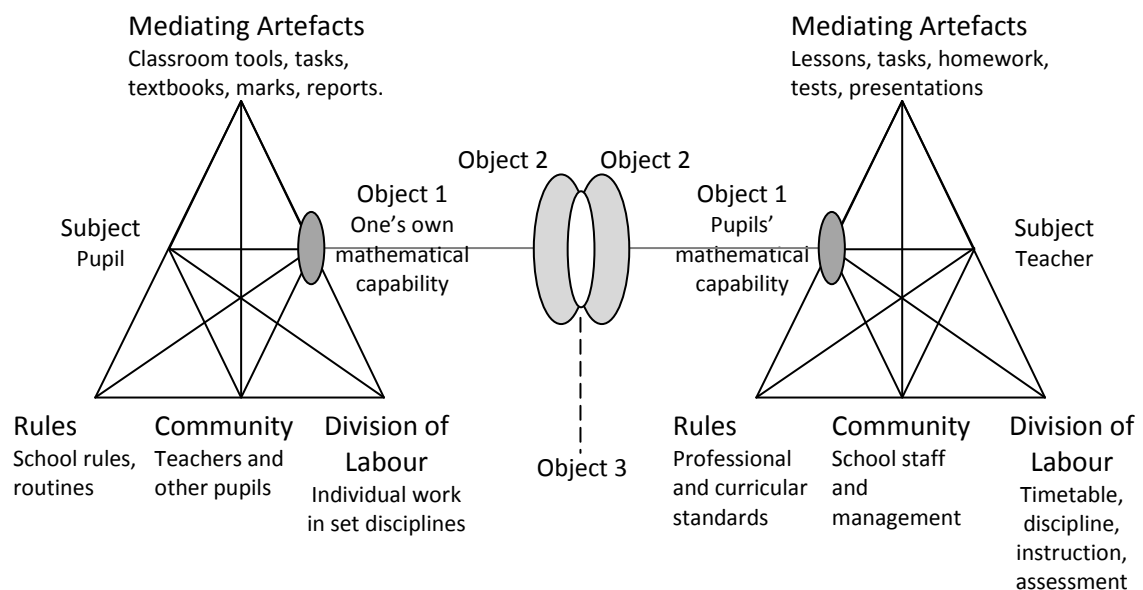


Figure 7: Tentative representation of mathematics classroom as a 3rd-generation activity system

In reflection upon my theoretical exploration I identified that my motivation lay in a concern for the acting person, and what it means to be and become in the world. In mathematics classrooms we create and endorse mathematical ways of being (Wiliam,

1998). Consequently my investigation would emphasise the positioning of pupils through their subject-object relations in the classroom activity.

5.5.1 The object in my research and the object of my research

In describing the activity systems that constitute the object of my research, I should have to convey the pupils' ongoing mediated relations to the object in the research. This would entail considering their varying agency in relation to the dual material/ideal characteristics of tools (Cole, 1996) in the mathematics classroom. In carrying out sign-mediated activity the subject must willingly engage with the sign and use it to operate on their self or their understanding (see discussion in Wells, 2002a). The use of tools is similarly dependent on the subject's active engagement and their choice to employ the artefact toward a specific goal. In opting to use an artefact as a tool, a subject imbues that artefact with a meaningful conceptual status, viz. 'the thing which does this job.' Conversely, the 'mental' artefact may have some physical manifestation or instantiation with which it is intimately related. In working towards transformation of their own understanding and capabilities, established mathematical facts and techniques are available as tools for pupils to use, but are known only through their representations.

However, these representations will be recreated in the production required for making concrete the latest addition to the object. Thus the production of the object becomes the tool for further production, and the distinction between tool and object might be blurred. Working toward my object would thus entail describing pupils' ongoing relation to their object. I saw the vocabulary of *visible*, *useable*, *constructible* offered by Kanes (2002b) as providing means of articulating the agency of the subject in the activity, along with the dependence on circumstantial conditions, as they come to construct the object of their activity.

Taking a developmental view of the classroom as an activity system means the object must always be seen as a "project under construction" (Engeström, 1999e, p. 65). For my purposes this is inescapably true. If the object of the mathematics classroom is pupils' mathematical capability, then it must be seen that the object is always receding from the

practitioner: once it has been decided that a pupil (or more commonly a class) has reached a certain level of proficiency within a particular area of mathematics, a new, more complex area is opened up: “as soon as an intermediate goal is reached, the object escapes and must be reconstructed by means of new intermediate goals and actions” (*ibid*, p. 65). The object of my research must thus capture a sense of this continual transformation and pursuit.

5.5.2 Buckets

The inherent methodological openness of CHAT (Nardi, 1996b) had sometimes resulted in applying a heuristic “buckets” method (Barab et al., 2004), as stated above. My explorations had shown that this was detached from the philosophical heritage of the theory, in which transformation is considered inescapable. Roth (2004) notes that activity theory places an emphasis on individuals’ power to change the conditions that mediate their actions, through engaging in activity. However, he notes that the key Marxian ideas of dialectical transformation are often absent from activity theory literature (*ibid*, p. 2), and hence the dynamism of the activity system can be lost in research. This observation stood as reminder to me to identify opportunities and processes of transformation in activity, or if they could not be found, to account for that. I saw third generation activity theory as a means of articulating the relationship between teachers’ and pupils’ conceptions of the mathematical capability, and how they combine in pupils’ production, in turn influencing ongoing activity. I saw addressing the “buckets” approach as a means of contributing to theory-building, as well as engaging with the empowerment of pupils in relation to transformative processes.

5.5.3 Analytical framework and reformulated questions

In order to move on I brought together the analytical tools I had encountered, to be co-opted into my activity. I articulated these tools as a framework which can be seen in Table 2, overleaf.

I aim to make sense of how pupils are positioned in relation to mathematical action, with regard to the analytical descriptors and vocabulary of

- pragmatic reasoning schemas and concepts
- visible mathematics
- mathematisation and demathematisation,
- visible, useable and constructible mathematics
- horizontal mathematisation and vertical mathematisation,

by investigating the mathematics classroom with Leont'ev's hierarchy of activity, the EMT and the five principles of 3rd-generation CHAT:

- artefact-mediation and object-orientation
- multi-voicedness
- historicity
- generative contradictions (which can be classified with regard to their structure)
- potential for expansive learning,

in order to describe the object of classroom mathematical activity.

This will be done from a philosophical basis of

- dialectical relations
- human relations to the object in its *Gegenstand* and *Predmet* aspects
- Marxian categories of value and production, consumption, distribution and exchange
- ascent from the abstract to the concrete
- action as transformation
- sublation,

and the learning principles of

- intersubjective action
- internalisation
- mediation
- the zone of proximal development.

Table 2: Analytical framework for my research.

Appreciating the mutual development of subject and object in the mathematics classroom had resulted in this formulation of my research questions:

What are the aims and values that shape and are served by social interaction and tool use in the mathematics classroom, and how does the resulting activity provide pupils with the means of locating meaningful mathematical action in the world?

How can CHAT equip me, as a teacher-researcher, to explore this question with reference to the subject developing in relation to an object of study?

My continued exploration into the bases of CHAT and the formative effect of participation in activity resulted in a reappraisal of the questions. I wanted to investigate the relationship between pupils' goals, their participation, and the curriculum and consider how they were positioned to engage in transformative activity. My guiding question also became more methodological, in response to the varied applications I had seen.

How do social interaction and tool use in the mathematics classroom position pupils, in relation to personal aims and cultural categories, to locate and value meaningful mathematical action in the world, constituting personal and practical transformation?

How can I best implement a methodology to trace dialectical subject-object development and contradictions in classroom activity using the framework of third generation CHAT and Marxian categories?

6 Methodological discussion

The adoption of CHAT does not automatically lead towards any particular methodology, being a “philosophical and cross-disciplinary framework for studying different forms of human development” (Kuutti, 1996), although understanding of the roots of CHAT leads to certain methodological priorities. In this chapter I explain the development of my methodology and research methods. With no explicit methodological prescriptions it is up to each researcher to interpret the general recommendations and apply CHAT as they see fit (Mwanza, 2002). In §6.1 I detail my research site, then in §6.2 I share the methodological consequences of understanding activity theory research as an activity in its own right. In §6.3 consider the interrelating activity systems of teaching and research and show that my position in the research was complex and constantly under negotiation. My position as a researcher in the research was a significant issue, as will be seen, and the process of engaging with the data was dynamic and evolving. In §6.4 I explain how a significant shift in methodology occurred in relation to the emerging data and the questions I sought to answer. This leads into a review of the incorporation of a grounded approach to the data in §6.5. I then consider the effect the development of methodology had on my research questions (§6.6).

6.1 Research site: school context and participants

This research was conducted in a boys’ independent secondary school (hereafter referred to as ‘the school’), situated in a borough on the edge of London. The school takes in pupils from 11 to 18, with GCSEs (or IGCSEs) and A-levels being the only courses on offer. The school is small, with fewer than 600 pupils. Entry is by selection examination and interview to year 7 for about two-thirds of the pupils. This year group typically comprises fewer than 70 pupils. The other third enter by examination and interview to year 9. More than 99% of pupils gain more than 5 A*-C passes at GCSE/IGCSE. The feeder schools are state primary and independent preparatory schools.

Participation in this research by eight self-selecting volunteers began in year 7. Over the course of the data collection, two of these eight left the school. At the end of the

research, each remaining participant chose a pseudonym. The two who left the school are identified by initials, to distinguish their data where it is used. Reference made to other pupils is pseudonymous. Over the course of the research, the participants were taught by five teachers in the mathematics department (never by me, see Appendix 1). Four of the teachers were available for interview during or towards the end of the process; each teacher is identified by an initial: C, J, K, M, W or P¹⁰. I have chosen not to make reference to the teachers' gender, as this was not a focus of the research.

Mathematics teaching at the school used a content-based approach, common in British schools, and would fit Boaler's description of "traditional" teaching (Boaler, 1998). Pupils and teachers used textbooks throughout years 7 to 11, which were used to co-ordinate work within and across classes, in relations to departmental schemes of work. Textbooks presented explanations of mathematical structures and methods and series of exercises in which the methods could be practised. Whilst teaching did not exclusively refer to textbooks, they were the predominant medium through which tasks were presented.

6.2 CHAT research as an activity

Engeström (1993) noted that CHAT provides conceptual tools that must be applied according to the specifics and nature of the object of the activity under scrutiny. Identifying the object of my research enabled me to derive and develop methods in order to realise that object as a research outcome, which could be presented in the *Tätigkeit* of this thesis. Placing my activity in relation to that of teachers and pupils in the school also helped me to articulate the development and conflicts within my teacher-researcher identity. In this section I first detail the implications of working with CHAT. I then consider the consequences of viewing my research as an activity.

¹⁰ In the data presented, teachers referred to by any other letters taught different disciplines and did not feature in further analysis.

6.2.1 CHAT research

Methodological responses to CHAT have been varied (see, for example, Brown and Cole, 1997; Kaptelinin and Cole, 1997; Bartolini Bussi, 1998b; Barab et al., 2002), and Engeström (1996) illustrates that the activity framework can be used to generate different descriptions of the same classroom behaviour, depending on the identification of the object of learning and the role of the community. Empirical applications of activity theory have identified the object at the scale of a specific task (Wells, 2002a) or of a period of learning activity (Blin, 2004). These applications reflect differing interpretations of the concepts of the activity theory triangle, and are accompanied by concepts reflecting the specificities of the research site, as well as the unit of analysis chosen (Hyysalo, 2005).

Engeström (1993) stated three principles which he considered crucial when operationalising AT concepts:

- Focus on a collective activity system as the unit of analysis
- Identification of the contradictions within and between components of the system
- Analysis of the historical development of the activity.

In order to satisfy these requirements, it was necessary for me to identify the activity systems of pupils and teachers in relation to broader systems of behaviour, and to identify the components of those systems. This began with my articulation of the objects of those systems, in order that the components could be derived, and the boundaries of the systems determined (Figure 7, p. 113). In this research I had to incorporate the developmental qualities of those objects, as instantiations of the pupils' development. Nardi (1996d) suggests that in analysing learning situations:

The research time frame should be long enough to understand changes in the object over time

Analysis should pay attention first to broad patterns of activity to reveal the overall direction and importance of the activity.

Varied data collection methods and points of view should be used in order to understand the activity system from different perspectives.

Analysis should lead to understanding from the subjects' points of view.

(Nardi, 1996d, p. 47)

As a cross-disciplinary framework, CHAT affords the inclusion of 'local' concepts to deal with specific activity systems, their contradictions and development. However, Kuutti (1996) identifies some basic methodological assumptions with which any additional assumptions must be compatible, originating in the recognition of the research process as an activity in its own right. Components of the activity and additional explanatory concepts should derive from concurring identification of the object. Consequently, the activity should be qualitatively studied in real-life practice, and researchers should be active participants in the process. Kuutti recommends that researchers "should constantly refocus their interest in order to provide different views but also to advance the activity [of research] as much as possible" (Kuutti, 1996).

With the aim of understanding the development of the object of study from the pupils' vantage point, I determined that my research plan would have to accommodate a variation of focus, in order for the participants to contribute their understanding of the mathematics classroom, and to permit participant validation of new constructs and inferences. Data collection would have to take place over an extended time frame, for the scope to explore any development in the activity of the mathematics classroom, and within the activity of the research. I intended that this would afford a multi-voiced description of the activity systems and their interrelations, whilst generating a language of description adequate for the circumstances, modes of interaction and implications of means of production in the classroom (Daniels, 2004). In permitting an evolving operationalisation of the concepts in this research, I aimed to produce a description that reflected pupils' ongoing relation to the object of their activity. I understood my methods to constitute using ethnographic tools, rather than doing ethnography (Green and Bloome, 1997).

6.2.2 Implications – research methods

A key issue in the data collection and analysis was to securely identify and distinguish the object and elements of the activity systems, as understood from the participants' point of view. Through lesson observations I would be able to generate descriptions of the action in mathematics classrooms, but in order to understand the meaningfulness of this action, I

would rely upon participants' descriptions of the values and purposes ascribed to the action. Beswick et al. (2007) note that "because activity systems depend on human consciousness and agency... affective self-report is informative" (p. 119). Researchers undertaking studies into pupils' perceptions and conceptualisations have made use of pupils' stories and images (Hoyles, 1982; Picker and Berry, 2000), questionnaires (Jaworski, 1994), and interviews (Kloosterman, 1996). The limited means for exploring complex phenomena provided by questionnaires meant that I did not consider these as a research tool. The main strands of data collection would therefore be classroom observations and interviews, which I would use in a "triangulation" (Kvale, 1996) process to make sense of each other. The focus of the observations would be the acting individuals in the activity system of the mathematics classroom; interviews would help to reveal how they saw this system as being constituted. In turn, observed classroom action would provide examples and reference points with which to make sense of participants' interview contributions. Classroom events were used to elicit descriptions and explanations during interviews. A schedule of interviews can be seen in Table 3, p. 123, and observations in Table 9, p.153.

In undertaking classroom observations considerations of observer interference and pupil reactivity came to the fore: I would not be able to pass as an unobserved or unknown observer. As Goodchild (2002) notes, being *au fait* with the procedures of the classroom, I was well-equipped to 'fit in' easily with the action; I could anticipate events in observing, but not helping the participants, whilst also being approachable by their non-participating classmates. I found that being known as a teacher in the school helped to minimise pupil reactivity (although not eliminate it entirely). The mathematics department had an open-door policy, and teachers would often pop in to each others' lessons. Formal lesson observations also commonly took place as part of school professional development routines. It was unexceptional to have a second teacher in the room, or for pupils to be called upon to explain their work during a lesson. These features of school life admitted on-the-spot discussions with participants, as long as I exercised judgement in timing and content.

Engaging with pupils during lessons could disturb their learning, if not conducted with sensitivity to the class teacher's aims and a participant's development. I worked to establish a research-based rapport with the participants before coming in to lessons with

Interview	School year: Month	Areas of discussion	Participants involved
I001	7: June	Primary school experiences; routines and relationships; using equipment; ICT in the classroom; teaching methods and teachers; good and bad behaviour; permitted behaviour and working habits.	Aaron, Alex, John
I002			Thomas, ZR
I003	8: September		Jake
I004			Jamie, MC
I005		Lesson routines and variation; teachers' rules and pupils' responses; misbehaviour; reading and writing; evidence for learning; Cross-discipline comparisons and projects.	Aaron, Alex, Jake, Thomas
I006			Jamie
I007	8: October		Aaron, John, Thomas, ZR
I008	8: November	Relation of the 'real world' to the classroom; pupils' attention to examples; teachers' planning; purpose of learning mathematics; exercises and practical tasks; motivation; potential connection between disciplines.	Jake, Jamie, Thomas, ZR
I008.1			Aaron, Alex, John (unable to attend I008)
I009		Working habits; collaboration; textbook examples; reading and writing; exercise structure.	Thomas
I010	8: January	Recreating examples and explanations; making sense; agreement; referring to teachers or peers.	Aaron, John
I011		Teacher response to class issues; understanding teacher's purposes.	MC
I012		Beginnings of mathematical work; the role of mathematics in everyday life; connection of classroom to world; teachers meeting individual needs.	Aaron, Alex, Jake, Jamie, John, Thomas, ZR
I013	8: March	Characteristics of mathematical action; use of vocabulary; role of equations; effort, time and tool use; precision and credit.	Alex, Jake, John, Thomas
I015 - 19	8: June	Individual interviews: textbooks; exercises; rules; links to future lives.	Aaron, Alex, Jake, John, Thomas (individually)
HIATUS			
I021 - 6	10:June	Individual interviews: respondent validation and reflection on inferences; role of the teacher and pupil, tasks and tools; motivation and connections to future life.	Aaron, Alex, Jake, Jamie, John, Thomas (individually)

Table 3: Schedule of interviews and features of activity discussed

them, so that they understood they were not being assessed by another teacher, but observed by someone who was interested in their ideas and judgements about the classroom. This aspect of managing my research identity (Foster, 1996) is discussed further in §6.3.3.

Observation data would supplement interview data by illustrating the means by which the activity unfolded, in relation to the object of study. However, observation data would not permit understanding of how pupils saw the elements of activity and I also had to be aware of my own perceptual biases and preconceptions. It would be necessary to disengage myself from any previously held notions of value and purpose that may contaminate my interpretations of the action. I would try to establish understanding from the pupils' points of view; therefore it would be crucial in the initial stages to take a "low inference approach" (Wragg, 1994): noting instances of behaviour and activity for discussion with the pupils afterwards (See Appendix 1.2 for example observation notes, and §6.5.2 for the use of the observation data in this thesis). Interview data would serve to redress any imbalance and offer a means of subtracting myself from the analysis. The opportunistic access I had to classrooms had the side-effect that none of the lessons observed had been specially prepared by the teachers, nor were they asked to change their practice in any way. The lessons were considered run-of-the-mill by the pupils and the teachers.

In order to access the participants' development, I worked to engender an atmosphere of reflective candour in the interviews, which required establishing trust with the interviewees. The research process therefore commenced with interviews and observations which focused on the classroom as a whole, rather than individual participation. This also helped me to understand the "broad patterns of activity" (Nardi, 1996d) before exploring specific events. In order to establish an historical context for the participants' object, it would be necessary to enquire of the pupils' experiences in previous schools. Once these layers of understanding had been developed, the research could then move on to a more fine-grained analysis, in which individual participants and their actions could be explored. Then, to understand how participants made sense of their

own experiences and actions, the interviews took on foci prompted by observations, or tasks related to classroom action. Throughout the research, interviews did not necessarily take the form of direct question-and-answer sessions. It was not desirable to pose my research questions directly to participants, as in order to make them comprehensible to pupils they would have had to be very explicitly detailed, thus inducing the “topaze effect” (Brousseau, 1997, p. 25). Following Kuutti’s (1996) recommendation that the researcher should constantly refocus their interest, the interviews involved some discussion which was not based directly on the observed lessons, but responded to issues about the purposes of learning mathematics. This was achieved through the use of additional tasks and discussions of specific elements of the experience of classroom action.

Interview	Areas of discussion	Teacher
I014	Motivation for teaching mathematics; encouraging interest in mathematics through pupil-teacher relations; sensitivity to individuals’ needs; doing mathematics in the classroom; school structures; freedom and constraint in relation to the curriculum; involving creativity; examination focus; mathematical authority.	M
I020	Motivation for teaching mathematics; mathematics as a formal pursuit and a means of making sense of the world; working within school constraints; the nature of mathematical action; responding to individuals’ needs; authority and reward in the classroom; objectivity; use of textbook and workings; reviewing work and making connections.	C
I027	Motivation for teaching mathematics; purposes of mathematical action; structure of lessons; scheduling curriculum content; classroom order; pupil-teacher relations; responding to individuals’ needs; encouraging productive participation; the functions of writing and methods; expectations when marking; setting.	W
I028	Motivation for teaching mathematics; purposes of mathematical action and achievement; personal development; lesson structure; difficulty of learning mathematics; empowering pupils; encouraging participation; the functions of writing; scheduling curriculum content; managing pupils’ emotions.	K

Table 4: Schedule of teacher interviews and features of activity discussed

To complement the pupils’ descriptions I also interviewed four of their teachers (Table 4, above). Two interviews took place in the midst of pupil interviews, two at the close of the data collection. The purpose of these interviews was threefold: to furnish a description of the teachers’ activity system; to uncover any differences in how classroom activity was

understood between teachers and pupils; to triangulate against pupils' contributions. The data from these interviews furnishes the CHAT description of the classroom in chapter 8.

6.2.3 Research as an activity

Davydov (1999, p. 50) proposed that the study of activity begins with identifying the object content, then defining "the structure of its collective form, the interrelations of its constituents, the methods of their exchange their various transformations, and the conditions and regularities of the emergence of individual activity". I aimed to work toward this over an extended period, using the contributions of the pupil participants. This involved meeting them as a group and individually, in order to explore the action and purposes of the mathematics classroom. It was desirable to establish shared understanding of the research activity so that participants could understand my interest.

All interviews took place in the school, but in settings such as meeting rooms to which the participants would never normally have access, thus placing our meetings out of the normal activity of school, and establishing a different forum for understanding between teacher and pupils. With respect to the developing history of the research group, asking the participants to reflect upon classroom activity could have generated tensions within the classroom, through the introduction of other priorities and points of view. This potential would have to be managed carefully, and is discussed in §6.3.4. Engeström's (1999e) construct of 'expansive visibilization' indicates how reflection upon an activity can be structured so as to develop practice. In contrast, my focus was on understanding how the pupils in classrooms come to a concrete concept of mathematical activity, rather than in generating changes in the activity. In interviews, I would craft myself as a facilitator, in encouraging descriptions of the classroom action and resultant conceptualisations, rather than a source of ideas for how to behave in the classroom.

6.3 Dual identity narrative: comparing activity systems

Engeström (1999e) quotes Margolis (1993): "when everyone in a community shares a habit it ordinarily becomes invisible for what everyone does no-one easily recognizes." My approach developed with the aim of making strange those classroom events which were

familiar. In this section I outline descriptions of the activity systems of teachers in the school and the part-time PhD researcher, in order to reflect on my role in the two practices.

6.3.1 The mathematics teacher in the school

I described the object of the teachers' activity as "pupils' capability in school mathematics"; developing this object was the aim of the teachers' work. This object inhabits a community which overlaps with that of the pupils, comprising the personnel structure of the school, teaching colleagues and the pupils. Different obligations and responsibilities sustained with regard to these parties and constituted the rules of teachers' activity which influenced classroom action. Teachers' duties included marking work regularly, assigning grades to the pupils' progress, writing reports and giving feedback in meetings with parents. The action of teachers in the classroom derived from their mathematical and institutional authority, but was subject to curricular and institutional constraints, in terms of content and scheduling. Teachers were also under pressure from stakeholders who expected outcomes of high public examination achievement for all pupils, regardless of their individual strengths or ambitions. At the same time, the teachers enjoyed a degree of freedom in their teaching, which enabled them to exercise creativity in the classroom, responding flexibly to new imperatives in teaching (Brown et al., 2000) as they chose, whilst exercising the responsibility to be sensitive to pupils' epistemological development (Jaworski, 1997).

At this stage, I described the classroom division of labour in terms of teachers choosing suitable tasks to assist in pupils learning the necessary curriculum content, providing explanations and guidance to pupils, and directing the sequence of action within lessons. Teachers were expected to deliver the curriculum with authority and through confident provision of appropriate tasks. The tools of teaching were the material tools of the classroom (texts, writing and drawing equipment, computers and interactive whiteboards, etc.) and the conceptual tools of pedagogy and mathematics.

6.3.2 The mathematics education researcher

As a novice social science researcher my focus was on the related object of the mathematics classroom as an activity system. The tools of the task were the research tools and instruments (including the researcher's self, discussed below), but as with other elements of the situation, these were discovered as the project progressed, rather than having been developed beforehand. The community in which the PhD researcher works comprises other researchers, at postgraduate and post-doctoral levels, with whom extended 'conversation' largely takes place through academic journals and papers. The expectations on the researcher are to produce regular written output as evidence of their progress; this output may be for the purposes of supervision meetings or for a wider academic audience, at conferences or through journals. As such, this means that the use of the literature as a tool becomes crucial, and the researcher's part in the division of labour is to work with those texts, in order to develop a position in the ongoing academic conversation, as well as to develop their plan of research and carry it through. In focusing on pupils' learning about mathematics, the outcome for which I aimed was the production of a thesis of suitable quality to satisfy the academic requirements of the institution in which I study. As such I am in a position of developing authority in the academic field, although this subject position was not without internal tensions: insights and arguments borne out of classroom practice needed to be suitably reformulated and examined in order to be acceptable in other arenas.

6.3.3 Resultant tensions and complementarities

The connection between pupils' and teachers' activity constituted the joint activity system which was the object of my research; exploring this connection comprises the rest of this thesis. The connection between the pupils' and researcher's activity has been touched upon above: the activity could not exist without the co-operation and contributions of the pupils, yet their engagement and identification with an activity that could detract from their classroom action had to be limited. In establishing a research identity with the participants I had to negotiate a delicate balance in shedding some of the trappings of institutional authority, but not all.

I took pains to manage relations between myself as a researcher and teaching colleagues in ways which could respect the research process without undermining their teaching or motivation. It was important that they not feel threatened by the research process or by conclusions I may draw. To mitigate against this, I remained as open as possible with them about the progress of the project, whilst respecting the pupil participants' confidentiality. In teacher interviews I recognised the hard work they did and emphasised that their approaches to mathematics teaching were valued and informative. I made it clear to them I was considering the ideas the pupils developed about mathematics and was not directly assessing or investigating their teaching. Classroom observations only took place when they were happy; it was continually reiterated that my focus was not teaching, but classroom activity. I believe we shared the "small deceptions" (Ball, 1990, p.157) that my opinions would remain unaffected by observations and our interviews, although in many respects these interviews were much like structured versions of conversations we had already shared as colleagues. An unexpected and sudden promotion to Head of Department meant that for an academic year I was not able to focus on this research project, and the study was formally interrupted. (The implications of this hiatus for data collection and analysis are discussed in §6.4.1.) The potential clash between roles could have been intensified when I received the promotion, and required due care when in the position of manager to the people I wished to approach as a researcher. However, I found colleagues still very happy to talk about their teaching. Transcribed interviews were handed to each teacher as soon as possible for checking and verification: none came back to me with any corrections or clarifications. At no point did any colleague express discomfort at the intrusion of this project into the department's work, and when questioned could not identify any difference made to the pupils' classroom participation.

Greater tensions arose between my simultaneous subject positions as both teacher and researcher. These were practical, philosophical and professional, often deriving from the contrasting loci of authority and experience in each. As explained previously, the first disruption to my teaching persona came about through reviewing the literature on differing conceptions of mathematical activity. Certainties I had held about mathematics

throughout my academic career were destabilised, and questions about my own epistemology and ontology of mathematics arose. I have had to develop ways of managing this lack of certainty which do not undermine my ability to teach with the authority expected.

In establishing a research persona with the pupil participants, I tried to shed those trappings of authority which could inhibit their willingness to participate or contribute. However “legitimacy frequently has to be won and renewed repeatedly rather than simply being officially granted” (Ball, 1990, p. 159). To do this, I continually respected the fact that the participants were volunteers who gave up time to contribute to my interest: interviews and lesson observations took place only when agreed with the participants. As a teacher I could have claimed the right to be in any lesson and insist that a pupil spend some of their free time talking to me. Time was given over in interviews to participants being able to ask their own questions of me; they were interested in the purpose of the project, my motivation for undertaking it and what it might mean for them. When explaining my research I positioned myself in the role of a learner within a formal framework to which I was accountable. On occasion, after some candid conversation regarding their peers, worries about confidentiality might arise: I did my best to allay these by reiterating the protocol that was explained at the beginning of the data collection and by explaining the procedures and rules of the university by which I was bound. It appeared to make sense to the participants when I cast myself in the position of the learner in an institution which laid down rules for how I worked. This was a situation with which they could empathise.

Sustaining this different persona was by no means unproblematic. During the interviews the participants were remarkably candid about past incidents of ‘poor behaviour’ and the role they had played in these. This was the source of some ethical concern for me as the researcher. Stepping away from the teacher’s persona had established an atmosphere of openness in which the participants felt able to be candid, but at the same time I could not dispense of my responsibilities within the school to uphold expectations of order. I had to make decisions as to how to respond to these reports which would not compromise the

covenant of confidentiality and respect, but without undermining my institutional responsibility to uphold standards of good behaviour. I made this decision by considering which system would best recover from the resulting disturbance. Had I broken the covenant of the interviews and their anonymity, it would have been extremely difficult to regain the participants' trust. However, failing to disclose to other staff members when boys had admitted misbehaviour in lessons was a relatively minor issue in the context of the school. In most instances, tales of misdemeanour concluded with the teacher concerned dealing with the incident, and thus there was no obligation on me to take disciplinary action. I judged that discussing past incidents of 'poor behaviour' with colleagues would have served only to undermine my working relations with those teachers and also break the trust placed in me by the interviewees. I chose to prioritise the activity of research, whilst making some small effort to redress the imbalance within the research. In the interviews I responded in the manner of a concerned adult, asking them to reflect on the appropriateness of the behaviour and drawing their attention to the purpose of lesson participation.

When interviewing or observing the pupils, I had to resist the impulse to correct, help or criticise with regard to their mathematical activity. I aimed to step away from my position of authority in order to learn from them, so would defer to the teacher (in a lesson observation) or would respond with questions to turn participants' attention back on to what they had done. In the final interview with each participant, we worked on a single textbook problem together (see Appendix 1); at this stage I occasionally had to use my mathematical authority, but this did not deter the participants from making their own judgements on the nature of the task.

The greatest practical problem created by this clash of positions was the management of time, in both the short and long term. The participants and I both had busy schedules within the life of the school, with pressures and demands which could arise with very short notice but could be neither ignored nor postponed, hence interviews and lesson observations were frequently re-scheduled and delayed. Regarding my self as the major research instrument (Ball, 1990; Jaworski, 1997), I desired space and time to prepare,

conduct an interview (or observation) and then reflect upon it. However, in the midst of busy school days, this was practically impossible. Time had to be managed tightly and reflective notes written down at the earliest opportunity afterwards. Some lesson observations were conducted with the aid of a video camera, which afforded the possibility of re-checking observations. In the longer term, having a full-time role as a teacher meant that only a small amount of time in any given week could be devoted to the research. This became particularly frustrating with respect to the data collection. In an ideal world, I could have devoted time to transcribe and reflect upon the data collected before moving on to the next instance of collection, but this was not to be.

Sustaining an atmosphere in which I could learn from the participants necessitated collaboration and flexibility when making arrangements. Consequently, participants sometimes forgot interviews or turned up very late, leading to rescheduling. However, I resisted the possibility of enforcing participation and decided that the dependence on them to remember arrangements was a price worth paying. Interviews also had to be accommodated within school timings and constraints. They were most often held during lunch breaks, and usually lasted less than 30 minutes. Some more substantial interviews took place after the school day.

6.4 Data collection and initial analysis: Adapting methodology

In this section I explain how the data collection was managed in order to accommodate my methodological priorities and practical constraints. I show how my approach to the data changed as a result of reflection upon the initial analysis, constituting theoretical and methodological development and a deeper understanding of the phenomena observed. The story of my data analysis is one of an evolving methodology, which raised questions about relationships between codes and theory. In a study that uses ethnographic methods, the main research instrument is the researcher and thus the researcher is central to the research (Burgess, 1985). I aim here to give some sense of the experience of undertaking this research in order to contribute to the relationship between methodology, methods and insights gained. As Ball (1990, p. 157) notes, “Reflexivity

provides the mechanism for relating social relations to the technicalities of data collection, and this is the basis of rigour in ethnography”.

6.4.1 Management of the data: collection and analysis

At the beginning of data collection, participants had recently moved from one school to another, so were in a position to make comparisons between different communities and reflect on how their participation differed in each. Coming to secondary school meant that concurrent experience of different teachers and teaching styles provided contexts they could compare across their experience, as well as historically. The participants were in the summer of year seven at the outset, and the summer of year ten at the close. Collecting data across a period of four school years was intended to reveal any change in the participants’ views or experience over time.

In the initial stages of data collection, the interviews were conducted with sub-groups of the participant group. These interviews were loosely structured in that I prepared interview schedules with points I wished to cover (Appendix 1), based on analysis of or reflection on previously collected data, but participants would often introduce unanticipated issues, interpret questions in unexpected ways or open up conversation between themselves which I judged it would be unhelpful to disrupt. My aim in the data collection was to learn about the activity system of the classroom from the subjects’ point of view: it was important to let the subjectivity of the participants come to the fore. Affording this freedom to the participants caused me some anxiety as a researcher, as I worried that I would not be able to collect the information I desired. Group interviews provoked discussion in which careful management was required to ensure all participants could express their opinions freely. Participants often disagreed and argued with each other in interviews, and at times I had to work to create spaces for one to explain himself if another devalued his contribution.

Time was also taken to work on transcribing and analysing interviews. Following a research protocol which called for reflection on data collected before moving on to collect more, also necessitated time between interviews. The burden of transcribing the

interviews was endured, in order to stay close to the data. I used a simple format of transcription, with units of meaning determining line separations and minimal notation used to indicate interruptions and cross-talking (See Appendix 2). Interviews were either audio or video recorded (or in some cases, both) in order to improve data capture. Careful transcription became a key part of the data collection, as the participants expressed themselves in hesitant, confused and disordered ways, often using vague or imprecise language and gestures to express themselves; in some instances the written words would actually seem to convey the opposite of what participants were trying to express. If in doubt, during subsequent interviews I would ask for clarification to ensure I had understood what the participant had intended.

Initial coding was aided by notes made during the interviews, although there was no attempt to close down the interpretation of a given interview based only on those notes – all comments had to be seen in the light of other interviews and lesson observations. Lesson observation data was used as a means of triangulation and support for the analysis throughout. It became useful as a method of avoiding an uncritically veridical acceptance of the interview data (Kvale, 1996, p. 221) and distancing the analysis from my own perceptions. As a researcher who also inhabited the research site it could have been difficult to prevent my informal observations from informing my interpretations. However the analysis of interview and observation data together revealed unexpected aspects of classroom activity and my structured insights as a researcher remained dominant over my informal suspicions as a practitioner. Adopting a reflexive approach (Ball, 1990), any assertions I wished to make had to be traced to data or relations within the data before committing to them.

My aim in the interviews was to develop a progressively tighter focus on the activity of the mathematics classroom experienced by these subjects, as it related to their position in the activity. Interview schedules thus first probed participants' primary school and early secondary education across all subjects, asking about the infrastructures they were used to and the teaching they had encountered. We then discussed the characteristics of different subjects within secondary school before focusing on mathematics lessons and

mathematical activity within them. A research instrument developed in a pilot study (Vosper Singleton, 2007) was used to elicit from participants the characteristics which might make an item or action 'mathematical'.

The departure of two of the original pupil participants presented me with the question of how to manage their data. Understanding that in group interviews their contributions had helped to shape those of others, I was reluctant to discard their data. Consequently, I used their input in establishing a picture of the generic activity of the mathematics classroom, but did not develop threads of analysis focusing on each of the two individually, as has been done for the other participants. Where their data contributes to the description of broad patterns of activity it is retained and reported here. In the latter stages of the data collection, interviews were held with individual participants. These had similar but not identical interview schedules, the variation a result of the need to check interpretations of earlier interviews and to pursue certain ideas further. Throughout the interviews it was imperative to question the aspects of school life the participants took for granted, and expose the knowledge they expected me to take as shared.

The hiatus in the project interrupted the data collection and formal analysis, but also provided a space in which to reflect upon the work I had done thus far. Standing back from the literature and empirical methods enabled me to reconnect with my aims of understanding the construction of mathematical capability and being able to speak directly to teachers on this issue. Reviewing the historical and philosophical background of CHAT enabled me to bring these aims together in a focus on the acting pupil, as they are produced in the classroom. When returning afresh to the data, I was able to give greater prominence to the pupils' voice in my analysis, which resulted in a significant shift in my analytical approach (see §6.5).

The hiatus coincided with the pupils' year 9, when the IGCSE course formally began. I was not able to investigate whether this resulted in any immediate change in classroom activity. I was also not able to track any gradual change in participants' contributions, as I had intended. When the project resumed my teaching commitments precluded any

further lesson observations and further limited the time available for interviews. Before conducting any more interviews I brought my analysis of the data to the point where I had extensive descriptions of classroom activity from each participant's point of view, including historical descriptions of their experiences, and had established a strong set of focused codes and their interrelations (see §6.5.2). I was then in a position to focus the final interviews on those points which required clarification and further exploration for differences as a result of time. In these I drew upon the historical descriptions, and asked participants to give summative descriptions of their current classroom experience. These descriptions came at the culmination of a long period of interaction between us and so discrepancies which may have arisen as a result of respondent validation had to be judged as corrections or the result of development in the activity.

These final interviews (occurring toward the end of the participants' year 10) offered an opportunity to challenge or support the structure of codes I had established, and to identify any change in the activity. Rather than tracking gradual change I would be able to compare two different states of the activity. These interviews contained wide-ranging questions about the nature and purposes of classroom mathematical action, and included discussion of a typical textbook problem and a set of recognisable everyday tasks. The problem was chosen from the pupils' year 9 work in order to pre-empt any issues caused by difficulty, but involved structures (trigonometry) which they were unlikely to have used outside school. Discussion of the problem and tasks shed light on the relationship between mathematical actions and operations, and the meanings sustained by these.

6.5 Grounded theory

My aim in data collection was to develop a picture of the mathematics classroom as an activity system, seen from the subject's point of view. To do this I had first to outline their school experience, and learn what distinguished mathematics lessons from those of other disciplines. In trying to assemble a CHAT description of the mathematics classroom for the subjects, I first applied the framework to classify the data.

Transcripts of interviews were initially coded following the “buckets” method for arranging data (Barab et al., 2004). I applied a line-by-line coding process, determining which node of the triangle was most pertinent. However I found that by following this process I had fallen into precisely the trap cautioned against. I had atomised the data into fragments which I could have assembled by applying my own pre-existing understanding of the classroom activity, and thus were offering me no new insights. I also felt as I read the coded data that I had accessed very little that was answering my questions. A lot of the data seemed irrelevant or unclassifiable, and the extent to which elements of classroom activity were co-constitutive and the means by which the subjects’ access to the object was mediated by these elements was difficult to discern. Evidence of the dialectic development in subjects simultaneously creating and accessing the object was also difficult to make out. This led to a halt in my analysis as I tried to intuit the source of this problem.

I stood back from the analysis and reflected on what had been achieved. After a frustrating period of doubt I realised that when telling me about the classroom, the participants had been talking about what was important to them. I had to listen to what they were telling me and not try to make their responses fill the “buckets” I had prepared. Simon (2003) writes that determining an appropriate methodology is problematic and “requires modifications in response to the nature of the data and the questions that drive the analysis” (ibid, p.161). Beswick *et al.* (2007) had identified the difficulties in determining the fit of data to the rigid framework of activity theory, and used it to cast light on the affordances of the theory. However, viewing my research as an activity in its own right, I saw this difficulty of ‘fit’ as a disturbance which could lead to development of my own activity through the adaptation of the tools I used. This shift in approach was a change in method as well as in methodology: adopting a grounded approach to the data (Charmaz, 2001) enabled me to establish a new perspective on the activity system, revealing what it looked like to the subjects, and also cast light on the use of the activity theory framework.

Reviewing the data from a grounded perspective revealed a very different understanding of the classroom as a social space. A significant part of pupils' talk referred to the establishment and maintenance of social relations. Taking a grounded approach revealed that the action I had observed had primary meaning for the participants in terms of interpersonal relations, constituted by exchanges of information and evaluation which reinforced the roles of teacher and pupil. These social relationships within the power structures of school set the scene in which mathematical development could occur. This echoes the observation made by Jaworski (1997), who noted the impact of classroom ethos and social interaction on the development of mathematical understandings.

With this established I was able to access more clearly the interplay of subjectivity and structure in the mathematics classroom, and get closer to the subjects' constituted relationship with mathematics. The work I had done previously considering how the object is created and developed in the classroom action became more tangible and the mutual development of subject and object could be described more meaningfully and with concrete points of reference.

6.5.1 Developing methods of analysis

Starting afresh, open coding was applied to the transcripts of the first five interviews (see Appendix 2.2.1). These initial interviews offered a breadth of data and access to all the participants: within them, four of the final participants were interviewed twice, and two once¹¹. Charmaz recommends keeping codes 'active' in order to describe action, reveal the structures that actions and statements take for granted, and explore how "structure and context serve to support, maintain, impede or change these actions and statements" (Charmaz, 1995, p. 39). This process yielded around 70 active open codes, which I then examined for their contextual support and connection. As the codes were explored for their logical or causal connections patterns and clusters of similar responses began to

¹¹ The first three interviews were open coded again approximately six months later, as a reliability check. Between 78% and 85% of codes agreed.

emerge (Miles and Huberman, 1994) (Appendix 2.2.2). For example, 'studying for a test', 'being examined', 'changing routines in anticipation of SATs' and 'getting SATs results' were brought together as 'Studying For'; descriptions of teachers giving instruction to pupils were categorised as either 'Teacher Directing Pupils' or 'Teacher Responding,' depending on whether the action had originated with the teacher or in response to the pupils' actions. Drawing this distinction within the teachers' actions enabled me to see the means by which pupils could influence the teachers' behaviour. This resulted in the emergence of the category 'Pupil Directing Teacher,' comprising the codes in which pupils had instigated action on the teachers' part.

Allowing these resonant data to coalesce led to the emergence of a set of 21 categories which described the action of the classroom. It is not asserted that these categories were mutually exclusive or exhaustive. These categories were then taken to the transcripts of the remaining 22 interviews as purposive codes. Once this coding had been completed, I gathered together the data from all interviews for each category (see Appendix 2.2.3), in order to articulate the conceptual relationships between the actions subsumed under each category. I drew up and filled in descriptive tables, answering these questions for each category, derived from (Charmaz, 1995, p. 39).

1. What process is at issue here?
2. Under which conditions does this process hold?
3. How do the research participant(s) think, feel, and act while involved in this process?
4. When, why and how does the process change?
5. What are the consequences of the process?

These tabular descriptions (see an example overleaf) were then re-written as summary descriptions of the code. This tabular process enabled me to further refine the focused codes and categories, as an aspect of the 'constant comparison' method (Glaser and Strauss, 1967). This led to the rejection of some codes. For example, 'sharing experience', 'individual experience' and 'making judgements' were rejected as they lacked specificity,

Category	Teacher responding
Processes involved.	<p>This code is related to PDT. Pupils make requests of teachers for explanations and assistance in solving tasks, but the way in which the teacher responds is not automatically determinable. Teachers also respond to pupils in less immediate ways, offering feedback on their actions through reviewing work and marking it, then returning scores and comments to the pupils. Teacher responses might not be purely factual or evaluative, but also incorporate punishments and rewards. The enforcement of behavioural norms through the regulative discourse, as well as mathematical expectations are in part declaratory, in part responsive.</p> <p>Teachers collect and review work, and make comment on the pupils' short-term and ongoing achievement. Classroom arrangements may be as a result of a teacher responding to the behaviour of the pupils - as seen by the pupils.</p> <p>Teachers correct errors, and explain mathematics through correcting misconceptions and marking work.</p>
Conditions	<p>TR takes place as a recognition of PDT, and thus may involve addressing behaviour; it also refers to the communication from teacher to pupil in which the pupils work is discussed. This pairing recognises the difficulty of disentangling the regulative and instructional discourses in school action. TR may be to recognise achievement, or to direct pupils to certain tasks, depending on their previous actions or achievements. For TR to apply as a code in this case, the pupil needs to be conscious of the responsiveness of the teacher's actions. Because of this, conversations between teacher and pupils may be either classified as TR or TDP. The distinction is whether the pupil sees that the teacher is responding to something the pupil has said or done, or whether the teacher has initiated the interaction.</p>
State during involvement	<p>When this process takes place, the pupil has made some demand on the teacher, or the teacher's role has determined that they should respond to the pupils' achievement. Hence the pupil is usually placed in a state of attention and expected to make effort to interpret and reflect on the information the teacher is giving them. How the dialogue proceeds would appear to remain largely in control of the teacher, the pupil is expected to still observe the over-riding relationship norms of the school situation. S/He may decide that a short answer or more in depth questioning are appropriate. (Written feedback may also be of variable depth and quality.)</p>
Process change?	<p>MF reported that TR changed when he had a teacher who "really had it in for me... We kind of didn't listen to each other anymore." He does not explain how this situation arose, but the change in communication between them then resulted in the teacher communicating with his parents, which he described as complaining. The pupil was held to account for the change, rather than the teacher. It would appear the teacher is held to be the consummate professional, acting out their role, whilst the pupil is the unformed and fallible human being, who is nonetheless held accountable for their actions.</p>
Consequences	<p>In principle the consequences of this process are that the pupil develops both as a student of the subject and as a human being, through the I and R discourses. However, as MF demonstrated, if the process fails or changes, it can have negative effects on the pupil's attainment and consequent ranking amongst their peers. This in turn can effect self-esteem and motivation in a self-reinforcing cycle. The manner in which teachers exercise their responses to pupils have longer-term effects on pupil affect, as demonstrated by JS explaining setting: a simple academically-motivated decision can have large labelling effects on the pupils.</p>

Table 5: Tabular articulation of developing category (example)

and were replaced with more precise codes detailing events in terms of mathematical actions and communal interactions. The above example was re-articulated in more narrative form, as below:

Category: Teacher responding (TR)
<p>Teachers do not only command attention and direct pupils' action in the classroom, but also respond to the demands of the activity system. They respond to pupils' explicit and implicit requirements, and also to the structure of the system in which they work.</p> <p>The ways in which teachers respond to pupils often fall into routine or recognisable patterns of behaviour. Work produced by the pupils needs to be marked, and the marking of work involves rewarding approved methods with credit. One teacher was reported as using coloured stickers with messages on as a means of alerting pupils to a need to speak to them or to reward them for 'good' work. Praising work or good behaviour was one way in which teachers responded to pupils, which made the pupils 'feel good about themselves'. In expositing to the class and in marking work, teachers were reinforcing the necessary methods to be shown. Teachers felt that in demonstrating the work to the pupils they were also demonstrating the effort required. This came into their individual explanations as well.</p> <p>Explaining or helping a pupil fell into several distinct patterns:</p> <ul style="list-style-type: none"> • Elucidating textbook explanations • Reiterating an explanation previously given to the class in more detail, or at a slower pace • Giving detailed answers to queries • Giving brief correct/incorrect feedback • Explaining the method required to solve a specific problem • Helping with a general method, in relation to a problem • Taking a pupil back to basics <p>In many instances it was noted that a pupil may initiate a conversation with the teacher on an issue, but then control of the conversation would be assumed by the teacher. They would ask questions, and the pupil would give answers. This was contrasted with just 'asking a friend' who will tell you the right answer. A teacher is not likely to do this, but rather will run through the deductive steps in solving a question until the sticking point is reached. Participants stated this was not always welcome, as they were made to reiterate their thinking as if being reprimanded, and that they knew the method, but wanted help with just one small point. Teachers stated that this approach was not only intended to get the pupil thinking through their method again and thus put them in a stronger position in terms of spotting their own mistakes, but also to act as a diagnostic tool, so the teacher could decide what needed to be explained. In discussing mathematics with pupils teachers reported modulating their language to fit the individual's understanding, and pupils reported teacher would encourage them to attempt problems which seemed difficult.</p> <p>A common mechanism for attracting the teacher's attention was to raise a hand in the air. This became a focus of discussion in one interview; the participants reported that to attract attention in other ways was likely to elicit a reprimand. If a number of pupils were to make the same query, then the teacher may halt the action of the class to address all those who needed help on the same issue. Sometimes teachers would use this structure to review a topic area, in light of a test or piece of work. Trivial queries (defined as such if they refer to a mathematical skill or fact which</p>

was considered elementary in the context of the current task) would be dealt with by telling a pupil to think it through or work it out for themselves, or to use a calculator for basic number problems. Sometimes pupils would be directed to refer to the textbook for an explanation, or in the case of English, to a dictionary. Participants understood this to be part of teaching them how to help one's self, without relying on the teacher.

The use of classroom equipment also became implicated in the ways teachers respond to pupils. Working on PCs was cited as a means of rewarding pupils for good work or behaviour (a 'treat'), but also brought into play different routines of observation and supervision, as noted by the participants. More mundane equipment (books, mathematical tools) was lent by teachers to pupils when they had forgotten to bring their own.

When pupils were asked to contribute to class discussions, teachers selected pupils to speak from those with their hands up (or not, in some cases), elicited explanations, and evaluated their comments and ideas, offering corrections where necessary.

Teachers' responses to pupils were also directed to reinforcing ordering and working norms, and as such illustrated how teachers themselves were constrained by these norms. If pupils acted 'out of line' teachers would make threats of punishment, or indicate how a hypothetical punishment may be administered. In some cases these might be whole-class or individual detentions, or might be just reprimands. Shouting out, forgetting equipment and being late were cited as likely to be met with a reprimand of some sort. At times teachers prohibited discussion and directed pupils to work in silence. Sometimes pupils would be deliberately ignored, suggesting a teacher anticipate their query would be vexatious. It was reported that teacher 'get cross', although this might refer to being told off, rather than the emotional state of the teacher. In an extreme case one participant reported a teacher contacting his parents to address behavioural issues. One teacher reported it a priority to establish a positive working atmosphere in the classroom before considering the mathematics, and participants reported their drama teacher instituting calming exercises before pupils were permitted to leave for a different lesson. In English, a teacher would use a fun activity to finish a 'reading lesson'. Motivating the pupils through varying activities was not uncommon. Teachers would also create tasks to aid memory.

Teachers also responded to the institutional constraints and affordances in their planning and execution of lessons. When pupils had been timetabled two maths lessons in one day, the teacher planned that the second lesson should be spent in the computer room. Taking a view of the development of the class as a whole was instrumental in whether or not the class moved on to a different topic. This was noted particularly in primary school, where teachers spent a lot of time on a single topic, "because nobody was very good at it". An individual's previous attainment also influenced teacher's decisions as to what work they should undertake, sometimes through the allocation of setting, but more frequently in the administration of extension work or a direction to aid pupils who had not finished the work so quickly. As such teachers were expected to have such work or strategies prepared. Changing tasks in light of group attainment was also reported; raising the bar of challenge for the pupils to meet. Sometimes teachers would enlist extra help in the classroom, in order to better aid the pupils.

Participants reported that if they questioned the purpose of their work they may be likely to receive a reprimand or punishment, although it was acknowledged that this may be as much to do with the manner of the query as the content. However, it was very unclear as to whether teachers did ever explain the purpose of what was being studied, either at the level of the discipline or just a specific exercise.

Table 6: Summary description of category (example)

I then used the data and the open coding to connect the summary descriptions with CHAT. Charmaz (1995, p. 38) contends that one can take a grounded approach to data even when coming from a particular theoretical perspective, with the caveat that “If you apply concepts from your discipline, you must be self-critical to ensure these concepts work.” In order to maintain a critical perspective, the open-coded lines were grouped according to reference made to resources used; classroom routines; interpersonal relations and expectations. These groupings corresponded to the CHAT concepts of tools, rules, division of labour and community, respectively, although the correspondences were kept deliberately loose at this stage. Data were used to cast light on the meanings of those concepts, as I made connection between the concepts and the grounded categories. These connections were then used to supplement the summary descriptions. It was important that the CHAT concepts were not applied exhaustively to the data or codes, with each category being atomised in terms of the six nodes of the EMT. Rather, the concepts of CHAT were introduced as the data appealed to them as a means of articulation (see the re-articulation of *Teacher responding* below).

Category: Teacher responding – CHAT terminology

<p>Teachers’ responses to pupils were not guided solely by the aim of providing pupils with the information they needed to progress on classroom tasks. Considerations of the activity system result in additional aims being imposed upon the teachers and pupils, with long-term and communal goals taking priority over the pupil’s immediate request. This negotiation of priorities reflects the tensions in the classroom.</p>
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<p>The routine patterns of teachers’ responses served multiple aims of encouraging participation and engagement: marks awarded by teachers might be related to the pupils’ object of study, but comments, stickers and praise were used a tools to sustain motivation, and encouraging pupils to identify their progress and develop their relation with the object. Giving the achievement positive social meaning could increase pupils’ perceived standing in the classroom community.</p>

<p>Responses to pupils’ mathematical queries fell into recognisable patterns:</p>

- | |
|--|
| <ul style="list-style-type: none"> • Elucidating textbook explanations • Reiterating an explanation previously given to the class in more detail, or at a slower pace • Giving detailed answers to queries • Giving brief correct/incorrect feedback • Explaining the method required to solve a specific problem • Helping with a general method, in relation to a problem • Taking a pupil back to basics |
|--|

<p>In assuming control of the conversation over the pupil, the teacher imposed their aim on the</p>

pupil. This depended upon the teacher having quickly diagnosed the source of the problem and relating this to the object of study. However if the problem originated in a failure to meet the *Ordering* or *Working norms* (not listening to an explanation, not having produced work correctly, failure to complete a previous task or have the right tools, etc.), addressing this point would take priority over the mathematical issue. The rules and expectations of the classroom were used as tools in relation to the aim of getting pupils working productively and in co-ordination with each other and the teachers' plans. Even initiating the conversation was done using recognisable social signals (hands up, queuing).

When reiterating deductive methods, there could be differing understandings of the object of the conversation; a pupil's focus on attaining the answer might not reflect the teacher's focus on generalisable methods. The conversation became a task (action) in its own right, with tools of diagnosis and explanation coming to the fore – teachers and pupils used the textbook, written work, whiteboard displays in these conversations. Language would be adapted to the needs of the specific conversation, selecting particular means of describing mathematical structures and methods, in order to connect the pupil's understanding.

Teachers had the right and means to disrupt pupils' work, if they judged a query deserved everyone's attention, as in the case of a common or unexpected difficulty, they might halt every pupil's action to share the explanation. Conversely they also sometimes withheld explanation, if they judged a pupil would benefit from persevering without their assistance; this related to establishing and maintaining the Working and Ordering Norms, as well as instilling productive mathematical habits such as referring to previous problems and similar worked examples and appropriate use of calculation tools (calculators). Here I see the overlap of Working and Mathematical norms: what counts as "work" in the classroom is informed by the means by which one can develop mathematical skills and productive working habits. Question the purpose of a task was treated as undermining the activity, a challenge to all three norms.

Less immediate responses to pupils' behaviour and attainment were reflected in the use of PC work as a reward; this placed an emphasis on compliance in the classroom as part of the object of the activity. Reinforcing the Working and Ordering norms formed a prominent part of teachers' responses. Reprimand and punishment (or the threat of punishment) were tools used in this regard, when the norms were disrupted. However, by their prevalence in the classroom as the pupils learned and adopted the social expectations, they became part of these norms. Pupils expected to be corrected and directed by teachers. Teachers rewarded this compliance with fun tasks and praise. A positive working atmosphere was desirable, but teachers and pupils might have different expectations of the division of labour (how much work and when) that constituted this. The focus on social relations by teachers might be seen as a means of "sweetening the pill" in motivating pupils to do work.

The position of the teacher in the classroom community entailed responding to the constraint and affordances of institutional structure and curricular demands. Teachers had to be adequately resourced (have sufficient tools) to meet the needs of pupils progressing quickly or falling behind. They also had to manage the time allocation and ensure pupils' attainment within set periods of time (managing the temporal division of labour in relation to the curriculum).

Table 7: Introduction of CHAT terminology to description of category (example).

Through developing the categories I was able to articulate further the “implicit, unstated and condensed meanings” (Charmaz, 1995, p. 43) which were shared or taken as shared by the participants in the interviews. This enabled me to make tentative interrelations between the categories which could then be explored. However, with 21 categories to explore, there were potentially over 400 pairings to examine. This was not feasible and so I worked with the codes to look for the strongest connections, in terms of shared data and logical connection, based on the interrelations that had been revealed by the analysis thus far. In order to make sense of these I used a diagrammatic method (Miles and Huberman, 1984) to first sketch them out (Figure 8, p. 146). This diagram produced 45 initial pairings of codes to explore.

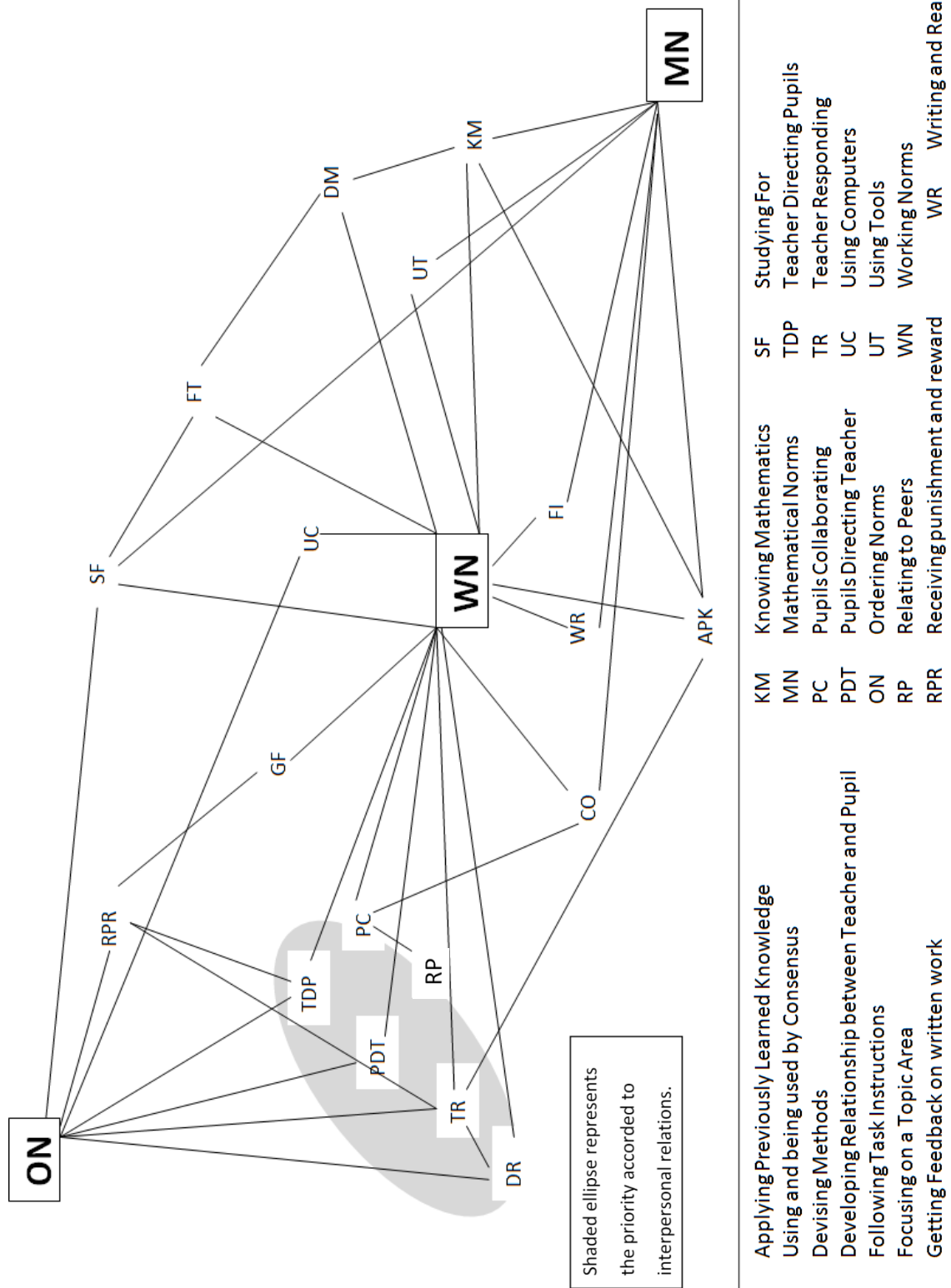


Figure 8: Interrelation of focused codes

6.5.2 Relationships between codes and theory

Having established a set of categories ‘from the bottom up’, it was necessary that their interrelations were developed in accordance with the CHAT framework. In articulating the summary descriptions with CHAT vocabulary I was able to foreground the dynamism and dialectic of the classroom and to reveal tensions. As Willis and Trondman (2002) note, “the nitty-gritty of everyday life cannot be presented as raw, unmediated data... nor can it be presented through abstract theoretical categories” (p. 399). They propose that the best form of the theory/data relation lies in generating moments of ‘surprise’ that one can bring to the other. They propose working towards a dialectic of surprise, in order to produce research that further informs theory and serves an interest in cultural policy and politics. My surprises had begun with pupils’ intense focus on the social (and the tensions this generated), and continued with the structure of categories (Glaser, 1992) that emerged.

I explored the relations between the categories by returning to the interview data; taking each pairing in turn, I identified the data coded by both categories, from all interviews. I then wrote memos describing the relationships revealed by these shared data (see Appendix 2.3.1). These memos were written by drawing upon the vocabulary of CHAT, without seeking to impose the EMT upon each relationship. These memos also drew upon my understanding as a practising teacher, and thus became a site in which I could articulate questions that would be used in the subsequent analysis, and initial identification of tensions.

In developing the relations between the categories, three of them emerged as axial codes, offering the greatest explanatory power. These codes were named *Ordering norms*, *Working norms* and *Mathematical norms*. My use of the terminology “norms” recalls the observation of “sociomathematical norms” (Yackel and Cobb, 1996; McClain and Cobb, 2001), which determined classroom actions that would be recognised as mathematically productive. These norms were seen to regulate argumentation in the classroom and influence learning opportunities. In my research, the *Ordering Norms* were identified as those normative actions which corresponded to the orderly maintenance of the

community. As such, they appealed largely to the Subject, Rules and Community nodes of the EMT. The *Working Norms* were those actions which were treated as appropriate in relation to classroom work. These related to the same nodes, and also the Tools and Division of Labour nodes. Finally the *Mathematical Norms* appealed largely to the Subject, Tools, Division of Labour and Object nodes. These norms related to those actions which were validated in the classroom as appropriate for dealing with pattern and structures, particularly in number and shape. The memo describing the *Ordering Norms* is given below.

Axial Code: Ordering Norms
<p>This code refers to the behavioural structures which order pupils' and teachers' participation in the activity of the classroom and school. These structures are an inherent part of the pragmatics of classroom activity, but depend upon the relative power positions of the protagonists for their establishment and maintenance. As was seen, however, the workings of the norms reflect back upon those positions within the power dynamic and serve to reinforce them. The regulative norms may bear little direct relation to the object.</p> <p>The emergence of the ON code was signalled within the first sentence of the first description of classroom activity in primary school: the action was described in terms of what the teachers did, with the pupils as passive recipients of the teacher's actions. The ways in which teachers controlled aspects of classroom activity were multiple and extended into all areas. In its most general aspect, teacher control was exercised in deciding what would be studied, when, and in what manner. Pupils had no say in this. This situation formed the basis for the relations and division of labour in the classroom, and was an unspoken shared understanding between the participants. Teachers had licence to control pupil participation in lessons, by choosing who could contribute publicly and who should listen, and in dictating what sort of task pupils would undertake.</p> <p>The ON around the use of various resources in the classroom also reflected teacher control. Teachers were described as using access to PCs (for a variety in working mode) as a 'treat' for good behaviour. As part of the ON, pupils were expected to bring certain equipment to class: it was explained how teachers responded to pupils failing to take responsibility for bringing classroom equipment, with reprimands or even financial penalties issued for forgetfulness or loss. Control of access to teaching materials was also part of the teacher's role in the RN: worksheets would be handed out at a time of their choosing, for example, or pupils given access to their textbooks when the teacher had finished speaking.</p> <p>In the wider school arena, the ON encompassed the allocation of pupils to separate classes dependent on their prior achievement ('setting' was referred to on at least five different occasions within the conversations). Freedom of movement around the school was also noted, in contrasting school experiences. Certain activities were discussed which were noted not to be suitable for school, specifically gambling, although this prohibition was swiftly attached to a specific senior teacher. Exceptions to normal routines (watching football World Cup matches) were controlled by the school, rather than the class teacher. The school's provision of a</p>

curriculum encompassing several subject areas, which all pupils were expected to work in was seen as a means of enabling pupils to discover their strengths and weaknesses. Sometimes school routines would become part of class life – preparing an assembly when it was a form group’s turn was mentioned.

The ON can be concretised in the set-up of the classroom, with the layout of furniture influencing patterns of behaviour and enabling different forms of access to other parties in the room. As a consequence, changes in the layout of rooms can offer opportunities to disrupt or evade the ON. The most distinctive way in which this was reported (and observed) was the different layout of the computer rooms, where supervision was not so easily maintained and pupils could communicate with more of each other more easily.

Disruption of the ON was indicated by participants to be part of the ‘game’ of classroom activity: they freely talked about the ways they might try to evade supervision, and at the same time recognised that it was the teacher’s role to enforce it. It was noted that teachers work at maintaining ON and supervision, and that their success in doing so depended on their character and the degree to which pupils respected or feared them. Discussion of the teachers policing the ON came to the fore at various points in the interviews, with variations in this seen as indications of the individual teacher’s personality. As noted, participants treated the disruption of ON as part of school life, and it was hinted that pupils test new teachers in this. Evading supervision was considered usual, and a teacher needed to be strong or strict in enforcing ON across the board, until they identified the “troublemakers” (Jamie). Once a disruption in a lesson had been initiated, there could be a chain reaction of further behaviour which pushed against RN, suggesting the ON is maintained largely by mutual consensus. Some pupils might even exploit a teacher’s responsibilities by pretending they did not understand some mathematics, in order to create entertainment. If a teacher was suspected of misattributing blame for an incident or treating a pupil unfairly, they ran the risk of undermining themselves, just as if they failed to carry out a threat: pupils would react against this “miscarriage of justice” (Aaron). Teachers might use displays of negative emotions or anger to enforce the ON.

Focusing on routines in school and class life helped to reveal the ON in action. The teacher was responsible for establishing routines for the class, and it was expected by participants that the teacher would determine the working norms for a group. Even breaking routines or establishing new ones was part of the RN: setting more homework in the run up to key exams (SATs) was cited as an example of when a temporary change in routine was instituted. Changes in routine such as changes in the working norm of a class (working in groups rather than individually) also prompted disruption to the ON [“we always take that as an excuse to be really loud”; Thomas, working in computer room]. Participants showed awareness of an association between behaviour and consequences, particularly in the case of pupils breaking ON, which would be followed by a punishment or reprimand from the teacher. Teachers might indicate that a breach was possible and try to prevent it with the threat of punishment. In some senses, the ON extended to even the smallest of actions: classrooms had approved means of attracting a teacher’s attention or initiating a conversation; the traditional ‘hands up’, or forming a queue at the teacher’s desk. Aspects of classroom life which teachers controlled or established standards for also included entering and exiting classrooms, seating allocations, the temporal division of labour in lessons, the homework routine, when pupils could access the teacher, what types of work were set, when pupils attended to discussion and when they could write examples down.

Teachers would sometimes prohibit further discussion or for pupils to ask questions if they felt the pupils were trying to disrupt or the time had come to work. At the end of 'active' lessons participants reported teachers would give the pupils exercises to restore order, by calming them down. Control by teachers extended into conversations about class work as well. Once a conversation had been initiated by a pupil, focused on a particular problem, the teacher would have an idea of what they wanted to say (M), and would then construct a Socratic dialogue to bring the necessary point to the pupil's awareness. This might entail going back to the beginning of the problem, but the participants felt that in this situation they were prohibited from interrupting the teacher, as that would be rude, and the teacher was the one in charge (John), even though this could have made the conversation more efficient and dealt with the pupil's actual, rather than anticipated query. A similar tension was seen in the practicalities of classroom life: the 'hands up' routine might mean a pupil felt as if he was waiting for a long time before his issue was dealt with.

The working norm is more closely related to the object than the ON, but if the working norm is disrupted, this places the teacher in the position of enforce, thus acting out the ON. The ON becomes embedded in the expectations pupils hold of their school life, as hinted at in various places above. Unspoken understandings included the role of setting, which was seen as necessary because pupils had to reach certain "standards" in each subject (Jamie). Participants expected teachers to enforce but also stick to ON, ensuring pupils were obedient or compliant, kept to a sensible volume, remained focused or 'on-task', avoided distractions, were punctual to lessons and put their hands up when expected.

Punishment and teacher/pupil relations emerged as central features of school life for the participants; this was echoed in a teacher's own recollection of school life (M). It appeared that participants recognised the purpose of ON, even if they did not always enjoy the consequences. To a certain extent participants said that pupils also policed the RN, particularly when they themselves wanted to avoid inconvenience or punishment, or were keen to gain reward. This assumed that the teacher could be relied upon to notice accurately what had happened in class (Jamie).

There was a difference in perception between pupils and teachers as to the degree of strictness in enforcing the ON.

Table 8: Axial code example (Ordering Norms).

Through the process of exploring the connections between categories these axial codes appeared with an order of priority in making sense of the activity from the pupils' point of view. The pupils' focus on social expectations and relations within the community prioritised the Ordering Norms, as processes within which these relations were established. Ordering Norms pervaded school life, and were broadly similar across all disciplines. Against this backdrop, the Working Norms represented expectations and behaviours that were largely common across disciplines and lessons. The Mathematical Norms distinguished mathematics lessons from other disciplines and illustrated the shared behaviours that were presented for pupils' appropriation. This order provides the

structure of chapter 7, in which the axial codes, categories and interrelations are elucidated. In chapter 8 a CHAT analysis is applied to this description: discussion of the relations highlighted by the concepts of the EMT leads to articulation of the tensions revealed.

I came to see the grounded method as a suitable methodology in an attempt to make the abstract concrete. By engaging in a constant-comparison approach which moved back and forth between my tentative theoretical vocabulary and the data I was able to come to a stronger realisation of the action of the classroom. Kaptelinin (2005), in reviewing notions of the object, insists that the relation of motives and objects determining an activity should not be taken for granted. Empirical studies should be open to clarification of this relation, in identifying the tools and means by which the object is accessed and transformed, and the opportunities for development which are thus opened up. I saw this as an appeal to investigate the extent of the overlap between the hierarchies of motives that may operate (Kaptelinin and Cole, 1997). The pupils' experience and data served as concrete instantiations of abstract notions of classroom action: together these concretised the objects of and in my research. The rigour in this research process has come through the constant comparison method, and continual reflection upon the connection between the social process of engagement in the field with the technical processes of data collection and the decisions that that connection involved (Ball, 1990). Concepts have been continually tested against the data, whilst the data has been interpreted through the processes of its collection.

6.5.3 Managing data

In writing this thesis I had to make decisions regarding the inclusion of verbatim data. Charmaz (1995, p. 47) explains, "Grounded theorists generally provide enough verbatim material to demonstrate the connection between the data and the analysis, but give more weight to the concepts derived from the data" Consequently verbatim data is included when explicating particular details of the grounded codes, and when illustrating analytical points made in the activity theory analysis. Within the results of the grounded analysis are also presented vignettes of classroom activity, resulting from the lesson observations (see

Table 9, p. 153). These are offered to give a flavour of the activity as observed and retold by the participants, demonstrating the mathematical tasks in which they were involved and illustrating the expectations and norms which surrounded these. These vignettes were distilled from my observation notes with the benefit of the pupils' information to reveal the salient characteristics of events. In this, I have attempted to connect my analysis with a "low inference" representation (Wragg, 1994) of the pupils' experience.

In this analysis, I make no claims to have produced an exhaustive understanding of the activity system as apprehended by the six full participants, but have aimed to produce a picture which reveals the action and contradictions of the observed classrooms. By doing so, I could explore the potential for development in the system and subsequently the nature of the object. A significant restriction on this research was the opportunistic access to only a limited number of classrooms, in just one school. My intention is not to extrapolate conclusions to all classrooms, but to develop a means of looking at the classroom with which I could engage in discussion about teaching practice, based upon similarity of experience:

Despite their diversity, individual classrooms share many characteristics. Through the detailed study of one particular context, it is still possible to clarify relationships, pinpoint critical processes, and identify common phenomena. Later abstracted summaries and general concepts can be formulated, which may, upon further investigation be found to be germane to a wider variety of settings.

(Delamont and Hamilton, 1984)

Observation	School year: Month	Vignette
R01	7: June	1 - The 100 Club (p. 159)
R02		2 - Angles in parallel lines: exercise (p.162)
R03		3 - Flight paths: bearings task (p.165)
R04		4 - BBC Bitesize (p.168)
R05		5 - Nets of 3-D shapes (p.171)
R06	8: November	6 – Constructing triangles (p.174)
R07	8: January	7 - 24-hr and 12-hr clocks: exercise (p.180)
R08		8 - Speed-distance-time: exercise (p.183)
R09		9 - Enlargements from a centre (p.185)
R10	8: February	10 - Similar shapes and enlargement (p. 187)
R11		11 - Test (p.189)
R12	8: March	12 - Scale factors and area (p.192)
R13		13 - Revision lesson (p.194)
R14		14 - Revision: circles and cylinders (p.198)
R15		15 - Conversion of units; metric and imperial (p.199)
R16		16 - Laws of indices: exercise (p.203)

Table 9: Schedule of lesson observations and resulting vignettes

6.6 Developing question narrative

The experience and reflection upon the data and analysis was not considered complete with the adoption of grounded methods. Throughout the remainder of the research, I continued to examine my methodology for its appropriateness and effectiveness, whilst remaining conscious that my methods would have a formative effect on any conclusions I might reach. Consequently, my questions took on a stronger methodological aspect. To the pairing of:

How do social interaction and tool use in the mathematics classroom position pupils, in relation to personal aims and cultural categories, to locate and value meaningful mathematical action in the world, constituting personal and practical transformation?

How can I best implement a methodology to trace dialectical subject-object development and contradictions in classroom activity using the framework of third generation CHAT and Marxian categories?

I added:

How can a methodology derived from CHAT and developed in the activity of research equip me, as a teacher-researcher, to discuss teaching practice with a focus on the transformability of subject and object?

7 Grounded analysis and description: the pupils' viewpoint

In this chapter I detail the results of the grounded approach taken to the data in my analysis, offering a “thick description” (Cohen et al., 2000) of the classroom. The hierarchy of explanatory power and dominance that emerged between the axial codes (Glaser, 1992) (Figure 8, p. 146), is used to structure this chapter. In §7.1 I recount the participants’ descriptions of the maintenance of order in school life (coded as *Ordering Norms*) and how these were interpreted by them in light of social relations. §7.2 details the *working norms* which were sustained in the mathematics classroom and §7.3 examines the *mathematical norms* established by productive participation in work. These three codes were not mutually exclusive divisions of the content of participants’ descriptions. Rather they are complementary aspects of the activity, relating to the multiple purposes and interpretations at play in the classroom. Consequently some features of classroom activity are revisited in the description and analysis, as they are treated within the different codes. Reflecting the emphasis placed by pupils on social relationships between individuals (before institutional responsibilities), I begin §7.1 and §7.2 with the descriptions of how teachers and pupils communicated and interrelated. In §7.4 I briefly review this presentation before reconsidering my research questions.

In this chapter, verbatim data are presented from across the pupil interviews in order to illustrate the concepts presented in the grounded description and demonstrate the connection between the data and the codes (Charmaz, 1995). To a certain extent data arose in an order reflecting the hierarchy of axial codes in my analysis: as I became better informed of the activity, I was able to direct the interviews more toward explicit discussion of mathematical matters, whilst tracing the effect of the *Ordering* and *Working Norms*. Where particularly emotive descriptions are presented, they originate with the participants, and are indicated as such. Through this presentation I aim to represent the “lived experience” (Charmaz, 1995, p. 47) of the mathematics classroom and “keep the human story in the forefront of the reader’s mind” (ibid). The vignettes are presented chronologically, at the end of each subsection, rather than in relation to specific codes or comments. Salient features of classroom action that emerge in the description can be

traced within and across all of these. Where particularly pertinent illustrations can be made, these are referenced within the description.

The picture presented here is an aggregate of the participants' contributions, with the aim of presenting the breadth of experience in the classroom; some observations might appear contradictory, although this is reflection of the multi-voiced nature of the activity, and is preserved in order to preserve "breadth, complexity and richness" (Denzin and Lincoln, 2000, p. 5). I reserve exploration of the data for chapter 8, in which data from teachers is included to complement that from the pupil participants in constructing the CHAT description.

7.1 Order in school life

Life in the school was structured by the imposition of order upon pupils' behaviour by teachers, across all disciplines, inside and outside lessons. In this section I first explain how the teachers directed pupils' behaviour, and then how pupils in turn directed that of teachers. After considering the teachers' responses in managing the pupils, I discuss how pupils' experience of the curriculum was structured. I conclude with a summary of the pupils' relations with their teachers. Observations in this section are drawn from lessons across disciplines, as the maintenance of order pervaded all, and the participants identified few distinctions with mathematics lessons.

7.1.1 Teachers directing pupils

The role of the teacher encompassed directing pupils' behaviour in maintaining standards of good behaviour in the spatio-temporal structure of school life. The *ordering norms* of the school varied little from the primary school experience of the participants: pupils' behaviour during their free time was monitored by teachers to keep it within school rules, although there was less restriction on movement and play than had been experienced previously. In all schools, exceptions to normal daily routines, such as sporting events or preparing assemblies, were determined and managed by the class teachers. However, there had been little movement between classrooms or teachers for different lessons beforehand; pupils were previously usually taught by the same teacher in the same room

throughout a given academic year. At the school, pupils frequently moved around the site to rooms specific to each subject, where teachers would be waiting for them. Teachers would also monitor the neatness of pupils' uniforms, prioritising this to the extent that they would interrupt conversation to address a pupil who was not dressed appropriately.

Pupils' behaviour was influenced through the layout of classrooms, as determined by the teachers. Desks were commonly arranged in broken rows to face the interactive whiteboard (IWB); this was the pattern for all mathematics classrooms. The focus of each mathematics classroom was a space occupied by the teacher, from which unobstructed communication could happen and supervision of the class was straightforward. Some other classrooms had 'horseshoe' layouts in which pupils faced inwards. In all rooms, the teacher's desk faced the pupils. Communication between pupils was partially controlled through these arrangements. In computer rooms, where pupils worked side-by-side observing each other's monitors, they would talk more and contribute to each other's work. Here a teacher would move around the room much more, in order to monitor conversations, see each pupil's screen and ensure they were doing the allocated task.

The routine pattern of lessons and the maintenance of order therein emerged as a consistent feature of participants' descriptions:

Aaron: well we come into the room well before that we line up outside the door

DVS: ok

Aaron: to ah to ah we wait until the teacher gets there and then we go in to the room and then we go into our assigned places which were given at the start of the term and then the teacher talks about what the lesson is a- what we're going to do in the lesson that really is dependent on what subject but if the subject- for example the history teacher gives us, tells us what to do and writes on the board what we're going to do and how we're going to do it etc etc and we just um we have to do it ... and uh then when the lesson's over, well after that we're asked to get our homework- if it's a homework day then we're asked to get our homework diaries out, and write in the homework which is always written on the board. Then we leave. You're dismissed and they usually go about it in a sort of that block are dismissed,

sort of you, you, you are dismissed, then you, you, you are dismissed and then that's about it.

(I005)

As well as the orderly management of the pupils during the lessons, teachers would also manage the manner and mood in which the pupils departed for their next lesson, ensuring the pupils were punctual and suitably attired. At the end of “active” (Thomas, I005) lessons such as Drama or P.E, the teacher would instigate cool-down exercises to restore calm.

For mathematics lessons pupils were expected to supply their own writing materials, rulers, compasses, protractors and calculators (hereafter, the ‘maths kit’) and to be responsible for bringing their exercise and textbooks to each lesson. Additional equipment such as small wipe-clean whiteboards was supplied by teachers as and when they saw fit, in relation to the tasks they chose. The decision when and whether to use PCs for work in mathematics lessons was solely in the teachers’ hands, as was the choice of software to be used on a given occasion.

Teachers controlled pupils’ participation in lessons, deciding what the pupils should be doing at different times and choosing those pupils who would publicly share their understanding in any class discussion. Questions could be asked when invited, but those which were judged to deviate from the point of the lesson would be curtailed. Teachers would judge when to cease discussion and direct the class to focus on their written tasks. Pupils usually worked individually, on their own versions of the same task, but would sometimes be given tasks to complete in small groups. Silence was rarely imposed in classrooms; quiet discussion on the tasks was usually permitted. The primary means of assessment for the teacher was through the pupils’ written work: collecting exercise books or sheets to mark formed an ordered part of the activity under the teacher’s management. Teachers would often lead the class in marking their own or each others’ work, in which answers could be verified and discussed, for the teacher to take in the books and review the written work at a later time.

Whilst the pupils enter the room, take their seats, and arrange their books and kit, W places sheets face down on each desk, repeating the instruction not to turn them over. When all the pupils are settled and attentive W explains that the class is going to do a “100 Club”, a task well known to the pupils (see Appendix 3.1). In this challenge pupils compete against each other to complete 100 times-table questions in the shortest time under five minutes. The questions are single-multiplication problems with numbers ranging from -12 to 12. W had judged that the class are not up to the expected standard of multiplication involving negative numbers and had made too many errors in this regard recently, so would undertake this task as practice. Once all the sheets are handed out, and upon W’s command (“On your marks. Get set. Go!”) they are turned over, and the pupils work on the 100 problems in silence. There has not been any recap of the rules of multiplication by negative numbers. Pupils write down answers to each problem, recalling or deducing the answers mentally. A stopwatch counting down the time is displayed on the IWB. Pupils make a note of the time if they finish all 100 problems before the five minutes is over.

When the time is over, pupils swap sheets with a neighbour, who marks their answers. W reads out all 100 answers; the pupils have to attend carefully to ensure they keep up. Once the sheets have been marked, W collects marks from the pupils. Going through the class register, each pupil calls out how many correct answers he has, and how long it took him. It is the practice to award a school commendation for 100 correct answers, and for improving upon a previous time.

7.1.2 A richer picture – pupils directing teachers

The above description is not given to suggest that school life comprised only the smooth running of routines, nor that pupils acted only reactively in response to the constraints of school rules and the delivery of tasks by teachers. Pupils often directed teachers’ behaviour by calling upon them for help, or behaving in a manner which disrupted the activity. Compliance with school rules and expectations was the normal but not exclusive state for pupils. Participants recounted tales of behaviour which had been deemed inappropriate and which resulted in action from the teacher to suppress it. Here I consider first those disruptions which were treated as minor by the participants, and then more significant ones. I then focus on how pupils negotiated their requests for help.

Pupils often inadvertently elicited an ordering response from the teacher in the midst of activity; disruption was not necessarily deliberate on a pupil’s part. The most common minor transgressions were: making an unacceptable amount of noise; not engaging with

the tasks given; talking to classmates, moving around the room or doing simple practical tasks (such as arranging their maths kit, sharpening pencils, etc.) when the teacher judged it inappropriate. Accordingly, restoring calm and quiet to a group, reminding pupils of expectations and addressing uniform issues were reported as part of the normal instruction from teachers to pupils. It was not uncommon for pupils to forget parts of their maths kit for lessons (particularly the protractor or pair of compasses) which often resulted in a request being made of the teacher.

Pupils expected well-defined tasks from teachers and clear instructions with which to comply. When working in small groups pupils would discuss ideas, but without a clear sense of what should be done or a strong personality to take the lead, they might not be successful in carrying out the task. Group work could become an argument over whose ideas to use, unless groups were closely supervised by the teachers. Difficulties arose in work if the expectations of the individuals were unclear, or the aim was not shared by all.

Trying to avoid work was part of the normal activity of the classroom; pupils would often rather talk to each other. When in a computer room, pupils could try to take advantage of the opportunity to play games on the PCs or access the internet. When doing this they tried to evade supervision by swivelling monitors out of the teacher's line of sight, or hiding behind them. Pupils often wanted to be able to see and hear each other in lessons in order to communicate. Managing this impulse was presented as a routine task for the teachers.

More serious issues, such as repeat offences or a persistent failure to meet expectations would elicit lengthier involvement from a teacher. Participants reported situations in which pupils had openly refused to follow instructions or to listen to the teacher when requested, or had tried to disrupt the work of the whole class. Incidents were noted in which this was sustained over a long-term period, becoming part of a pattern of interaction between teacher and pupils.

Testing how rigorously a new teacher would enforce order or "discovering the weak side of a teacher" (Thomas, 1005) was described as a normal feature of school life. Pupils could

derive entertainment from this, as if it were a game. If one class member seemed to be misbehaving without punishment or reprimand then other pupils would be likely to join in. Participants reported that disruptions had sometimes happened collaboratively in order to see the teacher become frustrated or lose composure, for example by pupils pretending they did not understand some mathematics. However, if pupils respected or feared a teacher (ZR, 1007) they would be more likely to adhere to classroom expectations.

Participants recognised that there was purpose behind the order of the classroom, even if they did not always enjoy the consequences of this regulation. Whilst disruption of order was part of the 'game' of classroom activity, it was the teacher's role to create and sustain order. It was explained that, when they wanted to avoid inconvenience or punishment, or were keen to gain reward, pupils also policed classroom order. However, this could result in further disruptions, if other pupils did not respect instructions to comply.

Asking teachers for help with tasks had to be done without disrupting classroom order. Pupils were expected to raise a hand and wait for the teacher to notice, or calmly walk to the teacher's desk and wait for attention. This could result in frustration if a pupil had to wait longer than they wanted, or if they felt their query could be dealt with quickly but were being overlooked. Occasionally queries were made which offered the teacher the opportunity to talk expansively about their discipline, taking in areas that were 'off topic' or beyond the scope of the curriculum. When they found the subsequent conversation interesting, pupils welcomed these disruptions. They also required attention when they had completed the set tasks more quickly than the teacher had anticipated. In this event, the pupil would usually let the teacher know so they could review the work and issue an extension task.

Vignette #2	Angles in parallel lines exercise	June, Year 7 All pupil participants
<p>During a 100 Club starter activity (see Vignette #1), W draws upon the board diagrams of various configurations of line segments intersecting pairs of parallel line segments. When the results of the 100 Club have been collected, pupils are asked to draw their attention to the board, and to volunteer which pairs of equal angles they can identify. Selected volunteers are invited to annotate the diagrams with pairs of equal or related angles, and give the endorsed name of each pair. Most contributions are correct, although one boy is chosen who does not correctly identify a pair of related angles: this surprises the teacher, who allows other pupils to offer correction. During this discussion, Thomas is reprimanded for chatting and sent to stand outside the open door of the classroom for five minutes. He accepts this without argument, and when allowed to return does so quietly.</p> <p>After this exposition of the rules of “angles in parallel lines”, the pupils are instructed to work on an exercise (see Appendix 3.2), beginning from question 4. Questions 1-3 had provided the diagrams on the board and had effectively been answered in discussion.</p> <p>Pupils ask whether they should copy the diagrams into their books. W tells them they should, and models on the IWB how a suitable solution should be written: the pupils are expected to record any arithmetic they calculate in order to find the answer and the rule which justifies that calculation. All pupils are expected to complete questions 4 to 9. When they have done so, the teacher hands out an extension sheet of ‘harder’ problems of the same form.</p>		

7.1.3 Teachers Responding

Connecting the picture of social relations in relation to classroom order required observing how teachers responded to direction from pupils. Teachers’ responses to pupils’ behaviour sustained a direct association between behaviour and consequences, particularly in the case of disruptions, which would swiftly be followed by a punishment or reprimand from the teacher. Teachers used the school systems of detentions and commendations: participants kept mental note of how many detentions they had received in a period such as the previous year or term, and the tally of commendations was recorded by a form tutor, with significant totals being rewarded publicly. A tiered system of detentions operated in which more serious or repeated misdemeanours would earn longer or more inconvenient detentions, supervised by increasingly senior members of staff.

Aaron’s description on p. 148 exemplifies the pre-emptive work done by teachers to limit the disruptions that could be caused. In the case of minor disruptions, before resorting to

punishments or reprimands, a teacher might re-impose order to the classroom by halting the action and reiterating expectations. This was exemplified in interviews with reference to M, who would insist that pupils listen and watch an exposition in full, before copying any examples or starting work. If pupils began to write, the exposition would cease until all pupils had put their writing equipment down on their desks, and were giving their attention. Alternatively, teachers might first threaten punishments. Typically, these threats would take the form of curtailing the free time the pupils would have, corresponding to the time 'wasted', by detaining them in the room at the end of the lesson or during a break time. Teachers had to follow through their threats or the pupils would not take them seriously.

Teachers appeared to judge their responses in relation to the severity of the misdemeanour and what they thought would be effective. Minor individual upsets, such as failure to bring the maths kit, were routinely dealt with by means of a light verbal reprimand. Similarly, talking out of turn or calling out could elicit a curt instruction to stop. A teacher might also include additional instruction, although this would not necessarily resolve the situation. If a pupil called out for help:

Thomas: Well, W normally shouts at us and tells us to ask the person next to us, and then tells us off for talking.

(I002)

This *Catch-22* situation originated in Thomas' need for help, but resulted in him receiving two reprimands, which he considered unfair. If a teacher was suspected of misattributing blame for an incident or treating a pupil unfairly, they ran the risk of undermining themselves, as pupils would react against this "miscarriage of justice" (Aaron, I005). The efficacy with which teachers administered punishments depended on how well they knew the members of the class. A teacher who was still getting to know the group would give whole-class detentions, but a more established teacher who could identify the "troublemakers" (Thomas, I005) would be able to target their punishment more effectively.

If a pupil stood out in relation to his classmates, he could expect to receive praise or admonishment. This could occur during a lesson or at the end, when the teacher might engage him in conversation to reflect upon his conduct. Behaviour which complied with expectations relating to conduct and productive work would be encouraged and rewarded by the teachers in various informal ways (such as praise, or sharing their work with the class), alongside the formal system of commendations. Pupils wanted their complicity to be noticed, particularly if they felt this made them stand out. The invitation to give explanations or insights publicly was treated as a reward for pupils, as it indicated they were expected to give the correct answer, which influenced their position relative to their peers. If a class satisfied a teacher over a period of time they might receive the “treat” (Aaron, I001) of having a lesson in a computer room. These lessons might be requested by pupils who felt they had earned it. Computer-based lessons typically involved consolidation of mathematical methods, with little or no formal written work (see Vignette 4, p. 168).

Teachers were also obliged to deal with long-term disruptions in a pupil’s participation. Sustained non-cooperation or a conflict between teacher and pupil might result in the teacher contacting that pupil’s parents or other members of staff in order to resolve the ongoing conflict. The teacher would report the incident or pattern of behaviour to these other parties who could then become involved in getting the pupil to observe the expected rules and standards.

Pupils’ requests for help would often be focused upon a specific problem from the set work or the examples given by the teacher. Once a conversation had been initiated by a pupil, the teacher, as the person in charge, would then control the dialogue to bring the pupil’s attention to a certain point. This might entail a longer conversation than the pupil desired. However, they were inhibited from interrupting or redirecting the teacher, as that would be “rude” (John, I010), even though the conversation could be more efficient if it dealt with the pupil’s actual, rather than interpreted, query. Rare occasions were reported in which teachers had responded expansively to unexpected questions. Pupils would enjoy

the conversations in which their studies were placed in a broader context, but often queries beyond the curriculum were curtailed.

Whilst pupils did not exclusively comply with expectations, the relative positions of teachers and pupils remained unquestioned, and were supported by the participants. They explained that without teachers, they “would not do any work or learn” (Jamie, I012). In comparison of experiences across schools and with their parents, the roles of teacher and pupil were described as self-evident. John explained that the ways in which teachers treated children had not changed in “generations” (I007). Teachers were expected to maintain order, which in turn meant that pupils were expected to comply with the order imposed by teachers.

Vignette #3	“Flight Paths”: Bearings task	June, Year 7 All pupil participants
<p>During a 100 Club challenge (See Vignette #1) those pupils who finish first are tasked with handing out a worksheet to the class (see Appendix 3.3). Pupils start the task if they feel confident, working in silence. After the challenge W collects the marks in descending order (but not the times), and awards commendations to those who had scored 100. W’s markbook, an excel spreadsheet, is displayed on the IWB for all pupils to see each others’ marks. The spreadsheet is used to show that the mean mark for the class has improved since last time.</p> <p>W then moves on to an exposition of bearings, explaining that they are needed to make sense of maps and to prevent becoming lost. W explains that there are three different sense of North (grid North, magnetic North and true North) and that it is important to know the difference. One pupil contributes that if you had a map, you wouldn’t be lost. W recognises and swiftly passes over this point. Observed pupils attend to the exposition intermittently. W explains the protocol for measuring bearings, as “clockwise from a North line” and draws over an image taken from Google Earth to illustrate the meaning of “the bearing of... from...” Pupils contribute how best to use the protractor to measure the bearing accurately. Pupils are then all expected to work on the sheet. W reiterates the procedure whilst pupils are working; also reminding them that bearings are written using three figures.</p>		

7.1.4 Order in the curriculum: setting, topics and testing

The mechanism of setting was a feature of school life in both primary and secondary schools for the participants¹². Arranging pupils into groups differentiated by previous achievement was described as appropriate for individuals to have the right opportunities to learn. Teachers “had to get you up to the right standards” (Jamie, I005), and could arrange groups according to pupils’ needs. Setting was largely determined by the teachers’ appraisal of the (regular and test) work each pupil had produced over a given period of time, but was also influenced by their enthusiasm and participation. This caused confusion when pupil perceptions of intelligence amongst their peers did not correlate with the set allocation. It was common to describe other pupils in terms of the sets they were in and how clever they were seen to be. Setting was seen as an implicit but unobtrusive judgement of a pupil:

Aaron: *Yes, in my old school we had sets, and if I was in a good set then that was fine, but if I wasn't in a high set... I wasn't quite happy because everyone was sort of showing off and ah, it's sort of like, all the good people were in one, and so sort of like, I'm a bad person and they're all good people. I didn't think that was the best way to deal with something, because ok, fine, I'm not, it's like they're really good but I'm really bad, and set two's in the middle.*

(I001)

Pupils were keen to elevate their setting position, as it was not seen as permanent and might change at certain times determined by the teachers, depending upon pupils’ attainment. Achievement within the discipline, as reflected in setting practices, also became a substantive feature of peer relations. Success with individual topics could become the basis for reciprocation of advice, but over the longer term differential achievement, enshrined in the setting practice, contributed to pupils defining themselves in relation to each others’ ability.

¹² This group of participants had experienced setting in relation to their mathematics learning in the final years of primary school, but in years seven and eight at the school were taught in form groups. They returned to groups differentiated by prior achievement for mathematics in year nine (see Appendix 1).

Pupils described the school's provision of a curriculum encompassing several disciplines as a means of enabling pupils to discover their strengths and weaknesses. Disciplines and topics within them were ordered over time by the school and the class teachers; pupils had no say in the arrangement. Topic labels were used to identify sequences of lessons, and pupils retrospectively identified and described their strengths and weaknesses in terms of topics. The beginning of a new topic might entail a recap of previous skills, by way of introduction, and the end would be signalled by summative assessment. These distinctions would be supported by the textbooks and other materials chosen by the teacher. Pupils would use topic names and timing within the curriculum, across year groups, in order to classify their development in mathematics. Undertaking work identified as belonging in a future topic was an indication of excellent progress, and good preparation for one's future school career.

Participants noted that teachers made their own work more efficient by having pupils working on the same topics (and tasks). This also enabled pupils to see the standard they were supposed to attain. Participants recognised that each pupil had to have sufficient experience of topic areas in order for them to achieve expected standards, within a manageable framework of time. However, this co-ordination of tasks could result in frustration if a pupil was progressing at a faster pace from the majority of his peers, and did not feel he benefitted from the continued practice.

Testing was a regular occurrence in lessons. Significant year-group tests would take place in order to inform teacher's termly reports to parents, but smaller 'class tests' would take place as a means of checking pupils' attainment on a topic or recent period of work. A test might be preceded by a period of time in which pupils were explicitly studying 'for the test'. During these times, focus was drawn tightly on revising relevant topics and types of problems which had to be understood for success. 'Fun' activities were less likely to happen and computer use diminished; tasks were focused on paper-and-pencil exercises which anticipated the problems in the test. With the tangible target of the test grade and the support of the teacher, the pupils were less likely to disrupt classroom order.

Vignette #4	BBC Bitesize	May, Year 7 All pupil participants
<p>This is the second maths lesson in the day, taking place in the last (ninth) period. The teacher habitually arranges for the class to move to the school's computer room for this period, to undertake some 'fun' activities. The class are instructed to log on to the PCs. Once they have all done so, and are attentively awaiting further instruction, W directs them to access the BBC Key stage 3 Bitesize revision website (via Google). They are told to click on <i>Shape, Space and Measure</i>, and then <i>Revise Co-ordinates and Bearings</i>. The site offers three pages of revision tasks.</p> <p>The tasks comprise short-answer questions which can be evaluated automatically. The website gives immediate feedback. Pupils are asked to plot and identify co-ordinates, use the properties of quadrilaterals to determine co-ordinates, identify quadrilaterals marked out by co-ordinates, and complete given shapes by identifying missing vertices.</p> <p>The pupils are allowed to discuss the tasks with their neighbours. W continually patrols the room to ensure pupils are on task, using the one permitted website and discussing only the work.</p> <p>Alex pinpoints co-ordinates with his fingers on-screen to identify values, whilst John copies the screen and pastes into the drawing software <i>Paint</i>, in order to annotate diagrams and find the desired answers. John shares this idea with Aaron, who is at first reluctant to use it. Toward the end of the lesson, W instructs all pupils to log off. No record of their work is made. W uses a number game to dismiss the boys one-by-one, whilst they stand quietly behind their chairs.</p> <p>Note: The web pages to which the pupils were directed have been revised. The specific task no longer exists on the site.</p>		

7.1.5 Community: Interpersonal relations

Throughout the interviews, the participants emphasised the social aspects of people acting together and developing relationships. In the early interviews, interpersonal relationships were prioritised in their descriptions, and were used as a means of making sense of the classroom.

The role of the teachers in maintaining and instilling order in activity was recognised by pupils. In return, they expected teachers to conduct their roles with equanimity, fairness and consistency. They interpreted the ways in which teachers acted out their roles as reflections of their personalities. Comparisons of how teachers enforced order were made in terms of their emotions and how they treated the individual pupils. Consequently, maintenance of order and working patterns informed the developing relationships between teachers and pupils. New teachers appeared to enforce rules and their

expectations more strictly, and were described as being “strict and stropky with everyone” (Jake, I005). Relations of respect (or even fear) could sustain, and pupils were sensitive to the fair use of a teacher’s institutional authority. Reprimands were interpreted in personal and emotive terms, often being described as a teacher ‘shouting’ at a pupil or ‘getting cross.’ If pupils thought someone had been treated unfairly this could be attributed to the teacher’s emotions getting in the way. Self-perpetuating cycles of disharmony and punishment had been experienced between pupil and teacher. Negative expectations would then lead to pupils failing to attend to and understand a teacher’s requirements, whilst feeling the teacher was not giving them fair attention. This would require reparatory action to break the cycle, on the part of the teacher.

Teachers’ failure to fulfil the pupils’ expectations was interpreted as a personal failing, rather than attributed to the pressures of the curriculum or the institution. Thomas recounted an instance in which W had made a commitment to take pupils to the computer room for “fun” lessons on a weekly basis, but this was sustained for only a short while, as the need to cover the syllabus predominated. When the commitment came to an end, this was described as “broken promises” (Thomas, I008) and his working relationship with W was damaged.

Positive relations were fostered by teachers letting pupils ask lots of questions and following their lead, or encouraging their contribution in public discussions. Pupils felt that they not only had an opportunity to display their capability, but also to develop their relationship with the teacher:

Thomas: *If you have your hand up and someone picks you, you feel a bit special, they’ve chosen you and they trust you to get the right answer*
(I005)

The approach pupils took to their work was mutually constitutive with the relationships sustained in the classroom. Teachers responded to pupils working productively by giving praise and extra attention, which in turn encouraged participation. Development of the interpersonal relationship functioned as a reward, alongside high marks and good grades.

In interviews, participants expressed the desire that other pupils all conscientiously contribute to the maintenance of the focused working atmosphere. However, it was also acknowledged that when with friends who were not focused on the work, they could act disruptively. This disruption could also occur as a reaction to the treatment of others, such as “miscarriages of justice” (Aaron, I005) or perceived favour. Pupils’ relations with each other were informed by their success in the classroom and how they related to the teacher, which cast light upon their own progress:

Jamie: Most of them, because they think they’re so up themselves they expect “oh I’m the best, the teachers don’t need to tell me anything”
DVS: that’s the pupils, you think?
Jamie: yeh they just think “oh, we don’t have to work ‘cause the teacher likes us the most”

(I006)

These comments were made at a time when Jamie was frequently reprimanded for lack of engagement in mathematics lessons and his achievement was falling behind that of his peers.

Understanding the purpose of studying a school discipline affected how willingly pupils complied with the ordering of activity, although the ease with which a pupil could enquire about this depended upon his relations with the teacher. Such queries could be treated as an attack on the purpose of study, or as a hindrance to getting on with the work:

Jamie: S, if you do ask that S will go “Why are you making silly questions, see me at lunchtime at one-fifteen for fifteen minutes.”...you can’t ask something like “Why are we learning Latin?”
DVS: Is that to do with the question itself or is it the way you ask it?
Jamie: it’s got to do with both of them really... And in Physics, I’m sorry I’m going on a bit but, in Physics D will be like-
Jake: “Just get on with your work.”
Jamie: yeh like D’ll go “There’s no need for discussion, there’s no need for questions, put your hand down.”

(I008)

As the pupils moved from year 7 through to year 10, their motivation for classroom participation shifted from maintenance of good relations to achievement of the best

grades possible. Contributions in later and final interviews indicated the priority of this aim. Good relations were sustained through sharing this focus with the teacher; the order of priority had shifted. This shift was sustained through subtle changes in the patterns of work.

Vignette #5	Nets of 3-D shapes	June, Year 7 All pupil participants
<p>At the beginning of this lesson, W explains the pupils will need to have their full maths kit, because the purpose of the lesson is to produce nets of 3D shapes. Pupils are issued with A4 sheets of squared paper, on which they will produce their work. W proceeds to lead the pupils through a process of sketching nets, then producing more precise versions. W works on the whiteboard, producing large diagrams with sketches that are not to scale. The first example projected onto the IWB is a triangular prism with a right-angled cross-section. The perpendicular sides are labelled as 5cm and 4 cm long. W explains that the precise diagram will have all necessary lengths annotated and right angles accurately drawn. A volunteer pupil is invited up to the board to draw his sketch of the net. This sketch is discussed and corrected where necessary. The pupils produce and annotate their own sketches whilst W talks. W explains that the pupils should then add the dimensions to all the edges in the net, including the hypotenuse of the triangle. This instruction leads into a calculation using Pythagoras' theorem. Some pupils listen whilst others contribute to a demonstration of the calculation, which W exemplifies with algebraic workings. This task is continued into the following lesson, in which pupils are expected to work more efficiently and with greater attention to precision.</p>		

7.2 Working norms

The behaviour described by the codes *Ordering Norms* created socio-temporal spaces in which 'work' could be done. *Working Norms* emerged as those legitimated actions which directed pupils toward developing capability in school disciplines. I restrict my attention to the mathematics classroom from this point in this thesis. In this section, I describe the characteristics of doing work, extending the coding structure which formed §7.1. I first consider how teachers directed pupils, then in return how pupils directed teachers. Teachers' responses are then considered, before looking at how teacher-pupil relations influenced work. I then move on from the participatory aspects of work. In the coding process, textual work came to the fore as a crucial constituent action: the code *Reading and Writing* developed for its explanatory power in making a connection between

participation in work and engagement with mathematical structures. I end this section by describing the functions fulfilled by reading and writing.

7.2.1 Teachers directing pupils

Here I consider the direction teachers gave to pupils in terms of the structured action that constituted work, in order to learn mathematics. Aaron's description of a generic lesson (p. 148) contained much of the kernel of what the pupils described as 'doing work' in their mathematics lessons: following expositions; copying examples; engaging with written tasks. Pupils were also accustomed to the teachers' use of the textbook, rehearsal of topic areas and doing tests. Here I consider each of these in turn, and the effect these had on individuals' participation.

Pupils expected first to follow a teacher's demonstration of the mathematics they were going to learn. The demonstration would involve the teacher writing up examples on the whiteboard or IWB, and include questions to and from the pupils who would be invited to share their ideas. Pupils might be called upon to contribute to the public demonstration of methods for others to learn. Teachers would generate appropriate problems to exemplify the necessary methods and explain variations.

The teacher would make clear the aims for the lesson and requirements of the tasks. When giving expositions, teachers would reiterate and stress certain information, whilst treating other concepts as assumed. The nature of the task might require lengthy introduction (for a new topic) or a minimal instruction (about revision). Pupils expected teachers to be sensitive to their needs with respect to a task. When giving their exposition, a teacher might require the pupils to take dictated notes or copy their examples, which acted as models for solutions the pupils were expected to produce. M would show pupils "the exact way they want you to write it down" (Alex, I018). This was justified by helping to keep the work neat, which meant that pupils made their answers clear, and later could easily review what they had done. Once the teacher judged the demonstration complete they would "set us to work" (Jamie, I005): pupils would begin some written work in which they would practise the methods shown and get to

understand the ideas involved. This written work usually consisted of an exercise from their textbook. In work, each member of the class would focus on their own attempt at the same task, whilst sharing ideas with neighbours.

In their textbooks pupils would find the specific exercises, task instructions and examples. Participants described the textbook as an efficient alternative (for the teacher) to producing worksheets or devising tasks for pupils. The pupils would have to follow the instructions therein and complete the tasks if they were to make progress. Sometimes a teacher would expect pupils to work through all questions in an exercise; otherwise a selection would be made. This would control the variety or repetition between questions, emphasising consolidation of a core method or more expansive application to different scenarios.

During a period of work a teacher might interrupt the pupils' action to call their attention to an issue which had emerged in carrying out the tasks. This would be likely to happen if a number of pupils encountered the same difficulty or presented the same query. The teacher would stop all pupils from working in order to give an explanation of the issue. This sometimes entailed reiterating the original explanation with new emphasis on specific details. Pupils would then be permitted to resume work when the teacher judged enough additional explanation and discussion had taken place. This was described as an imposition for those pupils who had not had any difficulty.

Following on from the previous lesson (see Vignette #5) W explains the class are going to practise constructing triangles precisely, using compasses. The pupils are instructed to turn to the same page in the book as they worked from yesterday. W explains that the focus of this lesson's work will be accurate drawing, using compasses.

W leads the class through an exposition in which a triangle with side lengths 8cm, 5cm and 6cm is constructed. W explains that this is only a recap as the pupils "should know how to do this already". Pupils with complete kit begin to construct this triangle whilst W hands out compasses to those pupils who need them. By the time W invites contributions from the class, some have already completed the task. W draws the triangle on the whiteboard, following successive instructions from one pupil. W interrupts the instructions to ensure that vocabulary is used conventionally (arc, intersection). W's triangle is drawn at a scale of 10:1, in order that it can be seen by all pupils, and is done using a metre rule and whiteboard compasses. More pupils complete the construction whilst the instructions are given. W congratulates the pupil who gave the instructions. Without further questioning, W then instructs the class to produce two more examples, with respective lengths 7cm, 6cm, 5cm and 5cm, 10cm, 10cm. Whilst the pupils are working, W invites explanation as to whether and why the arcs drawn should be erased upon completion. Pupils concur that they should be left as indication of their workings.

Turning to the textbook, W asks pupils to work through the exercise from the first question, producing accurate, rather than sketch diagrams. W provides sketches for questions 1 and 2, which are then reproduced as accurate diagrams on the board. W has no squares to guide the drawing, so estimates right angles. A missing length in question 2 is given by W, who decides it should be 5cm (see Appendix 3.4).

At the end of the lesson, question 9 is assigned as the homework task, with the inducement that the four most accurate drawings will earn school commendations.

7.2.2 Pupils working: exercising subjectivity

Pupils produced work at the teacher's behest, but explained that through doing so they developed proficiency and revealed their capabilities to both themselves and the teacher:

Thomas: Because um if a teacher like explains something and you say like, "I get it, I get it," but the teacher won't know that you actually do get it, so and you, you might say you get it but there might be something you don't, so you have exercises that make you practice so you actually make sure that you can do it, because if they are wrong you know you can't do it. You know there's something wrong.

(I017)

Productive participation resulted in pupils producing evidence of their current capabilities and constituted the experience through which they could learn new mathematics. Doing

work enabled them to identify gaps in their capabilities and address them. Reflection on the produced work then enabled pupils to be confident that they were making progress. Whilst in the course of doing uniform set work, pupils' individual understandings, aims and capabilities led them to exercise their subjectivity, which I detail here.

Using topics as a means of marking achievement enabled comparison between pupils, who could help each other in loose reciprocal arrangements across topics:

Jake: Uh, say for example if I was to sit next to Charlie¹³, I'm better at algebra and he's better at circle theorems and stuff and so we can help each other out in different questions. You know, like teach it to each other.
(I024)

Unless it had been specifically prohibited, pupils would often choose to co-opt the calculator into their work. The calculator was for calculations that were "long-winded or complex" (John, I021), but also aided efficiency and accuracy in short calculations. It was reliable but would compute only the instructions given; the pupil had to use their intelligence in planning the calculation. Pupils also had to meaningfully connect the output with the problem, which contrasted with using the device for a quick sum:

Alex: ...it's just using um your knowledge to punch in a sum, and things like that...to find out the answer, it's not really using your brain, you're using [the calculator] really to find out the answer.
DVS: so that's- so would that be different to using the trig functions on the calculator to find answers?
Alex: um, I suppose yeh you have to use your knowledge to find out how to use, how to find the answer, so there's tan and cos and things like that
DVS: m-hm
Alex: you have to know what that is in order to relate the information.
(I022)

With calculations that a pupil could not have worked out with pencil and paper, he had to know how they related to the problems being solved. However, it was not necessary to

¹³ Pseudonym.

know the mechanism by which the calculator produced the answers. Using the maths kit (in particular the calculator) was an integral part of solving many of the problems pupils encountered: problems in which their use had been intended could rarely be achieved without them.

Work on PCs often involved co-opting additional items to aid problem-solving. Pupils might use paper, calculators or small whiteboards for 'rough work' before inputting answers to the software. The availability of other programs could also aid pupils, who sometimes chose to produce rough work on-screen. Vignette #4 (p. 168) illustrates this. Aaron explained his reluctance to follow John's example:

Aaron: John showed me this thing, this program, that you can down, that you can copy and paste the umm co-ordinate map onto a, a program called Paint and then you can sort of, you don't have to think it through you sort of cheat, you uh get the co-ordinates, like 4-5, and put a dot there, and minus 3 and minus umm 2 and put a dot there and you can work out from there and then join them up, and you get your shape.

DVS: Ah ok.

Aaron: But it's sort of cheating because you can't do that in the exam ... you're sort of, you're not actually meant to do it and nobody else is doing it.

(I001)

Pupils had to decide whether the use of additional aids to support problem-solving would undermine the task; this was not always clear to them. Their judgement would be determined in part by what they saw others doing, but also influenced by their perception of the aims of the task. The pupils expected to develop the capabilities which would be pertinent to future summative assessment, and so would usually act in ways which matched the anticipated affordances of assessment tasks.

Whilst pupils were attending to a teacher's exposition, they were selective and purposeful in how they directed their attention. They would choose to ignore some of the information given by the teacher, if they felt it was "unnecessary" (ZR, I008). Necessity would be decided by observing fellow classmates, to judge what they attended to, and what questions the teacher dealt with. If one pupil was contributing at length to a

demonstration, others could use this time to orient themselves in the presentation, determining their aims in relation to the task:

Thomas: well one, like, what are we doing, and two, like um, is that right and do we agree with him and like three like you know um what's he going to do, what's W, what're we going to do next and I don't know if this is right but I always wonder what the teacher thinks about it.

(I009)

When a pupil requested help from a teacher, to clarify a method or task or resolve an issue, they would expect the teacher to ask them questions about what they had done so far and what they understood of the problem and the topic. These questions would require that a pupil reiterate the productive steps in his work thus far in order to reveal the source of an error or to make explicit a connection which would help him complete the problem. Observations considered irrelevant for the problem would be put aside in order to direct the pupils' attention to the necessary connections. The pupil worked under an obligation to make the connections anticipated by the teacher and demonstrate these in conventional terms. As such, the conversation became a task in its own right. When requesting a repeat explanation conversational control was willingly relinquished, as the pupil needed information from the teacher to complete the task:

Aaron: If you just weren't listening and you were playing with a ruler or something and then you realise when you get to the work then you go up and you actually listen.

(I010)

Material in the textbook could initiate a request for help, when a pupil needed assistance with an example or in interpreting a problem. These queries often placed the request in the context of producing responses to problems, and related to the form of exercises:

Jake: I think, that when you get on to the harder questions, for example if you were to look at something in the book, you wouldn't be able to do it just from the example and it telling you, you need someone to like give you the

best way to do it, how to do it, and like the quickest form instead of like having to look it up.

(I024)

It was not always possible for pupils to interpret from written examples the deductive steps they had to reproduce. In this case, they would request to see the process of applying a method or to have steps in an example explained, so that they could understand what they had to do.

The role of the teacher as arbiter of mathematical results would be called upon by pupils in the midst of their work. They might ask the teacher to check an answer or an intermediate step in the workings, hoping for a quick response. In this way, pupils sought support from the authoritative judgement of the teacher, from whom a 'right/wrong' response would be sufficient. When told they were correct, they would be content not to pursue further rigorous justification. When marking work as a class, pupils had the opportunity to cheat, filling in correct answers as the marking progressed (see Vignette 15, p. 199). However, they would usually cooperate honestly in this task, as it was important that the teacher saw their genuine work and awarded them the "right grade" (John, I010). The demonstration of capability lay behind the creation of work and the subsequent marking, alongside any classroom conversation.

Choosing whether to consult a teacher or a peer for help was in part guided by the type of conversation a pupil wanted to have. In contrast to calling for a repeat explanation, checking mistakes should be much more efficient:

Aaron: *but if you knew how to do it and got the basic idea but just made a silly mistake, like adding them wrong, or putting a three instead of a four then,*
John: *you don't really want to have a long conversation.*
Aaron: *yeh you don't really want, yeh.*

(I010)

Avoiding a "long conversation" was an efficient approach to getting the work done, shared by the pupils. If they judged they had a simple query, or the issue was a need to determine the minor error in one's current solution they would consult a classmate. The subsequent

conversation could become a comparison of the texts they had produced. Their focus was on efficient production of the correct work in response to the tasks given. The choice to ask a peer would also depend upon how confident they might be that they could answer the query correctly and the nature of their friendship. Choosing to work collaboratively on a series of problems depended upon similar considerations.

Such interactions between pupils were common: a cursory check that one's answers agreed with a classmate's was treated as sufficient reassurance that the work was being done correctly. Agreement on answers was seen as indication of valid reasoning in solutions; this use of consensus pervaded pupils' work. Consensus operated not only in one-to-one discussion between pupils but also when marking work as a class. If a declared answer matched a pupil's own, he could mark his correct before waiting for further discussion or confirmation from the teacher (see Vignette 8, p. 183). This was reinforced by the teachers' perceived habit of discussing incorrect answers, but only confirming correct ones.

Pupils would also refer to each other in order to make sense of tasks from a basis of shared understanding, rather than the teacher's assumptions of what they ought to know:

Thomas: ... sometimes you just ask a friend who's sitting next to you, and then they'll explain it. Because if you ask teachers they might think it's simple but it might not be that simple to you. But if you ask a friend then they'll put it into words that you'll understand, as they understand it.

(I025)

These explanations would also focus more explicitly on the productive expectations of the task, than on the mathematical structure which was its focus. In discussions, a pupil called upon another to demonstrate that he could do what was required of him in the work, and so a choice to speak to a peer was based on confidence that he would be able to answer.

Questions of purpose were rarely asked by pupils. This was in part due to the negative reception they had learned to anticipate when questioning the purpose behind a task or a discipline. However, as pupils grew older, they had a clear sense of a specific purpose

behind their studies. They became focused on attaining grades and the advantages these could bestow:

DVS: Do pupils ever actually question or ask why are we doing this particular bit of maths?

Aaron: yeh, well no they don't but they don't 'cause they just like take it... You don't actually, in life you don't actually need to, but it's just... You just have to know it for like for your A-levels and your GCSEs. 'cause those are, have required maths. If you want to get into a good university.

(I016)

Vignette #7	24-hr and 12-hr clocks: Exercise	November, Year 8 Alex and Thomas
<p>The class teacher (M) is away for this lesson, which is taken by a supply teacher. Once settled, the pupils are instructed as to which exercise from their textbook they all will work through (see Appendix 3.5). The supply teacher only supervises: no mathematical instruction is given. Thomas is slow to get to work, as he cannot find his textbook in his bag at first. Once ready and having written a date and title, he considers the first two problem sets. He looks at Alex's work and copies down the answers for these. He then works with Alex, from problem 3 to the end of the exercise. As they work, they read the problems out aloud to each other, and build the relevant calculations together. There is no explicit planning of the solutions or explanation between them, but the boys take it in turn to state intermediate results which build toward the answer. The answers are written down. Together the boys complete the exercise.</p> <p>After the lesson Thomas explains that copying the answers to the first problem sets was motivated by wanting to catch up and work with Alex, as people usually do better when working together. He describes his actions as "not cheating, because I would have got them right anyway," but explains that it was important to have those answers written down because the teacher would look over their work. Thomas explains the collaboration as "he says one bit and then I say another and he says another and then we get the answer."</p>		

7.2.3 Teachers responding

Teachers' actions in responding to pupils' queries in the midst of lessons were influenced through marking written work and the feedback given to pupils on their work. Teachers had licence to decide how to respond to queries: instances given by pupils were to give a quick answer to a closed question, to investigate the pupils' preceding action, or to deflect the query with referral to the textbook or calculator. A teacher might ask the pupil to think again about the problem, with minimal additional assistance, or create a new task

out of the query, by asking pupils to collaborate in resolving it. Pupils did not always anticipate correctly how a teacher would respond. They might underestimate the extent to which the teacher would want to investigate their understanding and work produced, or overestimate the amount of help the teacher was prepared to give on that occasion.

Teachers could focus on written errors and lead the pupil through a reparatory conversation. In these conversations, different attributions of importance could be made to the errors:

Aaron: When you usually make a mistake most teachers don't really realise that it's mostly something like actually you meant to write a two and you actually wrote a one. It's just they think you don't get it and they take you right back to basics and if it's something really complicated then the teacher's just telling you what you already knew.

(I010)

Teachers would use questioning to elicit step-by-step explanations of pupils' work, in order to reveal the source of an error and to reinforce the most efficient method. However, this might not directly address the issue which had instigated the query, focusing instead on constituent steps or the underlying structure.

In response to a pupil meeting persistent difficulties, a teacher might occasionally take a more flexible approach to the work, redirecting him towards problems which would meet his individual needs. The pupil might be directed to easier questions if he was finding the topic difficult; there also were more challenging problems available for pupils who required them. Extension tasks commonly consisted of doing an additional (harder) related exercise. Such exercises could come from the textbook already in use, or another source if the teacher had it available. In this case the pupil would also be charged with checking his answers himself. On some occasions, pupils had been directed to aid the teacher in helping other pupils. In this case he would find that he was using his own understanding in order to explain what to do. However, these discussions typically focused more on producing correct answers than on the need to show rigorous workings.

After pupils had produced their written work they would expect teachers to review and annotate it with grades, scores and comments. These embellished texts would then be returned to the pupils, who would be expected to reflect upon and reinterpret their own work in light of the teacher's input. When marking work teachers would award grades or scores 'out of 20' and add comments; credit was given for valid deductions presented conventionally, and effort often commented upon. However, marking would emphasise errors in deductions, arithmetic slips and incorrect answers, with corrections given or shown to be necessary. Correct solutions and answers would most commonly be passed over or merely indicated with a tick. Pupils became used to receiving superficially different styles of feedback from different teachers: comments could be easily ignored by pupils, so some teachers would use stamps or stickers to attract attention. These additions were described as being more fun and attracted attention better than written comments alone. There was no standing expectation that corrections should be undertaken to improve low-scoring work.

Through marking the class' written work, the teacher would gather information about their achievement, on the basis of which they could decide how the topic and tasks should proceed. This was reassuring for the pupils when they saw their achievement as commensurate with that of their classmates. However, in making a judgement for the whole class, each individual's achievements and subsequent needs could be obscured. High scores might not compensate for frustration felt at repeating work a pupil found easy. Conversely, the focus of work could move on before a given pupil felt he had achieved what he wanted to. A pupil could take some time to establish his proficiency in an area, and would then like to enjoy his achievement, producing work which would be noted as correct and gaining high scores.

After a test at the end of a topic, the teacher would usually lead a review of test performance. The pupils would then undertake brief practice of areas in which they had not performed successfully. Pupils would be given a range of materials, containing problems similar to those in the test, from which they should choose the appropriate tasks for them to practise.

Vignette #8	Speed-Distance-Time: Exercise	January, Year 8 MC, ZR, John
<p>Before the pupils enter, teacher M has laid worksheets face down on pupils' desks, which are they are not permitted to turn over. Pupils ask whether this is a 100 Club or a test. M explains that the sheets are a review of their recent work on Speed-Distance-Time and travel graphs (see Appendix 3.6). After a brief recap of using the formula $S=D/T$, pupils are permitted to turn over their sheets and work on the questions.</p> <p>John is sitting on his own in this lesson, and does not discuss his work with any classmates. He writes answers, but no workings, directly on the sheet. He works without break until he reaches questions which ask him to calculate <i>average</i> speed, when he puts his hand up to ask M what this vocabulary means.</p> <p>M listens to his question and explains that it refers to the total distance, divided by the total time. M then recaps the $S=D/T$ formula, writing it onto John's sheet, and explaining how it should be used for this question. John explains later that he did not need the extensive explanation, but had sought only a clarification of the use of the term "average" in this context.</p> <p>When M judges most pupils will soon finish the sheet, they are given a two-minute warning to complete. The pupils mark their own work, as M talks through the problems, inviting answers and explanations from individuals. John marks his work in response to the answers given by his peers when they agree with his. He does not wait for the teacher to confirm they are correct. In one instance (question c), this leads to him correcting his marking. He had put the same incorrect answer as had been publicly offered. After M corrected the answer and there was discussion of the mistake, John marked his own answer wrong. He continues to mark his answers correct upon confirmation from peers, rather than the teacher.</p>		

7.2.4 Teacher-pupil relations

Within the context of classroom order, the qualities of relations between pupils and teachers were frequently reflected in interactions oriented toward the work: good working relationships with a teacher were seen as being necessary to make progress within mathematics. Pupils had to understand the instructions they were given, which depended upon being willing to attend to the teacher. In return, pupils needed to feel confident that the teacher was attending appropriately to them and responding to their queries properly. These were identified as pragmatic moment-to-moment issues, which depended upon and constituted working relations. Clear communication between parties and a sense of mutual respect made it easier for the pupils to accept the requirements of a discipline. Productive relations between teachers and pupils were sustained if a pupil felt he was making progress on the work. However the notion of progress would be

determined by the work that the teacher set. Pupils compared the relative merits of different teachers in terms of how the teachers maintained focus on the work and how much they felt they had learned. Consequently there was a need for demonstration within the activity that the work being done was purposeful and leading to achievement in relation to the mathematics curriculum. Pupils could primarily judge this through the completion of tasks at a rate determined in relation to a their peers and the teacher's written feedback on their work.

Over the course of the data collection, pupils placed decreasing emphasis upon their relationships with the teachers. Whilst positive relationships were continually seen as important, their motivating power diminished. As pupils developed capability in mathematics this power was taken up by the need to attain and maintain high grades on their written work and reports, in anticipation of eventual public examination performance.

Vignette #9	Enlargements from a Centre	January, Year 8 MC, ZR, John.
<p>M begins the lesson by leading a review of the technique of creating enlargements of 2-dimensional shapes from a given centre. Near the bottom of the whiteboard (which has a faint grid), M sketches a square and places a centre of enlargement on a level with its lower edge, a little to the left.</p> <p>ZR talks M through an algorithm in which lines are drawn from the centre through every vertex of the original shape. As he does so, M follows his instructions on the whiteboard. M models what the pupils should do, with the exception that the object, lines and image are sketched, rather than drawn with a ruler. M estimates distances along the lines to create the image, by counting squares. It is explained that the pupils must use rulers to draw and measure in their diagrams. They can also count squares, as the problems they will work on have vertices of all shapes aligned with the square grids. M explains that the technique “Is like stepping stones; do one thing at a time.”</p> <p>The pupils have all brought pictures of cartoon characters into the lesson with them. When they complete the exercise (see Appendix 3.7), they will then stick their picture to a sheet of A3 paper and use the technique to produce an enlargement of it. Their homework task is to complete this enlargement.</p>		

7.2.5 Reading and writing

The dominant form of work was for pupils to read problems and produce written responses: here I consider these aspects in that order. Textbooks usually presented problems in exercises: series of themed problems invoking the use of a particular mathematical structure in increasingly complex scenarios. Pupils were familiar with the structure of the exercise, with earlier questions representing the focal method in its simplest application, and later questions developing complexity around this. Pupils valued the structure of exercises in different ways: some felt it was important to complete the exercises so that they had done adequate practice and could see the idea in different situations. The most difficult questions would model what they might meet in their most challenging examinations, and pupils could anticipate test requirements by tackling these questions. However, others felt that the most important part was the initial focus on the bare method. These pupils wanted to be confident that they understood the core idea before moving on, and considered the harder questions as “extension work”.

Pupils would find illustrations of the methods they had to learn in their textbooks. Written examples were presented which co-ordinated with well-defined and predictable questions. Pupils felt comfortable when problems could be completed by the recall and application of standard routines, and thus success could be seen in terms of the application of written algorithms. The subsequent rigorous application of the method would then be rewarded in the marking. This approach could be encouraged by the presentation of methods by teachers:

Alex: M writes down the question for example, and then shows you the exact way [M] wants you to write it down, so like simultaneous equations for example
DVS: ok,
Alex: M shows you like how to write down and do the calculation and then makes you copy it down so you actually know how to do it.

(I018)

However, when applying diagrammatic methods, pupils were obliged to reformulate the teacher's examples, reflecting the use of different equipment being used, and the degree of precision required and achievable. A pupil had to consider how to produce in their book what the teacher had done on the IWB. The teacher's display might be drawn with a precision only achievable with digital tools, and the measurements given would often be artificial, involving numbers upon which one could do simple arithmetic quickly. Alternatively, teachers would draw sketches whilst pupils would be expected to produce precise diagrams (see Vignette #9, p.185). Pupils would have to translate the demonstration into their own method, observing the constraints of *working norms* and the problems they met.

Participants identified the varying priorities served by producing written work, and saw this reflected in the use of different media. The use of different tools accompanied a different attitude to the working habits the pupils might adopt. This was made explicit in the comparison between different teachers:

John: ok, well actually, in maths um M prefers us to use pen, not pencil,
DVS: ok
John: last year W preferred us to do it in pencil because you can rub it out um

Aaron: *M likes us to do it in pen because that's how it's done in the exam.*

(I005)

Vignette #10 Similar shapes and enlargement

January,
Year 8
Jake, Jamie

W explains to the pupils that they will be learning about enlargements, and uses the IWB software to demonstrate enlargement of shapes, by copying, pasting and stretching the images. These shapes include an outline map of Great Britain, a squiggly shape drawn by W and a star, before focusing on more rectilinear shapes. He stresses that all lengths are stretched by the same factor “vertically and horizontally”; changes which do not scale all lengths equally are not enlargements. W then offers various examples of enlargements with some measurements given and asks the pupils to identify the scale factors used in each. The pupils then begin work on an exercise (see Appendix 3.8), writing down short answers or drawing correctly scaled diagrams. In this exercise they apply these new ideas to identify correct enlargements and scale factors, and then follow instructions to create their own enlargements.

7.3 Mathematical norms in work

In this section I offer a description of the *mathematical norms*: those behaviours privileged as the explicit purpose of work being done, and which distinguished mathematics lessons from those in other disciplines. I also consider the mechanisms by which they were privileged, in the interactions amongst teachers and pupils. As stated above, producing written responses to textbook exercises was the predominant form of work. I first detail the pupils’ understanding of textbook examples and exercises, and how mathematical action was delineated by these texts. I then detail how pupils perceived teachers demonstrating mathematical work in relation to texts, and how they responded to this. I then describe the production of texts by pupils in response to exercises. My description then turns to other forms of work, before considering the structures out of and in which pupils could make mathematical meaning and see mathematical action taking on meaning.

7.3.1 Working with instructional texts

Pupils predominantly took their problems and detailed task instructions from the textbooks and worksheets. Written instructions usually directed them to provide answers supported by deductive solutions. Suitable answers were usually numerical or algebraic

statements. Justification of those answers was exemplified in the textbooks and stressed in the verbal instruction given by teachers. In contrast, assessment materials contained relatively few instructions and no explanations: tests were presented as sets of problems, with little additional information. In later years, test sheets might contain the formulae that would be given in the IGCSE examination.

Textbooks were also used by pupils when they needed explanations or examples of methods. However, this usage depended on a pupil's capacity for understanding the written text, which created a paradoxical situation: If a pupil understood what to do, then he would understand the explanation, but then was unlikely to need it. If he did not understand what to do, then he would not understand the explanation. Additional support would be needed to translate a static written presentation into an active process, unfolding in time, and a pupil would call upon the teacher or a peer. When referring back to remind themselves of some previous learning, pupils would often rather consult their previous work for a reminder than look it up in the textbook. In reviewing their own work, they could recall the process by which it was created, rather than try to intuit the creation of the text from the end-product.

Studying for future tests was an unspoken feature of all mathematics lessons, and as pupils grew older they expected to understand how any given task contributed to their efforts in this regard. Completion of exercises and written problems embedded expectations of the requirements of future tests. Studying for tests and public examinations could become an explicit feature of mathematics lessons, with a focus on examination practice papers; this practice increased in year 10, when studying for the IGCSE examination. Pupils might also be given examination questions as an extension task if they had demonstrated that they were ready to attempt them.

The class are undertaking a test for which they have been prepared. The pupils enter the room promptly, and quickly settle themselves into position in readiness for the test. Thomas sits at the front of the room, at a teacher's desk, facing the class. M explains later that it had become necessary to manage Thomas' behaviour in class, in order to keep his focus on the work and not to disturb other pupils. This was a way of making him feel special which did not disturb the rest of the class.

M explains that the pupils' performance on the test will inform the grades the pupils are awarded in their reports, and what will be said about them at an upcoming Parents' Evening. M hands out the test, comprising questions on a range of topics that had been recently taught. Pupils work in silence, without collaboration or aid from exercise or textbooks. Pupils work on the test papers, writing their responses to the problems in the spaces provided. At the end of the lesson, the teacher collects the test papers in and dismisses the pupils.

7.3.2 Exercises

Exercises were regularly constructed of a short series of simple problems, recreating the examples shown by the teacher, followed by a longer series of problems of increasing difficulty or complexity. This increase was seen in (1) the incorporation of 'real-world contexts', (2) complications in the arithmetic required and numbers used, (3) increasing the number of steps required before or after dealing with the focal method, or (4) a combination of these factors.

Participants ascribed the increasing difficulty of exercises to an expectation that pupils push themselves to take on further challenges. The purpose was described as always to help pupils improve, as they routinely had to engage with increasingly challenging problems. Their progress could be seen in the success with which they dealt with the increasing difficulty. Pupils had to attend to the details in questions to observe how they had become more complex, as it would not always be obvious. The similarity of earlier questions was seen to "compound and test" the key method in its simplest form, whereas the variety and complexity within later questions was described as helping pupils understand how it all "fits together".

DVS: *Why would you have ten or twelve similar equations to solve?*

Aaron: *Just to like compound the, 'cause you may remember how it's done and you like test how it's done... Usually like the teachers don't tell you like the whole the from start to finish the whole thing so she gets like a question she just tells you what to do and you find out how amazing it is 'cause it fits together 'cause maths isn't like English it's like a machine and its sort of a and every single part fits in one way or another, you just need to know how to fit them.*

(I016)

By engaging in the work, pupils discovered connections which could be made with mathematical structures, supplementing the teacher's explanation with specific details relating to each question. Through completing these questions the pupils could build understanding of the mathematical structures and how to work with them. The process of consolidation and discovery was described as beginning with a "rough idea of what you're doing" and discovering how well one could use that idea in response to questions (Jake, I024). Thomas described the increase in difficulty as "they're trying to catch people out" (I009).

Pupils were well-versed in dealing with questions that contained 'real-world' contextualisations. They had become used to actively discarding the contextualisations placed upon mathematical structures in questions, particularly when temporal-physical or social contexts did not correspond to the world as they understood it:

DVS: *Did you think the questions in the exercise were... quite realistic? Were they the sort of thing you might have to work out?*

Thomas: *Yeh, yeh, 'cause it's not like saying 'you are in a canoe and you sail to New Zealand and it takes you twenty minutes', like-*

DVS: *#laughs#*

Thomas: *No in my old school our textbooks used to be like that and it was really dodgy questions. It was like weird...*

(I009)

Whilst pupils were aware of the artificiality of the contexts in questions, this rarely interrupted their capacity to solve them. The contexts could be ignored in solving the problems, as pupils would not have to incorporate any additional contextual information: applying the method was the focus.

Not all exercises were issued with the aim of developing new connections. Exercises were also given to consolidate pupils' understanding, in preparation for a test or to address perceived weaknesses. However, consolidatory tasks could achieve more than rehearsal of known techniques or results, as recall of results would be just the first element of more extensive deductive reasoning, and the application of additional understanding. Thomas described the "100 Club" challenge (see Vignette #1, p. 159) in a way which illustrates this. The task emphasised recall of results, rather than working them out anew. However, deductions were combined with recall in competing this task:

DVS: *so ... when you're doing the 100 club now... are you, say for example, working out seven times eight, or are you just remembering that seven times eight is fifty six?*

Thomas: *Yeh, well um yeh. But we don't really memorise ones like minus seven times minus eight, we don't remember those. We know it's going to be fifty-six but we don't remember, so we have to think more.*

(I009)

In contrast, when working on PCs teachers would usually direct pupils to websites or software that presented questions in a game or puzzle format and gave instant feedback on answers. Playing these games or solving the puzzles was distinguished from doing exercises by the presentation of the problems and the nature of the desired response. Game formats would use design, colour and fonts to enliven the presentation, and reward correct answers with an animation, or a score. Some computer-based tasks replicated more closely the form of exercises. The textbooks used by the teachers for years 7 and 8 had an accompanying website¹⁴, on which the pupils would work through exercises, and input their answers. This would also give instant feedback to pupils, and a percentage score at the end of each exercise. The choice of which task to use was the teacher's. The game format differed from exercises as they did not focus on as tightly as exercises on defined topic areas nor required detailed textual production from the pupils. They were

¹⁴ See, for example, <http://www.cimt.plymouth.ac.uk/projects/mepres/book7/book7int.htm>

described as helping “general knowledge” in mathematics (Jamie, 1004). Such tasks were used less as time passed and pupils progressed through the school.

Vignette #12	Scale factors and area	February, Year 8 Jake
<p>W guides the pupils through an exploration of the effect of enlargements on the areas of 2-D shapes. Pupils answer a short series of questions on the effect on lengths of increasing by a scale factor. W then sketches a 2x2 square on the board, and its image when enlarged with a scale factor of 2, then asks for observations regarding area. A pupil notes that the area has increased by a factor of 4. Another explains this by calculating the two separate areas and dividing 16 by 4.</p> <p>W then draws an enlargement with scale factor 3, and the new area is calculated. A chosen pupil (not a research participant) explains the nine-fold increase by calculating with the new lengths rather than the scale factor. W described this as working “from first principles”, and seeks an alternative explanation. A pupil asks “why don’t you just do four cubed, four to the power of three?”, but this question is passed over; W claims not to understand what the pupil is asking. W labels the diagrams with the scale factor and the proportional increase in area, and asks pupils whether they can spot a pattern. One contributes that the increases are square numbers; a second states that they are the scale factors squared. Each of these observations is reiterated by W to the whole class. W then checks pupils’ understanding by asking the areas resulting from increases by scale factors 4, 5 and 10. W is satisfied with the responses, but explains “we need to get you thinking in the correct manner.”</p> <p>W then illustrates the principle by enlarging 2x3 and 4x5 rectangles, and finding the areas of images with contributions from the pupils. He summarises that the new areas are found by squaring the scale factor and using this to multiply the original area. Two final examples (using rectangles) are given which the pupils are instructed to copy “word-for-word”. The examples calculate an original area, show the scale factor being squared, and the two values are multiplied to give the area of the image. One pupil asks whether it would not be quicker to use the lengths of the larger rectangle to find the new area. W agrees that this would be the case, but not with enlargements of more complicated shapes. Pupils are asked to work on questions involving compound shapes, finding the areas of enlargements by using the scale factors. However, many continue to calculate resultant areas from new dimensions.</p>		

7.3.3 Teacher work with texts, and pupils’ responses.

A significant part of the pupils’ work was in developing their creation of texts in response to the teachers’ texts. This could be aided or hindered by the presentation of the text; whether it was construction in a demonstration over time, or was presented in its entirety. Demonstrating methods step-by-step was seen to be crucial for pupils to be able to understand what they had to do. ZR explained:

ZR: *...teachers write down stuff for you. Say someone told you how to draw a picture. You'd find it really hard, but if they drew it for you, it would be easier.*

(I012)

When demonstrating methods, teachers would create texts on the whiteboard or IWB. Those texts would then act as an exemplar of the pupils' end product, but also enacting the constituent productive steps. Steps which the teacher judged could be assumed would be glossed over in the explanation, or reiterated verbally without an accompanying textual record.

Teachers would not explain methods in a uniform way. All teachers had different approaches, but a particular contrast was drawn between M, who would favour explanations of how algebraic manipulation was done, and W, who would use analogies and physical demonstration to convey structural relationships. Some participants saw the different viewpoints as helpful in filling out their knowledge. They saw the focus on written methods as helpful in revision and reassuring in terms of producing correct work, and the more imaginative descriptions as aiding them in remembering relationships. However, there could be clashes between the written methods they were expected to use, depending on how prescriptive the teacher would be and whether this was reflected in the marking. This could cause difficulties if pupils did not understand the reason behind the difference:

Alex: *It kind of confuses you, because... they're different methods... say W and M have different methods, then it's like, I'm not sure who to follow, and if W says there's one way of doing, like, fractions, adding fractions, and M says actually like a simpler way of doing it then you can sort of get caught like in two minds, like, in the test, what to do. So like... in M's test... the question is like something you've done for W, you're not too sure if you'd put W's answer or how he would do it, or like M's answer, so-*

DVS: *Shouldn't the two end up with the same answer, really?*

Alex: *They do end up with the same answer, but I'm saying like they're just different ways they get to the same answer, so it kind of confuses you.*
(1008.1)

As pupils developed experience and proficiency within the *working norms* and developed their own mathematical capabilities, they understood that in tests they could choose whichever method they liked, as it would have to be awarded due credit if it was correct.

Vignette #13	Revision lesson	March, Year 8
<p>M reminds the pupils that they have “known for some time” that they have a test coming up. They are told that their performances on the test will influence their position in the setting when classes are arranged for year 9. Their performance in the end-of-term test depends on how well they revise the necessary topics. In this lesson, they are provided with Revision Tests (see Appendix 3.9) from the school’s scheme of work. M points out that the tests are at the “Academic Level”; the middle of the three levels of difficulty that pupils can work on. Pupils can work on the sheets in any order, but are asked to do no more than three questions before checking with M that they have answered them correctly. Some pupils adhere to this instruction. Others work on rather than wait for confirmation that they are correct, as M patrols the room, offering assistance and confirmations when necessary.</p>		

7.3.4 Pupil production of texts

Participants were aware of how the production of texts helped them make progress, and their contributions revealed how textual production was identified with progress. Following or copying some text created by the teacher would sometimes be necessary in order to give pupils experience with a new mathematical structure or method; discussion could be inhibited until pupils had done some work. Applying numerical or algebraic logic enabled pupils to make sense of new methods, structures and problems, as they had had some experience of working within them, on which they could reflect (see Vignette 16, p. 203). Completion of a task and moving on to the next was the simplest way in which a pupil produced evidence that they knew the mathematics. Comparison against other pupils in terms of how quickly work was completed acted as a marker of one’s relative success, which took on greater importance as time progressed and when reviewing topics in relation to assessment, and in anticipation of public examinations.

Pupils accepted teachers' written methods as efficient means of solving problems and justifying solutions. Not only did these methods, if followed correctly, reach the correct answer, but they also offered a means of checking one's deductions for mistakes. There were no redundant steps in the methods shown, so they were quick without omitting the necessary steps required to gain marks. In practising recognised methods and checking the steps, pupils could be more confident of the answers reached. Practice also enabled pupils to recognise problem types and recall their associated methods. Relying upon a method became a guarantee of solving the relevant problems, but that guarantee depended upon the pupil computing individual steps accurately:

Thomas: There are lots of different ways, but this is the good one to do because it works. You know it may not work if you do a mistake, but this one works.
(I017)

In order for pupils to discover that a method worked, they would have to practise using it first. Once practised, they could rely upon the structure of the method to attain the answer, but had to attend to the arithmetic and algebraic details to avoid making mistakes in producing the solution. Following the method meant recreating each of the relevant deductive steps accurately, in order to reach the answer. Gaining credit for work depended upon getting the steps in the method correct, as well as attaining the answer. Producing a record of those steps would earn marks as a pupil progressed with a solution. Writing out one's methods became an important part of classroom practice, for the additional reason that one would have to do that in a test: reproducing a written method in its entirety became an imperative in terms of examination performance. There was some dispute as to whether a correct answer without supporting workings would receive full marks. It was believed by some participants that in examinations this was the case. However, when marking regular work, teachers would award credit where they saw fit, in relation to their current expectations of suitable method, as they had demonstrated.

'Showing workings' was a routine instruction to pupils. In doing so, pupils could demonstrate their deductions, observing conventional standards of rigour and justification. Careful attention to the steps in problem-solving, executing them correctly

and determining how to express these gave mathematical work an effortful character. Mathematical work should be consistently correct, based upon logical connections:

- ZR: *because maths is mostly pen and paper, and the other subjects you can explore*
Thomas: *yeh, explore and you can get things wrong and still be right, like in science we did like, making something up.*
Thomas: *You don't have to be, like in English you don't really have to think, you can just use your imagination, but in maths you really have to think to get the right answer.*
ZR: *Yeh. And like, if you come up with something, it's either going to be right or wrong in maths, but if you were in something like a drama lesson, you come up with something, it could be right either way.*

(I002)

In the exercises they had to deal with, pupils quickly moved on from routine application of methods, and had to determine how those methods should be used in more complex situations. This required reflection on the methods, but did not take on the character of exploration or creativity. Answers achieved in exercise work were characteristically either correct or not, and each stage in one's workings had to be 'correct' to gain marks. The notion of 'correct' had a dual sense of mathematically legitimate and pertinent to the problem being solved, being the next step toward the desired result.

'Showing workings' aided pupils by representing the structure in the problem in a form upon which the pupils could apply mathematical operations. Re-presenting the problem to themselves could also make implicit information explicit, which could then act as a prompt in determining the solution. Solving problems given in prose would be facilitated by drawing a diagram or expressing relationships algebraically. These reformulations could assist recall by making it easier to identify key characteristics of the problem. Conventional methods could then be applied, and marks earned if the steps in those methods were computed correctly and recorded appropriately. For those pupils who were confident with algebraic and numerical methods, using equations made problem-solving simpler, particularly if well-rehearsed forms of equations could be applied. Writing was also used by pupils if they did not know what to do in response to a problem. It would help to "jot

down” ideas (Alex, I018) and then reflect upon them in relation to the problem that had to be solved. This role of writing could be facilitated by the use of small whiteboards, on which pupils could test ideas informally then erase their jottings, before formulating a formal solution. (Teachers would judge when the use of whiteboards would be helpful.) The facility to efface their jottings was welcomed by pupil, who claimed it preferable to rough workings in their books. These workings would act as reminder of mistakes and corrections, even though they would be ignored by the teacher. The importance of devising methods in rough workings depended upon the aim in the task: if a pupil needed only to produce an answer, methods might be written down very roughly or not at all, whereas if there was a compulsion to produce a solution, then the workings might be kept neat or re-written afterwards.

Pupils’ expectations of future uses of their work informed their production. Workings were shown in order to convey to the teacher what had been done, what the pupils could do fluently, and how much they had achieved in any given time period. Pupils might present their workings in order to gain marks from the teacher, respecting conventions of layout, notation and clarity. Pupils could be content to leave rough working if they did not expect it to be marked. Pupils knew they would also look back over the work themselves; some would produce their written work with this future use in mind, and accompany their methods with reformulations of teacher or textbook explanations and additional explanatory notes.

The multiple functions of writing were summarised by Alex:

Alex: um, to show the teacher that you know your knowledge, that you know what you’re actually doing, to prove that you can not just think through it in your head, that you’re actually smart enough to actually show what you’re going to be doing, um also I suppose to um, to communicate your ideas, to figure out ways of doing things, especially when I’ve got lots of different ideas about how to solve a problem, which might not be right, but it’s always interesting to see what ideas you have.

(I022)

Pupils arrive late to this lesson in an excited and disorderly manner, so W imposes tight control over their behaviour. The pupils are made to line up calmly in the corridor, whilst W reiterates expectations and gives pupils instructions to enter and prepare for the lesson. Revision worksheets are handed out (see Appendix 3.10), as are compasses for those pupils who need them. W explains that pupils will need to have their own for the test. The pupils begin work on the revision sheets immediately, whilst W works through the first problem on the IWB as an example for those who need it. Some pupils ask questions to clarify concepts. W then walks around the room, checking pupils' responses to the first question. The work continues after W explains that completion of the worksheet will be that evening's homework.

7.3.5 Non exercise work – expressing mathematical understanding

Not all work done by pupils was in the form of exercises. Participants were familiar with other forms of tasks seen as belonging in the mathematics classroom. They reported optimisation tasks in which they had had to work with numerical and logistic information in response to certain constraints. These tasks required collecting information from websites and formulating plans of action, as a model of planning behaviour outside the classroom. They had also undertaken elementary statistical investigations into aspects of their lives in order to make both qualitative and quantitative comparisons. These were seen as exceptions to the normal pattern of work in both form and the function they played in their progress. Feedback from teachers on these tasks had been minimal in comparison to normal expectations, and pupils could not articulate what mathematical capability they had developed in doing them; they stood outside identifiable topics.

Pupils enjoyed alternatives to exercise work in which they themselves had to devise both questions and answers on given topics. This happened in the form of class quizzes, and what was known as the 'Marketplace' task. In the Marketplace, small groups of pupils would create posters detailing what they knew on a certain topic and posing problems. Members of each group would then go around the room, surveying other groups' posters, asking questions and taking information back to their own group. In devising questions one had to work out the answers as well, and the question had to make sense to the

pupils. In constructing problems which worked in these contexts, pupils had to be able to show they understood and had worked through the solution. As such, invented problems often recreated those the pupils had encountered when working through exercises in a given topic.

Vignette #15 Conversion of units; metric and imperial	March, Year 8 Jamie
<p>W displays two photographs of outer space, one very blurred and the other in sharp focus. When asked to guess their source, pupils offer jokey answers. W explains that the two pictures both came from the Hubble telescope. A pupil asks the question of whether they will be learning mathematics or physics. W tells a story that the original image was blurred because the telescope had been made using imperial measure, but the lens had been made to metric specifications. Errors in converting between the units had resulted in inaccuracies of thousandths of a centimetre and a defective lens. NASA had had to correct the lens, which required three space walks, at a cost of £20 million. The pupils are asked not to make this sort of error in the future.</p> <p>W then details various everyday examples of imperial units (ounces, pounds and stones; inches, feet, yards and miles) and the relations between them. Pupils are asked to calculate how many ounces in a stone and feet in a mile, and these calculations are shared. W illustrates how to set out workings, and the pupils then begin work on an exercise (see Appendix 3.11).</p> <p>At the end of the time spent working, W reads out the correct answers, and has written them on the IWB. Jamie copies down the answers to the last four questions (which he had not completed), and awards himself full marks.</p> <p>After the work has been discussed, W sets up a simple strategy game on the IWB, which volunteer pupils play whilst classmates offer suggestions. This is used as an entertainment after the period of work.</p>	

7.3.6 Mathematical structures

Actions by which pupils came to know mathematical structures became mechanisms which confirmed their claims to knowledge. A pupil's ability to devise connections and apply methods provided evidence for the claim that he knew some of the mathematics syllabus. The central form of work was problem solving, in which pupils had to make connections between given information and established mathematical results in both familiar and novel situations:

John: there's always something you don't know, and that you have to find out, using the information you do know, using some mathematical information you do know.

(I021)

This unknown was typically a number or a short formula. In finding the answers to problems and producing substantive solutions, pupils would simultaneously develop their mathematical capability and demonstrate it as mathematical authority. In contrast, contextualised qualitative problem-solving tasks such as those based on data gathering and optimisation enabled pupils to combine arithmetic or probabilistic considerations with practical and cultural needs. However, there was diversity of opinion as to the extent to which this work could be considered mathematical or just 'common sense'.

In working through exercises, pupils had to extend connections made between a basic method and increasingly complex scenarios. In doing so a pupil figured out "how it all fits together" (Aaron, I016). Building upon the application of standard responses to predictable tasks was supported by identifying structures within the problem, which aided recall of potentially relevant results. The means by which pupils did this could be influenced by the availability of different equipment and what they were permitted to do. Rough workings and annotations on diagrams acted as a means of revealing potential connections. Calculators could be used in an experimental way, with logic retrospectively applied once a pupil thought he had reached the correct answer.

Structuring solutions often required the recall of equations or application of algebra; once this had been done, it made a problem simpler, and recall of similar problems elicited models of solutions. In the process, recall of subsidiary results provided evidence that the pupil had learned the relevant preceding mathematical structures and methods. Successfully combining new methods and known results into one's working indicated the capacity to attend to the increasing difficulty of problems and develop more connections between topic areas. Figuring out the method to solve a problem might require decomposing it into constituent parts, and composing a solution out of those parts. These parts could be smaller problems, whose results were reserved until they were needed. In

doing so, pupils made connections and filled the gaps in what the teacher had told them (Aaron, 1016). Recall of elementary results was treated as common sense, but it retained its mathematical character as it had been established mathematically: one had to have done some classroom work to remember the fact and understand it.

Teachers ratified evidence of pupils' learning in multiple ways. Asking a pupil to contribute to the class discussion temporarily placed them in a position of explaining to the class, and producing the written example for others to follow. This indication that the teacher "trusted" the pupil (Thomas, 1005) to provide the explanation they wanted supported their claim to knowledge whilst implicitly valuing their previous performance. The pupil could also gauge from the responses of their peers whether they had made any obvious mistakes, whilst those peers also judged whether their conceptions matched with that which received the teacher's approval. For the spectators, understanding and anticipating the steps in an exposition also provided evidence of one's learning. When explaining themselves to the teachers, pupils had to recreate their application of a method and account for any adaptations they had made. In having their method reviewed by the teacher they received confirmation that not only their reasoning was appropriate, but that it satisfied classroom and topic requirements.

In order for written work to be marked correct, the form of the work had to satisfy productive expectations: traceable step-by-step solutions (omitting no steps considered important for the pupils at that time) which led to correct answers. In some cases, this could comprise a reproduction of an algorithmic routine, which would be credited as acceptable. The need to produce evidence and have it ratified by the teacher was a justification for neat work, but neatness also became a priority in establishing precision in diagrammatic work. It was a quality of 'good' work that one had taken care in its production, but also that it was mathematically precise.

Pupils often supported each other in applying their understanding as they worked, which in turn acted as evidence that they had learned some mathematics. Focusing on their own versions of the same problems meant that they could share ideas and contribute to each

other's solutions. Pupils felt they were more likely to succeed if they worked collaboratively, as they could repeatedly compare their partial solutions and justifications, which helped them to identify mistakes. When classmates had developed collaborative working habits, their assumptions and deductions were continually checked and affirmed in the midst of work (see Vignette 7, p. 180):

Thomas: well we were like working together and so like he said one bit and I said another and he said another and I said another and once we got our answers we checked them with each other... [I]t's more likely to be right if both of you agree than if just one of you agrees it is.

(I009)

However, if a classmate had produced a different method this would undermine a pupil's confidence in his own. The application of consensus could also apply to solutions, before pupils had compared answers. Variations which could not be immediately explained might need resolution and discussion before a pupil could continue with the work.

The dependence of problem-solving on recall of results privileged some mathematical structures above others. Some results had been memorised, such as times tables or key formulae, whereas others would be recreated in the moment, with the aid of the calculator or a short piece of mental or written arithmetic. Simple arithmetic at this stage would usually be done mentally, but marking practices would not distinguish between recall of elementary numerical results and quick written deduction.

Pupils' use of mathematical terminology developed alongside their capability. Technical terms could be used to identify topics, techniques or mathematical structures, but this was often informal, as an *aide memoire* to prompt further action. In discussions with each other, explanations would be given in informal language then rephrased into technical terms, if it was deemed necessary to shift from a productive focus to the structural justification. Producing written notes could also involve rephrasing explanations in one's own language and diagrams. Teachers would encourage the use of technical terms in explanations but rarely insist upon it, and would accept informal descriptions. Instances where a pupil could be penalised for failing to use a correct technical term were rare, but

became important when focusing on the IGCSE requirements. However, formal terminology identified those structures which were privileged within the activity as fitting items for investigation. Pupils had been equipped with the arithmetic or algebraic capability to investigate their properties. These structures and vocabulary then became central elements in pupils' descriptions of 'proper' mathematics.

In order to co-opt the calculator into action, pupils had to recall its capabilities and plan how to integrate its use. This was fluently done when pupils calculated a minor result as part of a larger solution. However, more sophisticated problems could remain difficult to solve until pupils recalled that the calculator had a pertinent function, and its symbol could be recognised. Use of the calculator distanced pupils' connection with mathematical structures: it was referred to in terms of asking a friend for an answer or as the calculator "creating" the mathematics (John, I021) at one's bidding.

Vignette #16 Laws of Indices

March,
Year 8
Aaron,
Thomas

After a communal recap of the meaning of index notation, with examples and mental calculations, M explains that the pupils will be able to progress "straight to the Academic questions"; the second exercise in the chapter, rather than the first. The laws of indices are written on the board algebraically, as they are in the textbook (see Appendix 3.12). After some brief discussion and examples of how to use these laws, the pupils begin the exercise. M has alternative worksheets to hand for pupils who find the exercise either too challenging or too easy.

Aaron and Thomas complete question 1, then move on to question 5. Thomas marks the intervening problems with question marks and they leave space on their pages for their answers to these. When they have completed question 5 they return to work on the intervening questions.

The focus on indices was sustained into a second lesson, in which pupils practised application of the laws with a similar set of problems.

7.3.7 Mathematical action: meaning and purpose

The characteristics of acting mathematically took form and inherited meaning and purpose from the structures of work. Teachers created orienting contexts through their expositions, offering possible interpretations of the purpose of the work. In some

instances the contexts represented action in the world outside the classroom, which was then related to the work. However, the power of a chosen context to generate interest or motivation would depend on the relations between the teacher and the class, their individual interests and the perceived of the pupils, as seen in §7.2.2.

Participants recounted incidences in disciplines such as Geography and Physics in which tasks called upon recognisably mathematical tools and structures. Working with graphs and statistical diagrams, and using equations to model physical situations had been done by the pupils in these disciplines. However, mathematics teachers did not make regular use of these potential connections. When the context given for the work was the curriculum itself, expressed in terms of topic connections, pupils had a clear sense of their aim. Teachers might explain that there was an imperative to improve the pupils' use of a particular mathematical method or make reference to achievement in upcoming or recent assessments. Revision of topics with test materials was welcomed by pupils who identified these tasks with the goal of their studies.

Participants could see uses for some topics they encountered in their lives outside and after school, specifically those relating to elementary shape, measure, time and arithmetic. Financial arithmetic was seen as an obvious use of the mathematics learned in school, but the ways in which this would develop from their school methods could only be conjectured by participants. The applicability of algebraic approaches to problems also came under dispute, as the participants could not construe social scenarios in which algebra (as they had used it) would be used. Hypothesised problems were insufficiently sophisticated to require algebra, which reflected the pupils' limited experience and the lack of meaningful contexts in which algebra had been applied:

Jake: *I don't know how algebra helps you to do anything at all. It's just equations.*
Thomas: *exactly!*
Jake: *it's not like in the bank they're gonna go "x equals five plus three equals two" (sic)*
Jamie: *you might have to do that if you're like a footballer, and they pay you and you have to work out how much salary you get in one week*
Jake: *that's not algebra*

Jamie: yeh but you could actually make it into algebra
ZR: and I was thinking say, say if you worked in a bank...could it be like if you're lending out a loan to someone of say a sum of money and then like you have say, they already had say a hundred pounds and they needed like nine hundred pounds you have to like work out say "x plus a hundred" which they already have, so you have to find out what x is.
Jamie: yeh but you're a banker and you've given them nine hundred and they already have one hundred, so you'd know what x was, it would be nine hundred.
ZR: exactly, if they
Thomas: it gets more complicated like, x minus five point three four
ZR: it's like if they say to you "oh I need like six hundred and fifty five pounds" and they have fifty five pounds, or something like that, you know x equals, x equals plus fifty five, right, no x plus fifty five equals six hundred and fifty five, you have to find out what x is. Which is like algebra.

(I008)

The use of number saturated pupils' work, and they identified that they had begun to act mathematically when learning how to count. This had been done at home, as had learning basic arithmetic. A lot of time in primary school had then been spent working on arithmetic and doing sums. However during the time of the research, pupils undertook written arithmetic methods only when working with 'big' or 'complex' numbers. Arithmetic was no longer considered an "important" part of their work (Alex, I022). The pupils' experience of arithmetic was in using it to solve other types of problems.

A defining feature of mathematical work was the production of answers which were either right or wrong. These answers were attained by making valid step-by-step connections between conceptual structures: the steps were written up as solutions in which each statement represented the result of such a connection. Through this transcription, the deductive method became a substantive solution. Pupils had to avoid making mistakes throughout their work, in order that their solutions would be correct from beginning to end, and the answers accurate. 'Being correct' stood as a central function of mathematical work in gaining credit for the pupil.

7.4 Summary and research questions

In this chapter I have offered a grounded presentation of the data collected through the interviews and observations with the participants, supplemented by vignettes of classroom activity. My aim has been to convey the activity of developing mathematical capability, from the pupils' point of view. Here I have shown the extent to which social considerations were a focus for the pupils. §7.1 showed that participation in ordered school life was placed by pupils within the context of social relations and personal characteristics. However, these influences did not merely frame the resultant action in mathematics. §7.2 shows how teachers and pupils together established and maintained the priorities which informed the characteristics of work. Mathematical action was shaped by expectations of others' reactions and their subsequent social impact, whether upon interpersonal relations or on a pupils' standing as seen in his grades and setting. Within work, mathematical norms (§7.3) emerged as those productive behaviours in which pupils were supposed to develop proficiency. The behaviours which were valued related to questions of purpose, but these questions were rarely addressed in the classroom, with greater emphasis placed on efficacy and satisfying teachers' expectations.

In the next chapter, I work with this grounded description to develop my understanding of the aspects of the activity which admit the emergence of the object in a structure of purposeful productive action. I uncover the contradictions inherent in the activity and explore their developmental potential. A list of mathematical characteristics identifiable in the methods devised by pupils and in the ways they produced evidence of their capabilities is provided in Appendix 4. However, these characteristics of action need to be seen in co-constitution with activities in which they take on meaning and purpose; understanding the potential for this to happen guided the development of my research question. I would consider the extent to which these socio-mathematical norms (McClain and Cobb, 2001) influenced the construction of the object, and the limitations these might place upon the pupils.

Through this grounded analysis, my research questions reformed to take account of the pupils' social focus and the primacy of social order in the classroom. The methodological aspect of my enquiry became concerned with preserving the pupils' viewpoint whilst still enabling the concretisation of the object of my research.

Before the grounded exploration of interview data, my questions had been stated as:

How do social interaction and tool use in the mathematics classroom position pupils, in relation to personal aims and cultural categories, to locate and value meaningful mathematical action in the world, constituting personal and practical transformation?

How can I best implement a methodology to trace dialectical subject-object development and contradictions in classroom activity using the framework of third generation CHAT and Marxian categories?

How can a methodology derived from CHAT and developed in the activity of research equip me, as a teacher-researcher, to discuss teaching practice with a focus on the transformability of subject and object?

In light of the emphasis placed by pupils on the meaningfulness of social relations, and the relative lack of emphasis placed on tool use, I had to reshape the questions to preserve the pupils' sense of meaningfulness in action. The apparent lack of change in the actions permitted in the social structure of the classroom also had to be addressed: subject-object development ought to entail shifting productive expectations, according to CHAT. The only change I had been able to observe was the shift between the value placed upon social relations and grade achievement.

I was also troubled by the extent to which disruptive behaviour from pupils, whilst not a frequent feature of the activity, was nonetheless considered part of the activity with which teachers had to deal, rather than a disturbance to it. This issue had to be explored for its theoretical implications as well as its meaning within my research.

My questions were thus re-formed, to reflect the mutual contributions made by the structure of activity and the pupils' own subjectivity:

How does the pupils' focus on social relations and grade achievement, as supported by the activity, position them to incorporate personal aims and cultural categories, in learning how to locate and value meaningful mathematical action in the world, constituting personal and practical transformation?

To what extent does the social stability of the mathematics classroom support or inhibit dialectic subject-object development, as expressed in the framework of CHAT?

How can I operationalise the constructs of the CHAT framework, from the grounded description of the mathematics classroom, to describe processes of transformation and the nature of disruptions to activity?

8 Applying CHAT – towards meaning and value

The grounded analysis of chapter 7 indicated that the *ordering* and *working norms* created a space in which *mathematical norms* could develop. In this chapter I re-present the mathematics classroom as the pupils' activity system, in order to understand how the pupils are positioned for their production in relation to the cultural transmission of meaning and the interplay of values. Adopting the Hegelian conception of the subject as the result of dialectical interplay between *Individual*, *Particular* and *Universal* aspects of activity (Blunden, 2007) entails recognising positioning as the outcome of both pupils' and teachers' actions. Aspects of the teachers' activity system are presented as they pertain to the formation of the pupils' activity system, as background. In describing the two systems I build upon the pupil participants' descriptions, with reference to data from teacher interviews. This enables the construction of a picture which is centred on the pupils' rendering of their experience, but accommodates a multi-voiced description in which I illustrate the dialectic interrelations between and within the systems.

In undertaking a macro-analysis of the classroom, I had to judge the extent to which my use of the theoretical framework raised intra-personal issues such as individual motivation, mathematical ontology or epistemological development. This was not the aim of my research, yet I could not discuss the constitution of the activity without some reference to these issues. They are discussed only to the extent that they inform the description of the pupils' production within and by the activity.

In §8.1 I discuss the roles of teacher and pupil in the classroom communities and their interrelations. I then consider the ways in which the division of labour relates to the ZPD, and how pupils both use and develop mathematical capability in lessons (§8.2). I then turn attention to the values and meanings embedded within mathematical tasks pupils undertook (§8.3). I conclude each section with discussion of the tensions which were revealed in the analysis.

Connecting the theoretical framework of CHAT with the grounded description entailed a simultaneous operationalisation and exploration of the concepts of CHAT, whilst pursuing

my aim of understanding how the pupil is produced in the activity. In offering a concrete description of the pupils' activity system I explore the affordances of the EMT. This continues the theoretical engagement narrative. The axial codes that structured chapter 7 offered an order of priority in which to consider the data, enabling me to preserve the pupils' attributions of meaning and importance. In turn, the concepts of activity theory enable articulation of the mediation of pupils' action through the tools and stable structures of the classroom. In §8.4 I summarise the tensions and reflect upon my use of CHAT, before reviewing my research questions (§8.5).

8.1 Classroom communities

In this section I explore the mutually constitutive roles of teacher and pupil. I show how pupils are positioned relative to the object of study and to practices in which that object can be made concrete. Through this exploration I begin to substantiate the constructs that form the EMT. In chapter 7 the split between teachers' actions and those of pupils had emerged from adopting the pupils' point of view. Here I briefly consider the positions of teachers and pupils in their respective activity systems before bringing the descriptions together to explore their dialectical formation. I consider how roles were constituted in relation to mutual expectations and working together toward the object.

8.1.1 The pupil as subject in a community

The division of school experience into a range of disciplines was largely accepted by pupils as a framework in which they might develop their individual interests and talents. In the school, compliance with institutional expectations was the norm: respecting the allotted timetable and institutional bounds; being adequately equipped for lessons; engaging with tasks issued. Pupils' part in the division of labour, under direction from teachers, was generally fulfilled, and they usually complied with the constraints placed upon social interaction in lessons. These limitations applied to all pupils, regardless of achievement or setting. Pupils appeared complicit in their subordination to the structure of classroom activity and the institutional arrangements. However, perfect co-operation with the expectations of the classroom did not happen automatically, but was learned through

personal or observed experience. Pupils could also react against limitations on behaviour and take opportunities to undermine these restrictions, creating disruptions to classroom action.

In contrast, teachers' roles in the pupils' community were constituted by the ability to tell pupils what to do, and the relative liberty they enjoyed in determining their own actions. Teachers had authority to interrupt and redirect pupils' behaviour as they saw appropriate, in relation to school rules and communal expectations, using the scale of institutional sanctions. Teachers freely inhabited the school, and could exert their authority at any time or place in the school day. Enforcement of the communal expectations of the pupils provided a means for teachers to establish working patterns in their classrooms. The teacher's role was supported by a range of institutional mechanisms, and nuanced in space and time: teachers could exert greater authority when conducting lessons in their own discipline and their own classroom, laid out to aid communication and supervision in a manner of their choosing. Setting and syllabus coverage were managed by teachers, as were day-to-day concerns such as the choice of tasks in lessons, availability of extension tasks and the use of specialist tools. Pupils' achievement was evaluated and guided by the teachers.

Within their own activity system, the position of the pupil as subject emerged as one of subordination to the structure of the activity, and to teachers.

8.1.2 The teacher as subject in a community

Whilst teachers occupied a degree of freedom within the classroom community, their work was constrained by institutional expectations and curricular demands. Responsibility for establishing and maintaining pupils' participation in line with institutional and curricular arrangements derived from the object of the teachers' activity system, the pupils' mathematical capability.

Teachers felt they had responsibilities pertaining to pupils' social and emotional wellbeing, alongside their mathematical development: "you're *in loco parentis*, as opposed to a

sergeant major” (K, I028). This consideration informed classroom relations, the nature of expositions and the teaching of topics:

W: *If I feel one topic flows through another topic or they might want or need a clean break, so they can almost have a clean slate for it, then I'll choose my topic carefully.*
(I027)

In arranging topics within the scheme of work, topicalisation became a tool by which teachers managed achievement and engagement. However, decisions were influenced by the arrangement of topics within the curriculum materials, and the availability of alternative resources. Such materials supported the teacher in supplying structured tasks but also constrained the choice of topic connections and tasks that could be offered. The content of lessons was determined within a context of available resource and time, while teachers tried to respond to individuals' needs:

K: *Given all the resources in a perfect world you would have the correct worksheet for every individual child and write on them. But sometimes you have to improvise in a way that you think in a perfect world you wouldn't. And that could be due to resources or time.*
(I028)

This responsiveness reflected concern for the pupils, but also the belief that they would achieve more in a positive “social-psychological climate” (Haladyna et al., 1983).

The teachers worked “as part of a team” (W, I027) who provided mutual support and advice and co-ordinated their teaching within schemes of work. Colleagues learned from one another as teaching and classroom management ideas were shared. Schemes of work and the scheduling of topics in anticipation of common tests both supported and constrained their choices. However the common focus still afforded variety amongst approaches, and the teachers tried to convey their own particular interests: K considered mathematical study to be part of an enterprise towards discovering truth; C spoke of making sense of the day-to-day world; W emphasised using mathematical modelling of situations to solve physical problems; M focused on learning mathematics as a personal achievement. Teachers recognised that they encouraged slightly different written styles

and would anticipate dealing with any differences that might arise as pupils changed teachers:

W: *... [pupils] very much get used to that style, layout and structure and thinking about it. When a pupil comes from a different set, you find that they've been approaching it in a slightly different way. Not massively different, because it's maths anyway, we work from the same principles.*
(I027)

It was believed that the pupils benefitted from a variety of approaches amongst teachers, enabling them to experience different values within the discipline.

Teachers adopted long, medium and short-term approaches to planning, anticipating connections that the pupils should be able to make in their future work:

M: *um I think obviously my main aim at the end of the day is although they're only year eight is them achieving the best grade they can at GCSE so I do tend to make, to think if I'm doing something "is this an IGCSE topic?", "what kind of route do they have to take with it?"*
(I014)

Teachers felt that time pressures were tight, and in meeting the needs of the pupils and the institution, practice was guided by ensuring the pupils reach "the level they have to get to" (W, I027). Expectations of high levels of examination attainment were sustained in the school. Curricular obligations also extended to the elements of teaching practice: there was an expectation that all teachers at the school would offer pupils appropriate opportunities to work using ICT.

The above concerns created nuances in the activity without challenging the curricular basis or object focus: the pupils' achievement had to come about in a communal system, as calibrated against the visible mathematics of the syllabus. Teachers' own interests could only be conveyed implicitly, not as the aim of pupils' work. As will be seen, ICT use complemented, rather than informed, the working norms.

8.1.3 Community relations: enacting roles

Exploring teacher-pupil interrelations revealed the dialectic relation between their roles, informed by mutual expectations and responses. Teachers' approaches to establishing and maintaining productive relationships with pupils appealed to a mutual recognition of these roles. This was served by teachers establishing a classroom persona before relating to the pupils as individuals:

M: ...I think in the first week or so, it's very formal... I think I would stand more and talk at the board rather than talk to the child. I think once you get to know the child, that to me is such an important thing, 'cause you bring it down to their level.

(I014)

Institutional and classroom obligations could then be seen as shared by teachers and pupils, rather than imposed by teachers. This approach stood in opposition to the younger pupils' focus on personal relationships: positive relationships could only develop within a framework of recognised social order. Once roles and expectations had been established, they became the basis on which a teacher could get to know the individual pupils. This enabled teachers to tailor their mathematical explanations and the extent to which they became involved in each pupil's work, which in turn furthered their understanding.

This knowledge could be seen to operate in choosing who should participate in tasks such as class discussions. In encouraging participation from all pupils, teachers reported they would not pick only those they could guarantee would give the correct answers:

W: Um, one I'd get a very false idea, assessment of how the class is performing. And also I think it would be quite demoralising for the other pupils, basing their progress purely on the pupils who are getting everything right... If you're only asking the pupils who get things right, the pupils who do not get things right won't be happy to have a go at things if they're not sure they're going to get it right.

DVS: So, encouraging their participation?

W: Yes. Having a go at things, and making it okay to get things wrong, as long as they're putting the effort in.

(I027)

Teachers' responsibilities coalesced around engendering productive participation. This need took priority over displays of perfect mathematical reasoning; teachers understood that making mistakes and reflecting upon them was a feature of learning. Pupils were recognised for the quantity of their object-oriented production and the effort it was assumed to indicate, before its mathematical qualities.

Exploring the mutual constitution of classroom roles revealed that all parties appreciated aspects of regularity in classroom activity, although not necessarily in the same ways. Pupils and teachers recognised the need for teachers to maintain pupils' engagement whilst managing their conduct and keeping their interest and enthusiasm. However, the authority invested in the teacher had to be employed with sensitivity to pupils' development. The pupils were understood to be developing skills of participation in social activities, and thus were not expected to enact their role with exclusive focus on the object of study. The incomplete overlap of pupils' and classroom goals (Kaptelinin and Cole, 1997) was understood by the teachers. This issue could be most pressing during those periods in which pupils were not studying for formal assessments nor focused on attaining a particular report grade. The focus of classroom action did not "always have to be mathematically interesting" (W, I027); teachers provided contextualisations of topics and methods, related to the perceived interests of the pupils, and offered games and activities in which it would be entertaining to participate. In my analysis, sustaining productive participation emerged as a primary aim for the teachers in developing pupils' capability.

Pupils' expectation that their compliance and effort be recognised and rewarded balanced their acceptance of the teachers' dominant role. Failure in this regard could destabilise a teacher's authority and lead to unproductive pupil-teacher relations. Good relations also related to perceptions of how much progress was being made and to whom this could be attributed. The teachers were aware of the emotional impact of classroom action. C reported that pupils would "put their hand up when they know they are right, just to be told 'yes, you are right'" (I020). The potential for teachers to affect pupils was related to

their institutional and mathematical authority, which together could be used to make pupils feel “dumb”:

K: *And a teacher shouldn't go round making people feel like that... Because we're totally capable of doing that. There can be an extraordinary condition of humiliation in our [work]... I always talk about it, the sheer importance of power... And so you have to be super-careful to try to dispel that and sometimes you can see in their face, their face kind of relaxes when they realise that you're not doing that.*

(I028)

Pupils' inclination to focus on social relationships and impute personal motives to teachers' actions overlooked institutional obligation or curricular priority. Teachers understood that making pupils feel inadequate was not only counter-productive in terms of engendering participation, but also undermined pupils' confidence and concentration on mathematical tasks. This concern derived from the *Gegenstand* aspect of the capability the pupils were expected to adopt: notions of correctness could not be avoided and pupils' production was influenced by the extent to which valid structural connections had become part of their *Predmet*.

The asymmetric positions of teachers and pupil were reflected in other interrelations within the classroom community. Teachers and pupils shared relationships bound by obligations and institutional expectations. In contrast, each individual pupil sustained degrees of friendship with his peers, with personal expectations informed by institutional particulars and participation, as seen in collaboration over work and comparisons of achievement. Pupils' achievement in individual lessons and across time influenced friendships through positions in the classroom community. Extension work and a high setting position conferred a relative status of success, as did permission to help others, which involved a temporary elevation of the pupil's position in the community. Conversely when pupils conspired to misbehave they undermined the teacher's role and reinforced their friendship structures. These acts revealed shifting allegiances amongst pupils, continually reshaping around shared foci and expectations. The classroom community would also occasionally be extended by the teacher to include other teachers and the

pupils' families, through formal reporting and Parents' Evenings. If necessary their input could also be sought to engender more compliant behaviour and engagement with classroom tasks. These temporary extensions to the community served to reinforce the activity structure, in response to any action that might undermine it.

The teachers' role required that they respond to disruptions with repair actions that re-established focus. These repair actions might constitute intervention or issuing alternative instructions for a pupil to follow and were the result of on-the-spot decisions. Transgressions considered minor were routinely addressed with a brief reprimand, whereas behaviour which prevented others from working or which the teacher judged unacceptable would result in halting the pupils' behaviour to give a serious reprimand or impose punishments. If a pupil found difficulty with his work and appropriately made a request for help, he could expect to receive a supportive response from the teacher. However, considerations of social order would predominate over mathematical concerns;

W: If a pupil was to interrupt another pupil's work or call out in order to draw attention-seeking purposes [sic], then that would have to be dealt with, in a disciplinary manner.

(I027)

All interactions between teachers and pupils took place within the framework of school rules (both explicit and implicit). In responding to pupils' behaviour teachers usually suppressed transgressions which threatened the balance of roles in the classroom and the ability of pupils to remain focused on tasks. Focus was maintained through the imposition of order; predictable minor transgressions were pre-empted in teachers' classroom management. Reference to school rules and classroom expectations enabled teachers to maintain order, alongside the use of school mechanisms of punishment. The rigour with which teachers upheld their expectations influenced pupils' adherence to the implicit and explicit rules of the classroom. As they exercised their agency and observed others, pupils learned the limits of acceptable behaviour and how each particular teacher would enforce those limits. In acting out their subjectivity, the pupils' transgressions could be understood in relation to their social focus, arising from the available tools, the complicity of other

pupils and the ease with which the teacher could supervise the action. Over time, as the overlap of goals (Kaptelinin and Cole, 1997) converged upon examination achievement, disruptions diminished.

Community roles provided the basis of working norms through which pupils could develop their mathematical capability. Pupils participated in tasks oriented toward an object which they would discover only through participation in those tasks: their participation depended upon the understanding that the teacher would issue tasks which were appropriately oriented. However, pupils had little means of judging this suitability and had to trust the teacher. This relationship of trust was explicitly prioritised by M as the basis of productive relations:

M: absolutely I mean my first port of call is to gain their trust and then if I've gained their trust they're going to trust what I'm telling them.
(I014)

This trust was sustained when pupils understood a purpose of work they were doing, or believed that they were making progress: a shared focus on attainment in tests contributed to good working relations. However, requests for explanations of broader purpose were treated as if questioning the purpose of the discipline and the teacher's aims; pupils were expected to trust the teachers. However, teachers' varying expectations of pupils in relation to different settings and tasks, if not clarified, had the potential to undermine this trust. Failure to sustain commitments could have a negative impact on pupil-teacher relations.

In relation to the aim of pupil participation, my analysis revealed the mutual constitution of roles and rules. The rules did not operate unless upheld by the teachers and observed by the pupils, whilst the teachers used the school rules to establish and maintain their position in the community. Pupils recognised that teachers could enforce their will, had institutional support for their decisions and could impose punishments for non-compliance. In my analysis I came to see the rules of the community as tools the teachers could use in relation to the aim of participation.

8.1.4 Establishing working norms

Within a framework of good working relations and mutual contributions to classroom order, clear communication of expectations enabled the establishment of the *working norms*, which contributed to achievement in lessons. Sustaining working norms thus depended upon the pupils' productive participation, adding to the value of their compliance in the activity.

In the medium-term, setting based upon prior achievement established productive expectations for pupils and teachers. Teachers would prepare tasks and plan lessons with reference to the position of the set. These plans responded to and anticipated pupils' productive capability, across periods of time, with topic coverage and declared expectations of attainment being used to manage pupils' development and interest. Consequently, I saw setting as a tool used by the teachers which mediated the pupils in relation to mathematical capability, producing and distributing them throughout the school. Setting reflected pupils' prior productive participation being used as a predictor of their future participation and capability.

In the short term, teachers also responded to pupils' production in the midst of lessons. This would be necessary if the productive requirements of a task were too challenging or too easy for some pupils, requiring repair actions. These actions would take the form of extra, unanticipated support, such as interrupting the pupils' work to share and discuss a common difficulty or point of interest, or directing the pupils towards different tasks. Establishing and maintaining this intersubjective understanding informed medium-term planning in teachers' judgement of the class' achievement. The experience of the pupils was mediated through whole-class judgements such as these. For example, talk about the wider applications of mathematics would occur only if the teacher judged the class' achievement created a suitable opportunity.

Conversations with individual pupils, under a teacher's control, brought individual achievement into shared focus, and acted as the means by which teachers and pupils

sustained productive relations. Understanding a pupil's logic aided the teacher in guiding the pupil to extend his capability:

K: ... to basically try and give the right clues... to try and help him make the link. Because every time he makes a connection himself, then you've won, right, because you're empowering him. If you make all the connections for him, you're not doing anything for him.

(I028)

However, the teachers' need to manage and maintain the ZPD prevailed, in spite of the potential for intersubjective understanding to develop. The teachers knew which aspects of mathematics practice pupils had to focus on, and offered examples and problems which served this need. The teacher was at liberty to generate and supply questions, but in doing so, retained control over the pupils' exposure to mathematical constructions. In the need to observe the constraints of the visible mathematics of the curriculum, dialectical formation of subject and object was inhibited. If pupils were invited to introduce problems, this could introduce unexpected complexities or problems which could not be solved within the bounds of the current ZPD. Pupils' queries which went beyond these bounds were managed carefully: questions would be judged for their suitability in relation to the aims of the lesson, and possibly curtailed. In response to a question, a teacher might decide to "pare it back... if the answer isn't going to be comprehended" (W, I027). This could engender negative reactions from pupils and undermine productive relations, but the authority of the teacher lay in the knowing curricular requirements and understanding what capability could be constructible within the constraints of time and resource in a given lesson.

Through their supervision of the activity, the teachers established and preserved the distinction in classroom roles. In doing so, they unified their mathematical and institutional authority. By respecting this connection, pupils were able to orient themselves in the institutional and classroom order. The activity was presented as a static entity, which the pupils should observe.

8.1.5 Discussion and tensions revealed

Having explored the mutually constitutive roles of teacher and pupil as they were enacted in the classroom, I was able to describe the pupils' position in relation to the object of their activity. Pupils had no choice other than to accept their own distribution between disciplines as offered by the school. There were no means of establishing alternative arrangements, and failure to comply with the distribution within the school engendered negative consequences for the pupils. This distribution also happened in relation to developing mathematical capability with setting practices. Pupils were in a subordinate position to the distribution of themselves and their behaviours, and to become successful they had to align their personal aims with those of the school. Teachers responded to this position of subordination with sensitivity to individuals' developmental needs, within the constraints upon their choices, and with classes of pupils in mind. Productive participation was identified closely with individuals' development; engendering this was seen as communally beneficial in sustaining achievement, and required sensitivity on the teachers' part to those elements of mathematical capability that could be used as a concrete basis for further connections. However, teachers were constrained to developing pupils' useable mathematics and establishing expectations of constructible mathematics with respect to the distribution of structures within the visible mathematics of the curriculum. Public examination grades represented the standards against which pupils' production was assessed; pupils were distributed throughout the system with reference to these standards.

To this extent it would appear that the pupils were positioned by the school in subordination to the distribution of mathematical capability, but describing the community relations revealed the importance placed upon the maintenance of roles in the activity. Compliantly inhabiting the role of the pupil was encouraged and valued in classroom relations before correct mathematical work or insight. Social order predominated as pupils learned the "dance of agency" (Pickering, 1995), both within the activity and in relation to the object of study. This expectation was sustained by both pupils and teachers. Pupils were expected to follow the teachers' lead as they exercised

their authority in relation to developing mathematical capability. In return, teachers were expected to fulfil their responsibilities, aiding the pupils in making identifiable and demonstrable progress and distributing them appropriately. From this basis of trust, pupils sustained their role through participation, enabling their distribution throughout the school and the emergence of working norms. Non-production of work did not interrupt the distribution of pupils; rather it contributed to the evidence used in the distribution.

Teachers' and pupils' expectations of constructible mathematics were mediated by the setting distribution, placing the pupils in the midst of collective judgements regarding their capability. These judgements might become nuanced during lessons, as a teacher engaged with an individual's production, but were set against the development expected of the class. The ZPD, as constructed by the teacher with the available resources and time, had to be maintained and traversed as expected. Divergent actions were curtailed. As such, both the activity and the anticipated object appeared impervious to influence by the pupils. Both the subject and the object of the activity were distributed within the school, and so revealed as subordinate to the purposes of the school.

Through exploring the positioning of pupils in relation to the object I identified communal systemic tensions. The predominant tension which came to light related to the organisational presumption that the mathematics classroom was oriented toward developing mathematical capability (see Kaner and Lerman, 2008): engendering participation emerged as a primary aim. Teachers encouraged participation by appealing to extrinsic motivations when possible. However, the curriculum predominated: teachers were aware of the effect of the *planning paradox* (Ainley et al., 2006) and the object of study could not always provide entertainment. In these circumstances, pupils were expected to comply with the provision of the school and patterns of action in the classroom without dispute. The use of extrinsic motivation offered the potential for interplay between overlapping goals in relation to the object. Teachers supported pupils' engagement by according value to the produced work: the tangible aspects of this were in

the exchange value of positive relationships or good grades. I refer to this tension as the *Orientation tension*.

The responsibilities of the teacher resulted in a need for classroom order, in which the pupils had to play their part. The need to establish this order preceded and sustained throughout the mathematics teaching. Teachers explained this as the part they played in the adolescent pupils' social development, and recognised the multiple motivations that lay behind pupils' behaviour. In their compliance with (or rejection of) the activity, pupils produced themselves and contributed to their distribution throughout the school system, which could introduce conflicting motivations. When pupils exercised their subjectivity and disrupted the activity this was curtailed by the teachers, which was anticipated as a routine part of their practice, which would take priority over mathematical development. I refer to this as the *socialisation tension*.

Several disparities reflected the pupils' subordinate position in the classroom. Teachers could halt and redirect action when they considered it appropriate, yet the pupils could not. Punishments and reprimands were deliberate disturbances in community relations, used as repair actions to maintain complicit participation. Pupils had no recourse to such manoeuvres. Pupils were expected to trust that the teacher would make choices that contributed to their development and to submit their will to those choices, but this trust could easily be broken. This related to the pupils' inclination to impute personal motives for teachers' actions, rather than curricular and institutional ones. I refer to this as the *subordination tension*. This tension relates to the *socialisation tension*, but also reflects the issue that object-orientation was difficult to sustain without a strong idea of the object in its concrete relations (*Predmet*).

The role of the teacher as the member of the community who had the responsibility to maintain order encompassed managing and overcoming disruptions. Consequently, disturbances could be seen as an inherent part of the activity and, if managed effectively, become subsumed within the flow of action. This presented an analytical difficulty in distinguishing between disruptions which might be accommodated as a matter of course,

and disturbances which indicated tensions in the activity. I saw this analytic indeterminacy as reflecting a tension between the roles of teachers and pupils: in anticipating and overcoming disturbances caused by pupils' behaviour, teachers maintained order and the stability of the activity. Acts of pupil subjectivity could unintentionally emerge as disturbances, indicating the inherent strains of individual membership in a communal activity. Teachers' notions of object-orientation predominated, supported by their institutional authority and awareness of the relations between the required visible mathematics and constructible mathematical capability. At this point, I posit that the teachers' position and obligations within their own activity system inhibited harnessing the generative potential of such disturbances, and oriented them toward maintaining the balance of roles in the community. I refer to this as the *stability tension*.

Describing this tension deepened my understanding of the roles of teacher and pupil in the classroom. The object of the activity as I construed it was the pupils' capability in mathematics; a feature of the subject himself. However, as an anticipated object in thought (*Predmet*) pupils were aware that they would have to incorporate the logical restrictions of mathematical structures (*Gegenstand*) into their capability, which would be learned through experience and production. This continual imperative toward personal transformation meant that the subject was constructed in the activity as inherently deficient. Accepting object-orientation meant that the subject was expected to modify their behaviour to fulfil their role, taking the lead from the teachers; authority could only be established on the terms of the classroom. The imperative to assess pupils' production was derived from and reinforced their position as error-prone learners. In both working towards the object and learning how to participate in the system, the subject would make mistakes: their subjectivity was inherently tensional. I refer to this as the *pupil fallibility tension*.

8.2 Working toward the object: zone of proximal development

In this section I consider the mutual constitution of division of labour and community roles in relation to the ZPD through exploring the *Working Norms*. I first explore how communal

positions influenced the formation of the ZPD, then how pupils exercised their subjectivity within it. I next consider how the production of written work operated as both the mechanism by which pupils made progress and as evidence of this progress. I then consider the development in pupils' authority before considering the tensions revealed.

8.2.1 ZPD and division of labour.

Within the order of the classroom, teachers' dual authority was demonstrated in showing the mathematics the pupils were expected to learn and what had to be done to learn it. These requirements were expressed simultaneously in expositions which combined mathematical expectations and productive requirements. In explaining the "pretty standardised" pattern of lessons, W recounted the aims of expositions:

W: I explain a starter, [to] get them to think, use their previous knowledge in a different way...then apply that knowledge to a slightly different situation, to extrapolate or take that knowledge a bit further. And then put into context to how to practically use it to answer mathematical questions, and that sort of thing in the end what they actually have to do.

(I027)

The dual instruction thus outlined a ZPD for the class, with productive action constrained within the classroom order. That ZPD was also delineated in terms of the knowledge and skills that were assumed, implicitly expressed through the omissions and inclusions of any exposition. Sensitivity to the long-term aims for pupils' learning entailed delivering tasks through which current mathematical actions could become operational, and available for use in making further connections.

As such, the ZPD was established from the teacher's appreciation of the pupils' current and potential capabilities, derived from produced work, assessments and pupil responses, and was a 'best fit' response of what the pupils should learn next. The teachers had to take into consideration all pupils' achievements, not only those who put themselves forward (see W's comment, p. 206). The teacher would take into account the capability the pupils were expected to develop and also the most typical problems they would encounter, connecting the constructible capabilities of the pupils with the visible

mathematics of the curriculum. In managing the development of groups of pupils, the teachers effectively maintained a collective ZPD. This ZPD sustained a conventional relationship between the actions and operations the pupils were expected to adopt and positioned the pupil uniformly in relation to their own developing capability.

Teachers illustrated paths through this ZPD by modelling the production required to develop fluency in methods. Working through examples with pupils not only demonstrated required methods but also showed teachers “getting involved” in mathematical action. In doing so, subject development was constructed as productive participation: internalisation of a method could only happen from this basis, so practice became a priority. Such demonstration illustrated the practice of solving problems and also revealed the use of writing in mathematical action:

M: I think they see us getting involved a lot more in maths that way...I think that teaches them a teaching point of showing working out,... even if it's something simple I'll write it down and they can highlight, "ooh that's where you could have gone wrong if you'd have done it in your head."
(I014)

The collective approach oriented teachers toward exemplifying methods the pupils should use in response to specific problems, which they could be taught as a routine of written operations whilst only briefly considering the structural justifications behind them. Teachers would make judgements as to whether fluency with and recall of a method could be prioritised over understanding of the underlying connections. This priority came to the fore with pupils in lower sets:

K: If I'm teaching a bottom set year ten... I think that recall is everything, so revisiting things all the time is very important... And I think that with more experience you get, you're more aware of the things people are likely to forget and so they're the ones you revisit more often. You find over time that people forget them more easily.
(I028)

Teachers' awareness of curriculum and assessment expectations structured pupils' relations to the object, placing differential value upon their production. In developing

their capability, pupils learned when results and methods would require justification, or could be applied without explanation.

Periods of recap and rehearsal before tests would recreate the pupils' movement through the ZPD, with the aim of reinforcing their productive capacity. Teachers prepared pupils for tests by anticipating the test requirements and ensuring pupils had had ample practice of specific methods. Revision was seen as constituted by productive rehearsal, which could be done without frequent appeals to teacher authority. The teachers would then use test achievement in planning further action for the class.

Pupils' contributions to discussions and expositions indicated them determining the structure of the ZPD (see Roth and Radford, 2010). Requesting clarification of elements the teacher considered omissible or withholding contributions until some productive work had been carried out were means of comprehending the given start point. Pupils' queries also clarified the productive actions that were required of them, alongside the structural understanding; they needed to understand their role in the division of labour in order to be able to fulfil it. In this conversation, pupils and teachers worked to establish intersubjective understanding of the concrete basis of the tasks, and the means by which as yet abstract notions could be concretised. Mathematical queries which elicited adjustments to the flow of activity by the teacher reshaped the ZPD in response to unforeseen difficulties, whilst maintaining direction toward the intended goal. Such changes to the ZPD would be prompted by the cumulative force of individual queries and required the teacher to make a judgement on every pupil's behalf.

Once teachers had made clear their expectations, it became each pupil's responsibility to produce their response to the task, in the expected form. Progression through the ZPD depended upon such participation, which in turn depended upon understanding those expectations. An individual pupils' production subsequently influenced his position in the division of labour. If a pupil successfully completed a task ahead of his peers, the extension work issued was, by definition, not necessary for him to do at the time, and so conferred a different status upon his work. It sat outside the current collective ZPD and

was likely to be attempted with minimal support from the teacher. If he was asked to assist other pupils, the focus of his task then became his peers' mathematical output, rather than extending his own. This placed the pupil at the 'horizon' of the others' ZPD, as he assumed authority to advise them.

Through producing written methods, pupils attempted to recreate fitting solutions to problems, producing evidence of their capability to complete allotted tasks. Teachers' written feedback would then inform the pupil how well he had met curricular expectations, and what he could do to improve. The marks awarded to a pupil would be recorded by the teacher for comparison and as a basis of future judgements. A pupil's writing (in class work and tests) thus became a tool by which the teacher would judge their mathematical capability, and the teacher's response a tool to aid the pupils' reflection. Through this system of textual production, exchange and consumption, the pupils themselves were produced within the activity system, represented in the texts. This production became the basis of their distribution within and between classes, reinforcing the exchange value of written work.

In this analysis, the ZPD and the division of labour emerged as mutually constitutive. The means by which pupils influenced and negotiated their way through the ZPD suggested it should be seen as jointly constructed by teachers, pupils and texts (Roth and Radford, 2010). This observation is supported by considering the pupils' subjectivity in lessons.

8.2.2 Subjectivity within the division of labour

In working towards an object they would discover only through their participation, pupils could not anticipate the needs which guided the teachers' decisions in structuring the ZPD (*the subordination tension*), and so had to accept their guidance. Nevertheless, pupils found ways of exerting their subjectivity within the division of labour, whilst submitting to the teachers' expectations.

Pupils' questions were designed to maximise the efficiency of their participation, positioning themselves in a compromise between subordination and subjectivity. When following expositions they would identify information they would need, and which

concrete mathematical structures formed the basis for the work; they could ignore other information. Their production of work would often be guided by minimal expectations of written explanations. This approach might, however, require repair action if they had misjudged what was necessary. Calls for help were similarly directed by the extent to which a pupil wished to retain control of an interaction, in relation to the assistance he wanted. A quick question to a peer, focused on productive expectations, could be controlled by the pupil, whereas conversational control might be willingly relinquished when requesting a second explanation from a teacher. This submission of subjectivity could be seen as a form of repair action to a disturbance created by a pupil's own inattentiveness or misunderstanding.

Pupils exercised judgement in deciding what it was appropriate to ask about, but in doing so, an individual's ZPD was exposed in relation to that established by the teacher. Teacher's responses were informed by knowledge of the individual, the task, the pupils' previous experience, current achievement and their productive expectations. Pupils would anticipate as best they could the teacher's response, but might receive a refusal to help directly:

C: *So if it were early questions I would go back to an example and reiterate the initial information that's been introduced however if it's a more challenging question I wouldn't necessarily want to – well I would encourage them to think about it more themselves.*

(I020)

Pupils called upon the teacher's role in the division of labour to negotiate the structure of the ZPD (Roth and Radford, 2010), but the responses constituted the teacher re-directing the pupil toward the anticipated requirements of the original ZPD, whilst making a judgement about the individual pupil's needs.

Teachers justified taking control of conversations as they served a diagnostic purpose, enabling them to determine whether a query indicated a misconception or gap in understanding, or derived from an operational error. All teachers spoke of the importance

of working through problems with pupils who were having difficulties, in order to understand their logic and remedy any misconceptions:

M: I always go back to the beginning of the problem, and work the problem through with them anyway, even if they've got to a certain point because... their work looks pretty strange and I don't see how they've got to that answer and sometimes the kid's gone on such a tangent.

(I014)

However, these conversations could close down pupils' enquiries, by addressing a diagnosed problem rather than the actual query, and redirecting attention toward the teacher's aims. In some instances this could oblige the pupil to articulate structural relations more explicitly than he had anticipated. Alternatively it might result in suppressing the pupil's interest, if that did not correspond to the task in hand. Pupils might not recognise or share the justifications of teachers' actions in this regard.

This guidance could be aided with a focus on written mathematics. Teachers might emphasise the function of writing as a means of expressing ideas, or recognising forms of solutions. Writing was treated as an artefact on which a pupil could reflect and act. Working through problems with pupils gave teachers the opportunity to show the effectiveness of using text to help reasoning, whilst also satisfying the need to produce a written record of operations. In the division of labour the pupil would be aided in adopting habits of reflection and explanation. As such the pupils were positioned to use writing to produce representations which could become the basis for further production.

The general pattern of action in a lesson was treated as self-evident: solving simple problems which closely fitted the model shown by the teacher should be followed by tackling more complex problems with less support. Pupils were expected to tackle these having learned from the early problems. C's comments above indicated an endorsement of exercise structure, and a focus on ensuring "early questions" were tackled successfully by pupils; reflection on these should provide support for more challenging problems. In doing so, the object of pupils' activity became a tool for their further production, in a pattern determined by the visible mathematics of the curriculum and teaching materials.

Exceptions to the pattern of maintenance of ZPDs could occur: when pupils presented difficulties in answering problems teachers would occasionally adopt a more flexible approach in their use of the textbook. K spoke of adjusting the “gradient” (1028) of difficulty in response to a pupil’s needs. In this case, sensitivity to the pupil’s needs and their individual ZPD took priority over communal practice and order, and rested upon the teachers’ judgement that expected standards would eventually be reached.

8.2.3 Written work: process and outcome

The actions that constituted work were those seen to be productive: pupils were not allowed to be idle, and produced mathematical texts to show not only what they could do, but that they were doing it. (This applied even if they had completed the necessary tasks in a lesson.) The production of written work was constructed as purposeful in itself. Through producing written artefacts, doing mathematics was constructed as a conscious and effortful mode of activity, and one in which pupils had to adhere to conventional reasoning and expression. Interim judgements of pupils’ progress would be made (by teachers and pupils) from the quantity of their productive response to exercises. The imperative to be working, and the provision of extension work by teachers, meant that a pupil always had work to do which could stretch his current capabilities. In this, transformative participation was identified with production. However, the evidence of transformation resided in a static artefact hence had to be continually recreated in order to retain value.

In working through the ZPD, writing had multiple valued functions in production and consumption. For the pupils, writing was a tool to aid their development, offering concrete representation of mathematical structures upon which they could reflect and act. This happened in the first instance by observing and reproducing the teacher’s examples, creating their own versions of endorsed solutions. By reflecting upon the reasoning which connected one step with the next, pupils worked to produce responses to increasingly complex or novel problems involving those connections. Through this process pupils developed their use of conventional mathematical representations and algebra, wherein representation and manipulation shifted from being actions to operations.

Having a method written down facilitated reflection, thus enabled co-opting that method as a tool. Producing the written record of a solution aided the pupil in checking the operations it comprised. In a pupil's written methods, he could see the dependence of his actions on correct operations, and the meaningfulness of those operations in the context of the specific method. Producing and reflecting on written work aided recall of results and methods. However, over the course of pupils' development an increasing range of mathematical connections could be taken as assumed, and omitted from explicit workings. Some operational arithmetic and algebraic steps could be excluded without diminishing the credit that would be received. Transformation was seen in adopting such connections as 'common sense' operations, thus constituting the vertical mathematisation of pupils' actions.

The cyclical nature of the curriculum entailed continual revisiting and development of topics. However, teachers did not always successfully convey different productive requirements in relation to anticipated pupil development. Pupils' choice between written methods could be inhibited when they preferred one method, but felt they ought to give another method their teacher favoured: credit would primarily be given for reproducing the methods upheld as appropriate for the pupils at that stage in their learning. As such, trying to fulfil their place in the division of labour could conflict with their own developing mathematical authority. Pupils had to work to produce themselves in relation to continually shifting structural expectations, in advance of their current capabilities. This could threaten to destabilise their burgeoning authority as expectations changed: pupils had to remain continually sensitive to the changes and continuities in productive requirements.

8.2.4 Revealing the object: developing authority

M explained that mathematics teachers held a unique authority in relation to their discipline, demonstrated in the ways that written examples could be presented to the class:

M: A lot of us now will go into a year eight lesson and go 'right, we're doing equations today' and we'll be able to say 'equations with exes on both sides,' put problems up on the board and go through them like that and that's when we get involved in doing them... I've often heard boys say 'Where's your maths folder?' 'cause they see that a lot in other subjects.
(I014)

The capacity of the teacher to generate examples on the spot acted as a demonstration of what the proficient mathematician could do, and to what the pupils should aspire. Through following and recreating the teacher's examples pupils could begin to establish authority, but they had little means of appreciating the limitations the teacher placed upon such examples in maintaining the ZPD. Complexities in number or algebraic structure could not be anticipated or negotiated by pupils without further experience. The teachers' authority rested in their understanding of the *Gegenstand* and *Predmet* aspects of the object of activity, within the framework of the visible mathematics curriculum. Teachers were able to negotiate the structural constraints upon problem creation, in relation to what was useable and could be constructible within lessons. Pupils' authority developed as their capability accommodated these considerations.

As pupils progressed through the ZPD, their proficiency with types of problems and mathematical structures developed, and thus they could assume authority over what they produced as evidence of their capability. However, pupils could become caught in a clash between the teacher's and their own mathematical authority, which I saw as indicating two aspects of development: understanding the equivalence of various algebraic and arithmetic methods, and being confident to choose between them in producing work. Idiosyncratic written methods, informed by pupil's personal judgements, would receive less credit. This could be seen in non-standard notation or methods, or when a pupil felt confident of his operational work to the extent that it would not be written down. If the pupils' useable capability was not expressed in accord with the visible mathematics of the curriculum, the constructed mathematics was not valued. Personal development of mathematical authority was seen when a pupil could avoid this potential conflict.

However, this required attention to be paid to conventional expression of the concrete basis of a pupil's ZPD, rather than to work towards its horizon.

Teachers saw pupils' experience of varying approaches to mathematical action as an opportunity to form a nuanced understanding of the values of mathematics. The differing emphases in action experienced during the data collection period enabled pupils to develop values related to truth, efficacy in problem-solving and personal development. However, problems generated and supplied by teachers inhabited the practice of the classroom and existed purely for the purpose of the pupils learning how to solve them. Pupils could only ascribe meaningfulness to problems or establish truths within the activity. Purported use values sustained only within the school system and mathematics curriculum and as such reinforced the exchange value of mathematical achievement outside the school.

8.2.5 Categories of mathematical action and tensions revealed

The delineation of appropriate work within the ZPD gave meaning and value to mathematical action in terms of whether it consisted of valid structural connections (i.e. correct or incorrect) but also whether it conformed to productive expectations. Action was valued not only in relation to attaining high grades, but also to changing productive requirements (such as when justification was necessary or recall sufficient). Fluency in mathematical methods was valued throughout pupils' work. The most salient aspect of pupils' transformation was in developing fluency as actions became operational and methods could be treated as tools. This constituted concretisation of mathematical structures; properties previously the focus of problem-solving became constraints to be negotiated in further problems. A pupil's success in this transformation contributed to their distribution and value within the school, as seen against the other pupils.

Pupils' responses to this positioning represented a compromise between goals (Kaptelinin and Cole, 1997). Their focus on efficiency of production and participation was an attempt to appropriate the transformative process, but was continually constrained by the teachers' expectations. As such, pupils were obliged to accept the route through the ZPD

offered by the teacher and teaching materials, which placed a predictable shape upon their productive actions. The activity sustained a trajectory of encountering mathematical structures through solving sequences of increasingly challenging problems; pupils adopted this as a model of transformative behaviour.

Transformation and production were closely identified, which meant that the role of the pupil (as one being transformed) existed in productive participation and had to be continually re-enacted. Pupils' increasing sophistication with regard to the activity was shown in their responsiveness to the changing productive expectations of teachers, understood as reflecting developing mathematical capability. This resulted in the value of reproducing conventional texts. Sensitivity to the combined requirements of mathematical structures and the visible curriculum, in relation to their expected capability, constituted the pupils' mathematical authority. Values in mathematical work sustained within the classroom, where this authority could be exercised.

The efforts of the teacher to maintain the ZPD for the whole class entailed selective consideration of variations in development, and remained in constant tension with the pupils' self-identified aims. Pupils were positioned to make uniform progress, particularly in sets arranged by previous attainment. These arrangements constituted the *Gegenstand* aspects of the object, alongside the logic of mathematical structures. Pupils were in a subordinate position to the object in that they had to acquire conventional methods and expressions of reasoning; success came through aligning personal development with communal aims. Limitations placed upon discussion and action contained the object of study within the predetermined ZPD, and could abnegate a pupil's interests within or outside mathematics. Subordination to the object and maintenance of the ZPD together inhibited subject-object dialectic. Consequently progression through the ZPD related to recognising the multiple authority of the teacher in a stable community structure and accepting their guidance. I refer to this as the *Collective ZPD tension*.

The co-ordination of action across individuals and in relation to timings within the syllabus resulted in a focus on efficiency. This saw pupils' learning constrained and propelled by

expectations of what had to be constructed within certain time periods. Teachers worked to resolve this tension through placing focus upon fluency in production of solutions in response to classes of similar problems. In exemplifying a topic area, teachers introduced classes of problems and suitable solutions together. In discussion of such methods, pupils' contributions would be passed over unless they corresponded to the desired solution. These expositions presented the problem-with-solution as an instantiation of the object, in which algorithmic responses to problem types could be learned without recognition or understanding of their operational components or relation to the underlying structure. These operations in turn might not then be amenable to application in different problem types. There was potential for such methods to acquire a resistant (*Gegenstand*) character in this process, not being amenable to adaptation and becoming concretised in concert with the structure to which they pertain. A pupil could 'trust' a teacher that a method worked, and gain reward for applying it diligently. Recognising a problem of a certain form and applying the method uncritically could thus operate as a pragmatic reasoning schema. I refer to this as the *problem-with-solution tension*.

The notion of trusting the teacher revealed a tension in the nature of personal historicity of subject and object. In the historical development from one mathematical method to another, pupils were incapable of understanding beforehand why that next method would be an improvement, or why a successive teacher offers different instructions for similar problems. The concrete knowledge of how a new method would be 'better' could only come through using it in circumstances that require it. However, for the pupils those circumstances only arose in the mathematics classroom, and not out of their own aims. Trust in the teacher comprised belief that tasks issued were fitting mechanisms for developing new skills and the skills therein developed were appropriate. The notion of trust intruded upon the values of mathematical action: work based upon deduction and rigorous justification should require little in the way of trust. Learning mathematics in the classroom thus becomes a model of *learning in advance of development* (Vygotsky, 1962), which I came to see as inherently tensional.

In their expositions, teachers' demonstrations comprised both a representation of the solution to a problem, and the artefact the pupils were expected to produce. This artefact would function as both solution and evidence of the pupil having produced a solution. Similarly, textbooks represented both the means and the result of pupils' work. These texts thus laid claim to multiple attributions in the activity system. The use of texts as artefacts on which to reflect, and as items of exchange in the classroom, conferred tool status upon them. The need for pupils to learn how to produce these texts and the role they played as instantiations of the pupils' mathematical capability places them as an instantiation of the object. The *roles of text tension* consisted of the continual shifts required in pupils' focus on written and diagrammatic mathematics. Through productive participation pupils were positioned for the use of conventional representations to become operational and for the shifts in attention to become redundant. This happened only after frequent focus upon them as the object of study, in expressing pupils' useable mathematics as the visible mathematics of the curriculum.

The focus on compliant production of responses to problems, coupled with the pre-determination of progression, deprived mathematical authority of personal meaningfulness. Pupils encountered new mathematical methods in an order determined by the curriculum, rather than by their personal development. Developing authority entailed accepting the pre-determined classification and expectations of their own achievement and production as more or less successful pupils. This production was informed by social needs, alongside mathematical relations, over individual aims. Thus social convention was seen to dominate in the transformation of the subject, and a pupil's achievement marked how he had developed in relation to the activity, as well as to the object. I refer to the *personal meaning tension* to observe that the meaningfulness of mathematical actions resided largely in their exchange value.

The potential for mathematics in the school to take on the character of a *Tätigkeit* was strong. All disciplines were bounded in space and time, and distinguished from each other through kit, texts and teachers involved. Within mathematics the inclination of pupils to disregard contextualisations or information on which they judged they would not be

assessed encapsulated the object of their attention in the methods demonstrated. The mathematical structures within contextualised questions were coherent for the pupils, but there was questionable potential for translating the valued mechanisms of production and exchange into other practices. To produce meaningful responses to these problems entailed constraining one's action to the logic and conventions of the discipline, as done in the classroom, thus isolating the actions from the world outside. Here I note that efforts to locate mathematical action in wider productive practices could be actively ignored by pupils who wished to focus on attainment as it was defined in the classroom. Pupils' focus undermined opportunities for meaningful mathematisation of real-world scenarios. This then contributed to a conception of mathematics as defined by the expectations of the classroom and constrained within the division of labour, rather than as a means of dealing with quantitative relations in the world, or of personal transformation. Internal curricular distribution of mathematical action could also be seen: the notion of 'extension work' suggested that some development was necessary for pupils at a given time, whilst other development was not. The work produced by pupils was exchanged, distributed and consumed only within the practice of the school, and the production of the pupils within lessons did not relate to their social standing outside. I refer to this as the *bounded practice tension*.

8.3 The tools of the mathematics classroom.

In this section I consider the tools of the mathematics classroom in order to explore the values implicit in their use. In my analysis I discovered various features of the mathematics classroom that could be considered tools, such as the practice of setting, which distributed pupils and informed productive expectations in the division of labour. However, I regard setting as primarily a tool for the teacher. The participants did not speak of pupils using setting in their work towards developing mathematical capability. In this section, I restrict my attention to those features of activity which emerged in the grounded analysis as tools for the pupils.

These tools are related to, but distinguished from, the tools of mathematics. I first focus on the mathematical tools to understand the relationship between the two sets of tools and their roles in productive action. I then consider use of the textbook. In the grounded analysis the exercise emerged as a predominant task form; I consider this form as a tool used by teachers and pupils. I then continue the exploration of writing as a tool, and compare this with the use of PCs in the formation of the object. I then consider the use by pupils of consensus and close this section by focusing on the tensions revealed.

8.3.1 Tools of mathematics.

Pupils first encountered the maths kit in primary school or during year seven, in which the only formal teaching of these material tools took place. This teaching presented tool use as a subsidiary component of working with mathematical structures such as angles, shapes or functions. Ensuring that teachers had spare items available impinged upon the division of labour, although department policy meant teachers were less likely to lend out calculators. The teaching of calculator use comprised the identification of relevant symbols on the keypad and the order in which commands should be programmed to compute calculations. Calculator use was largely treated as operational: it was seen as a reliable tool which would compute what the pupil asked it to. In common with the other tools, operational use of the calculator assumed the integrity of the tool: mathematical relations would be correctly retained through correct use. The pupils' task of devising a calculation in connection with a problem was an action, the result of which would then be transferred to the calculator. The result of the calculator use would then be incorporated in a solution. Similarly, pupils depended upon the integrity of the items in the maths kit when solving problems in which measurement or construction were constituent operations. The values made explicit in using these tools were those relating to precision and accuracy, but pupils also worked from an assumption that each tool conveyed structural relations correctly. Tool dependence could result from the persistence of the relations embedded in each tool.

Leont'ev's hierarchical structure of activity offered a means of describing how mathematical structures became tools for pupils. Initially the focus of action, connections

are made between new (abstract) structures and old (relatively concrete) ones. These connections concretise the new structures, placing them in systems of relations which make them amenable for use in further problem-solving. As the pupil internalises the structure in its relations, the connections are recreated in use as operations. This movement has the character of tool acquisition. The use of writing in mathematics has been discussed above: pupils were positioned to use writing to aid their reasoning and reflect upon structures and solutions in attaining answers. Values of deduction, justification, rigour and accountability were sustained in mathematical work in this way and embodied in rules associated with representations. In the case of routine methods, learned with classes of problems, there was potential for the structure behind a method to be related only to a specific representation.

8.3.2 Tools of the activity: Instructional texts and topics

The textbook was used in the majority of lessons, as a source of explanations, examples and problems. The use of the textbook by the teacher conferred an authoritative status upon it: provision of problems was delegated to the textbook and reference would be made to the prepared examples when helping pupils, who could also refer to printed answers. In the busy commerce of the classroom, the pupils would be expected to consult the textbook before speaking to the teacher. The use of the textbook also generated a uniformity of experience for the pupils. Through this analysis I came to see textbooks and exercises performing a role greater than that of mediation. Through authoritative use, they conveyed the intentions of a third party in the division of labour, supplementing the role of the teacher, supporting their authority and structuring the pupils' progress through the ZPD. This contributed to the *roles of text tension*.

Productive requirements of pupils were represented through the authoritative textbook. Textual reproduction and representation could be seen as a primarily meaningful action in the activity and the pupils' role as determining the process of reproduction. However, when reviewing textbook examples pupils often required assistance in determining the connections behind the production, from the presented end product. Asking teachers for clarification of textbook material placed both the written expression and its mathematical

content under scrutiny. The teacher's advice would aid the pupil in making sense of examples, and thus there was a mutual reinforcement of authority between the teacher and the textbook.

The teacher and textbook mediated the pupils' engagement with their own capabilities by simultaneously presenting endorsed sequences of action and representations of that action. This purpose also mediated the teachers' action, in responding to what they saw as useable and constructible. The issue of translating between representation and action was negotiated by teachers in their presentation of written examples. Steps would be notated one at a time, with accompanying explanation and opportunities for pupils' contributions and questions. Pupils saw the process and the result develop together. The pupils' need for written examples to be translated into action was reflected in the preference for referring to their own examples. Reviewing these was a means of re-enacting that productive action, offering the possibility of reflecting upon the connections previously made. Similarly, when they had reformulated teachers' examples in their own work, they did not need to renegotiate that text in later readings.

Topicalisation enabled teachers to quantify, record and report pupils' achievement. The textbook defined topics and substantiated their content, through examples and appropriate tasks, structured in predictable exercises. In terms of Vergnaud's *concept*, topics provided representations that became associated with particular structures. The arrangement of syllabus content into topics aided pupils' sublation, through offering classifications of connections in anticipation (and formation) of use.

8.3.3 Tools of the activity: the exercise

Different uses of exercises were imputed by pupils and teachers. Exercises could be set in order for pupils to make connections and discover applications of a method. The predictable form of many exercises anticipated a particular progression: reproduction of written methods exemplified by the teacher would be followed by 'applying' those methods to contexts in which the focal mathematical structure was embellished with additional details. Pupils recognised this pattern and would use it to make judgements

about the value of their own production, in relation to the quantity and qualities of problems completed. The pupils' compliance was required to make the exercise an effective tool. From one question to the next the challenge increased, and it was the pupils' task to manage the information given; only by identifying the additional features would they make progress. Only by "compounding and testing" their own capability could they develop a "rough idea of what you're doing" into a productive pattern in solving a certain class of problems, making connections between structures (Aaron, 1006). Exercise form embodied the "thinking with the basis" approach (Greeno, 1992); the predominance of this task form positioned pupils to accept this as the best way of developing mathematical capability.

The pupils were positioned to extend their capabilities through engaging with problems, and therefore the problem-setter's role was to provide increasing challenge. Through engaging with an exercise pupils could develop fluency with a method whilst acquiring experience with anticipated problems and results. Aaron's description of using the exercise to see "how it all fits together" placed a stronger emphasis on the pre-existing structures which a pupil was expected to discover. However Jake suggested that working through problems was also a process of self-discovery. These descriptions shed light on the exercise as an agent in the division of labour. The exercise mapped out the ZPD, defined in relation to a specific mathematical structure, and could also perform the role of the more able peer in supporting the pupils through it. Exercise form embodied the values of personal development, discovery and reward for effort: these values were best appreciated by the pupil who was compliant in productive participation.

Exercises were also administered to consolidate pupils' productive facility or to develop operational fluency. Through revision exercises and tasks such as the '100 club', rehearsal led to fluency with methods and recall of results. This placed a value upon pupils' recall and recognition of structures. Frequent and increasingly rapid connection with results conferred upon them the status of 'facts' that did not need to be justified. Within the classroom, value was placed upon a pupil having many of these facts to hand, which offered evidence of sublation of mathematical structures. The use value of these facts

within the activity became transformed through production into the exchange value of the object of activity.

The notion of learning as ascent from abstract to concrete offered a means of understanding the formative effect of working through structured exercises. In applying a key method with increasingly elaborate problem scenarios, that method had the potential to be concretised through multiple connections. However, in the activity of the classroom, whilst the exercise was presented as a tool for this concretisation, the method itself was a tool for completing the exercise and gaining credit. In the system of production and exchange, the value of pupils' written work derived from the credit it would earn. Thus concrete understanding was not necessarily the focus of pupils' transformative action; it could be supplanted by the need to complete tasks.

8.3.4 Tools of the activity: Writing

Recorded operational steps were used as an indication of reasoning by both pupils and teachers. Consequently, solutions were developed within the use of conventional expressions of reasoning; the object encompassed the capability to express oneself conventionally. The division of labour directed pupils towards internalising the checking, judging and editing role of the teacher. Consequently, written work was used as a means of communication not only from pupil to teacher but also from pupil to his future self. Pupils learned to treat their own output as a record of their action, and to stand apart from their previous selves (Roth, 2010) in order to evaluate their work. This reflection was also encouraged when teachers publicly used pupils' mistakes as means of exposing misconceptions and revealing the necessary logic of a method. Pupils' work could become a focus for class discussion and contribution and as such was used as a tool in the articulation of mathematical ideas in an intersubjective space. This usage mirrored that of teacher's examples, in which attention would be drawn to common mistakes and potential pitfalls. In these situations, the rules of written representation were unified with the structural logic as the *Gegenstand* of mathematical action, whilst a pupil whose work was shared was produced as a helpful member of the class for having offered the illustration.

The value placed upon deductive solutions expressed conventionally made it meaningful for pupils to reproduce teachers' examples in their own work, before adapting or embellishing them. Prepared examples and marking together promoted efficiency, by excluding some operations which were valid in response to the structure, crediting only those which would lead directly towards an answer to the given problem. The communicative use of pupils' written work impinged upon the role of writing in discovering new connections. As pupils anticipated the teacher's use of their work, they concentrated on producing those steps the teacher wanted to see and would reward. Preparation for examinations drew attention to the production of written forms that would gain credit in particular circumstances, thus placing a limitation upon the action. The emphasis on efficient solutions could lead to reluctance to commit deductions to paper until a pupil was confident they were appropriate and correct. Ability to produce such solutions could be identified with "knowing what to do" (Alex, 1018), producing the pupil who could as successful. Pupils were reluctant to commit tentative ideas to paper in the midst of work that would be assessed. Consequently the role of writing in representing mathematical structures for reflection was inhibited. This then led to a conflict with the teachers' appraisal of quantity of written work as an indication of engagement and achievement.

All teachers conveyed the importance of the end-product as a solution to a problem. However, different relations between writing and action could sustain, as the comparison made between M and W showed (p. 178). Working in pencil enabled pupils to experiment and record their intermediate thoughts, making them available for reflection and editing. Work done in pen obliged pupils to resolve their action before committing it to paper, and having in mind an image of the form their solution would take. Both approaches placed a privilege on the production of the final artefact, whilst valuing different aspects of mathematical action. Working in pen encouraged pupils to plan ahead, but impinged upon their willingness to produce tentative work for reflection. In contrast, a focus on editing pencilled written steps emphasised the expressive and reflexive purposes of writing, by encouraging pupils to review their output and change it where necessary.

The cycle of production and exchange saw problems recreated as solutions which were then reformed with feedback from teachers. The classroom authority of the teacher meant they were entitled to annotate and correct the pupils' work, and their mathematical authority meant that their writing was worth attending to. However, feedback could easily be ignored by pupils, and thus the cycle terminated if it did not inform their subsequent work. Teachers encouraged paying attention to their feedback through awarding grades and supportive comments.

8.3.5 Tools of the activity: the PC

Access to PCs was controlled for practical and pedagogical reasons, as was the choice of software pupils could use. Instruction in the use of software could be a distraction from undertaking mathematical tasks, so teachers would choose programs that could be used effectively with minimal instruction. The long-term aim of teachers' work was for pupils to develop mathematical capability that could be reflected in reproducing hand-written responses to textual problems. PC use occurred only when teachers wanted to focus on aspects of capability that did not necessitate extensive written rehearsal, such as revision and recap of results. This usage reflected the tension in PC use noted by Hardman (2005b): use as a revision tool focused on a narrow aspect of object-orientation and reinforced the concrete basis of the ZPD, whilst initiating little transformative action.

The reduction of the valued part of a pupils' work to a single numerical answer or button-click made PC tasks more enjoyable, removing the requirement to produce justifications which could be inspected by teachers. Enjoyment was also built up by game and puzzle formats, which sustained the exchange value of mathematical action. Pupils developed their capability and fluent recall of mathematical results in engaging with the tasks. Whilst there was a relative lack of supervision by the teacher, instant feedback demonstrated the program's mathematical authority over the pupil.

8.3.6 Tools of the activity: Consensus

A principle of consensus was used by pupils to guide their action. Agreement on answers was a necessary feature of mathematical problem-solving in the activity, but was treated

as a sufficient condition of correctness, used to avoid justifying answers. In their individual written work, pupils could edit their texts, based on comparison with the work of their peers, without reference to structural justification. This approach would also be seen in their conversations with each other when the completion of tasks would be described in informal instructions relating to the production of the text. The use of consensus could be seen as a response to the need to produce quantities of work, as in Thomas' actions in Vignette #8 (p. 183). The mechanism of class marking also allowed consensus to operate as a guiding principle. Pupils would mark answers correct on the basis of agreement, rather than justification and confirmation, sustaining a focus on the answers to problems.

I saw consensus emerging as a pragmatic reasoning schema in response to the uniformity of tasks and the predominant use of closed problems which had single, predetermined correct answers. Together pupils positioned themselves as co-operatively working towards the same targets, each producing their own version of a task response, which had only minimal capacity for subjective influence.

8.3.7 Categories in action, transformation and tensions emerging

Through adopting the use of material tools and the calculator pupils assumed and developed the concrete nature of mathematical structures. These tools enabled structural relations to be used without investigation, and established a dependence upon them: pupils did not need to know why all useable connections sustained, which placed a bound on their transformative activity. The use of writing as a tool enshrined structural relations in lexicographic manipulations, and similarly liberated pupils from investigating those relations. Contingent categorisations of mathematical structures and methods as 'topics' bestowed meaning upon pupils' recognition and recall of facts, which could be used as prompts to action. An emphasis on quantity of production, placed a high value on fluent and rapid production, sustained by revision tasks (such as the PC exercises). The emergence of consensus as a tool reflected its use as a category of mathematical action: from agreed premises and rules, "right answers" should be attained and mathematical authority established.

Authorised mathematical texts were treated as exemplary indication of the pupils' desired actions: to do mathematics was to be active in recreating such texts, and reviewing one's own work (once produced as fitting and correct) was a means of reliving that action. The "thinking with the basics" (Greeno, 1992) approach conveyed by exercises framed pupils' transformation in terms of the extent to which they progressed through an exercise. Their mathematical action was substantiated by the fluency with which they could recall and apply mathematical 'facts', and use the products of their recent action in further production. This was propelled by and contributed to the exchange value of a completed exercise. Understanding the multiple uses of their written production enabled pupils to orient themselves effectively in the activity, supporting their authority. Their claims to mathematical capability were sustained within and substantiated by the norms of the activity, informed by the visible curriculum and teachers' endorsement of their production.

Pupils benefitted from the assumption of tool integrity in that they were able to work with structures such as powers, roots and trigonometric ratios, without investigating underlying calculations, and thus make progress in extending the range of problems they could solve. However, this progress was dependent upon the tool, and obscured the structural relations that lay behind the outputs, which might otherwise have been a focus of action. I refer to this as the *tool dependence tension*. Subject development in the vertical mathematisation of the curriculum was aided by time saved in these relations being assumed and treated as 'facts'. The visible mathematics of the curriculum placed use value upon these facts but not their justifications.

The *bounded practice* tension was sustained by the use of exercises constructed around rehearsal of a specific method, and by the integrity of problem-solving routines. The appropriateness of a given routine for a problem was assured by the task form and the fixed and pre-determined nature of the problems the pupils encountered. The information that would be useful from a given problem was indicated at the outset, by the task context of the method. Pupils could expect that the information given would fit the requirements of the method, and that rigorous application of the method would

inevitably lead to a meaningful answer. They understood that problems rarely contained any redundant numerical or structural information and should not be ambiguous or insoluble. However, these conditions could only be guaranteed within the classroom, where the learning constituted matching the given information to the method. The meaningfulness of the attained answer also resided in its use in gaining the pupil credit, rather than in the proposed context. Any mathematising action undertaken served classroom purposes, rather than making sense in the world. In rehearsing methods in this way, the otherwise dialectical relation between necessary information and valid methods was undermined twofold. Whereas given information should lead to the choice of a method, in the case of exercises the method was predetermined. Conversely, the need to find information in order to be able to execute a known method was pre-empted.

Focus on classroom relations and grade achievement sustained the *exchange value* of mathematical capability. This tension existed not solely in the teacher's practice or the pupils', but in the use made by pupils of the teacher's production and distribution of their mathematical action. The communication of achievement through test scores and grades continually emphasised the exchange value of pupils' production (Lave and McDermott, 2002), made tangible for the pupils within the school system. This use foreshadowed future exchange value, presaging the use of the pupils' ultimate grade in public examinations. Tension between *use and exchange value* pervaded the activity. The *use value* of pupils' mathematical capability would come from the flexibility and appropriateness with which it could be applied. Consequently experience in dealing with representations of real-world problems was essential in order to understand how mathematical modelling could work and how to translate physical or temporal relationships into the language of mathematics. The exchange value of the mathematics qualification, however, would depend on it being understood in the world outside school, which in turn would depend upon the qualification and curriculum having a coherent structure. Thus examples and practice from the world outside the classroom had been refined in order to avoid distracting structural relations or complexities, and to constitute fitting visible mathematics for a school curriculum. However, such tasks had limited

meaning for the pupils; they had also become fluent in ignoring contextualisations that had little substantive effect on task requirements.

This tension was also reflected in the roles of text. A tension arose between the aspects of writing as a tool for determining mathematical ideas (use value) and in representing the completed work to a teacher (exchange value). This tension could also be framed as a clash between writing mathematics as an action and as a record of action. The abstract notion of how to solve a problem became concretised in production of a specific textual form, which would receive reward from the teacher, without guarantee that the pupil could justify the operational steps in the form. I refer to this as the *production versus development tension*.

8.4 Theoretical engagement narrative

Whilst I had not stated *time* as a feature of my analytical framework, it became clear in this preliminary analysis that the passage of time was an underlying feature of CHAT. Roth (2010) proposes that time has been under-theorised in CHAT, treated as something external to activity and culture, thus inhibiting the identification of transformation. He proposes recognising the non-self-identity of things in relation to time, i.e. tools, communities, subjects, rules, etc. which continually change in the passage of time (diachronicity) and in social uses within time (synchronicity). He notes “This non-self-identity, this immanent contradiction, is the internal engine that brings about history and historical change” (ibid, p. 208). In my exploration noting the passage of time within lessons and across series of lessons and years was crucial in identifying change and stasis. This required being sensitive to the enduring features of activity that enabled moments to be identified across time, in and between different scales (Lemke, 2000), and enabled differences to be tracked.

In building upon the grounded description of activity I found that data did not match exclusively to specific nodes of the extended meditational triangle; just as behaviours refracted multiple aspects of classroom practice, so data often appealed to more than one node. These multiple correspondences reinforced the shift away from the “buckets”

approach (Barab et al., 2004), leading toward the use of CHAT categories as a vocabulary of description rather than as the framework of a “totally structured” (Beswick et al., 2007) activity. This new usage aided the identification and description of dialectical relations in the activity, and observation of multiple motivations and production. This was facilitated by the structural presupposition that each of the constructs was directly related to each of the others. My aim to make sense of the constructs each as a “concrete unity [of] mutual connections” (Ilyenkov, 1960) contributed to this approach. In articulating tensions within the community which sustained across particular instantiations of the activity (lessons), I became aware that these tensions could become apparent through various aspects of pupils’ productive participation. Tensions could become manifest in the pupils’ and teachers’ actions in relation to the division of labour, the tools and the object itself. Consequently, I decided to put aside the ordinal classification of tensions (Engeström, 1987).

In my exploration of the classroom, it became clear that the activity endured across time despite the interplay of subjectivities. To reflect this I have adopted the terminology of *tensions* rather than *contradictions*. This better conveys the non-terminal nature of these strains, as revealed by the grounded analysis. Adopting the grounded approach had revealed these tensions, but also their limited transformative effect. Freeing the description of the tensions from Engeström’s (1987) ordinal classification also reflected my observation that tensions might be seen in the production of work, but were suppressed in the relations framing the cycles of production, distribution, exchange and consumption, i.e. the production of the pupil.

8.5 Developing question narrative: towards meaning and value

Through exploring the action of the classroom I uncovered multiple interrelated tensions (see Table 10, p.253) and began to uncover aspects of the transmission of value through productive participation and tool use. In preserving the pupils’ position in a multi-voiced description I began to detail the production of the pupil in the activity.

Pupils were positioned to develop through compliant participation, focused on reproduction of textual responses to classes of problems. Pupils' transformation could be seen in their developing fluency with the mathematics of the curriculum, as presented to them through the structure of lessons. In the mathematics classroom, subject-object development consisted of revealing the structural relations of mathematics that were valued by the curriculum. Transformation resided in these relations becoming 'common sense' tools, amenable for use at the subject's will. The meanings (and hence applicability) of these tools were restricted not only through the reference to logical structures, but also through their value in terms of relations of authority.

It is a feature of CHAT that focus is placed upon productive action. My analysis has raised the question of how the activity produces mathematical capability as classroom authority. I saw this distinction as relating directly to my question of how pupils locate mathematical action. A person's capacity for locating mathematical action was constructed in the classroom as his recognition and determination of logical structures, capability with the tools that aid his action with these structures and recall of objective mathematical results. Within the classroom this capability is produced as mathematical authority, as the pupil becomes empowered within the structures of the activity. These structures act as constraints upon his action, determining "proper" mathematical knowledge and acceptable means of justification. A pupil is well placed to locate mathematical action when he can transcend these constraints in his behaviour outside the classroom; preserving values and structures as he chooses, using tools to establish authority in situation-specific terms, alongside developing additional and temporary meanings.

In terms of the development of the object, the notion of sublation admits the use of two metaphors simultaneously. Pupils could be described as developing an *emergent* capability in mathematics, or they could be seen to be *convergent* upon the content of the curriculum, if their authority remains within classroom activity. My further exploration would be concerned with the best description, in terms of these metaphors, of the development of the object of the activity.

Describing the pupils' position in relation to the object of activity and revealing tensions raised questions as to how or whether these tensions became manifest in the developing object. Awareness of these tensions, and attempted resolutions, would reveal the values the teachers emphasised in classroom action, which could become manifest in the object if adopted by the pupils. These observations and concerns formed the basis of my continued exploration, and contributed to the evolution of the research questions, rephrased as:

What are the values that sustain in the activity of the mathematics classroom, as revealed by the tensions and teachers' efforts to overcome them?

To what extent do the tensions within the mathematics classroom infiltrate the pupils' developing notions of mathematical action, and the cultural values contained within?

To what extent should pupils' mathematical capability be described as an emergent property of the subject or convergent upon a fixed set of skills?

Community Tensions	Orientation	Engendering pupils' interest whilst preserving the integrity of the curriculum.
	Socialisation	Pupils playing their part in the social order for the benefit of all, whilst pursuing individual development.
	Subordination	Pupils occupying a subordinate position to teachers in their own community.
	Stability	Preserving the working of the activity system in the face of pupils' subjective development.
	Pupil fallibility	Pupils' subjectivity constructed as inherently deficient; working towards an ever-receding and incorrigible object, with only temporary and contingent endpoints.
ZPD/division of labour Tensions	Collective ZPD	Individual development channelled along communal and pre-determined lines.
	Learning in advance of development	Pupils worked towards an unknowable object, developing skills whose purpose only becomes clear through application. Trusting the teacher to illustrate those skills.
	Problem-with-solution	The notion of fluency in mathematical problem-solving engendered a focus on efficient methods, rather than exploration of mathematical structures.
	Roles of text	The functions of text as a tool to aid in problem-solving or as a representative of the pupils' development came in to conflict.
	Bounded practice	Institutional structure, classroom tasks and syllabus arrangements presented mathematics as a bounded productive practice. Socially meaningful, but disconnected from other practices.
Tool/Object Tensions	Personal meaning	The structuring of pupils' action offered little opportunity for pupils to develop any personal meaningfulness for mathematical action outside the classroom.
	Tool dependence	Dependence upon the reliability of tools, derived from the resistance of mathematical structures, enabled pupils to avoid investigating and learning about those structures.
	Use and exchange value	The exchange value of mathematical achievement was seen within school practices (reporting, setting), whilst the use values remained difficult to illustrate convincingly to all.
	Production versus development	Marking practices, emphasis on solution methods and notions of quantity conflate pupil development with fluency of production.

Table 10: Tensions identified in classroom activity

9 The object of pupils' activity

My analysis in chapters 7 and 8 enabled me to form a description of the mathematics classroom which revealed the systemic tensions of the activity. In this chapter, I explore the developing object of study, considering:

- The influence of those tensions on the emergent object
- The values imbued in the production of the object
- The extent to which the developing object is best described as emergent or convergent.

I structure my exploration in order to connect with my initial aims for this research (§1.3). I first consider how pupils' developing mathematical capability was produced and recognised (§9.1). I consider the values which sustained and how these related to the teachers' management of tensions and pupils' application of their own mathematical capability. I then turn to the pupils' descriptions of mathematical action, and discuss the aspects of activity which allowed characteristic features to persist (§9.2); data in this section come largely from the pupils' later and final interviews. I trace the extent to which systemic tensions and values were preserved within the object by focusing on the elements of production and exchange. Together, these two sections enable me better to understand how pupils were produced to locate mathematical action, inside and outside the classroom.

In §9.3 I explore the characteristics of the resultant object in relation to the aims of mathematics education, and views of mathematical action encountered in chapter 2. I consider the potential for the meanings and values of classroom activity to inform pupils' transformative action in their future work. In §9.4 I reflect briefly upon pupils' capacity for locating mathematical action, in relation to the distinction between capability and authority.

9.1 Subject development: capability and authority

In this section I review the processes by which pupils' development was valued, by themselves and others. I begin with a description of the results of production which were valued, how these values were conveyed and how they influenced ongoing action, in relation to pupils' notions of purpose. I then consider how values in productive action were revealed by teachers' attempts to resolve some of the tensions identified in chapter 8. In exploring the productive action, I consider pupils' use of their previous mathematical work in furthering their development and review how the pupils were themselves produced in relation to their mathematical capability.

9.1.1 Values in mathematical action

Examining the focus on productive practice revealed the mathematical values which were endorsed through the system of exchange in the classroom. Production of logical chains of reasoning, using conventional notation and layout, indicated pupils' effectiveness in problem-solving. Through this practice, pupils developed their facility in algebra. Algebra was also treated as a topic within which manipulative skills could be developed and assessed. This development could happen dialectically with problem-solving across topic areas: applying the logic of mathematical structures enabled pupils to develop their skills in algebraic manipulation, and dependence upon the rules of algebra aided them in making progress within topics. Pupils were encouraged to exploit the resistant nature of mathematical structures, in recalling and applying routines in appropriately refined problems. They were expected to manage information from such problems in working systematically from premises to answer. The effectiveness of mathematical modelling was implied through working with 'real-life' scenarios, in the context of exercises of developing complexity. Within this work, pupils had to translate between different representations of a problem in order to solve it and make sense of the answer. In working with mathematical structures to produce new information in relation to problems, pupils created experience on which to reflect and further develop their capability in recognising the structural relations of the curriculum.

These values were explicitly communicated from teachers to pupils through assessment and in classroom interactions. Marking encouraged the development of fluency in conventional written algebra as part of valid deductive solutions, through the focus on errors and necessary corrections. Similarly, in conversations directed by teachers greater focus was placed on pupils' errant conceptions than on their correct ones. These values were also conveyed implicitly, through an absence of interaction. Whilst a pupil worked uninterrupted by a supervising teacher he could assume that he had understood the task, his reasoning was correct and was acceptably written down. This pattern of interaction contributed to the *pupil fallibility tension*, reinforcing the roles of teacher and pupil.

Deriving the above description from the data pertaining to the mathematical norms alone, my analysis might suggest that the production of mathematical work was guided solely by the values of rigorous deduction and justification, efficacy in problem-solving and intellectual discovery. However, as seen in the nested relationships between *ordering*, *working* and *mathematical norms*, participation in the activity shaped the mathematical action, thus infiltrated the resulting mathematical values. Pupils' compliant productive participation was valued first and with greater importance than their mathematical work. At all times, pupils were expected to adhere to the predetermined social order of the classroom, in their role as learners. Recognition of this feature of activity underlined all other values and associated actions. Within the *working norms*, teachers' expectations were first satisfied by pupils doing work at a sufficient rate. These norms entailed a focus on problem-solving structured by exercises, which valued the production of answers, justifications of those answers, and the pupils' progression from one problem to another. These productive expectations applied to all pupils at all times, and represented the necessary alignment of pupils' purposes with those of the classroom.

The confluence of teachers' and pupils' intentions resulted in a shared emphasis on efficiency. The pupils' focus on efficacy in problem-solving entailed a tendency toward satisfying minimal productive requirements in producing solutions and answers. Complementing this, by rewarding precisely those deductions which were pertinent to the desired answer, teachers encouraged pupils to produce 'streamlined' solutions with no

errant or superfluous workings or revisions. Awareness of examination requirements engendered teachers' confidence in those aspects of the syllabus which could be taught as *problems-with-solutions*. These conditions permitted sequences of operations to cohere into a routine method to be applied as a tool in problem solving. Reliance on these tools depended upon preserving their integrity ('showing all your workings') and in doing so such methods would be concretised in connection with classes of problems.

However, completion of tasks and progression through exercises often required that pupils build upon the routine application of methods. In doing so, value was placed upon managing information, recognising mathematical structures and making connections between topics. When working on contextualised problems pupils had to accept the validity of mathematical models of the 'real world'. In working with such contextualisations, the pupils were expected to accept the pertinence of the model, and the refinements made to fit the constraints of the classroom and the current topic. Participants had learned to suppress any objections they had to contextualisations they might have considered artificial or difficult to understand. The *bounded practice* and *personal meaning tensions* sustained. However, pupils still expected answers to be sensible in the proposed scenario. Failure in this regard undermined both the contextualisation and the legitimacy of the method in that context. In the feedback given to pupils, the process of translating from context to mathematical representation was not explicitly discussed. Pupils were rarely asked to justify models or generate novel ones, without the context of a supporting exercise.

The values of classroom achievement were seen in pupils' interpersonal relations; pupils chose to work with those upon whom they could depend, and could happily engage in an exchange of support and information whilst they worked. A combination of mutual dependence and support indicated pupils validating each other's mathematical capabilities and benefitting from joint efforts in attaining answers. A shared focus on productive requirements saw pupils correct each others' mistakes with minimal justification, as they supported each other in attaining credit.

In summary, the values associated with the development of mathematical capability centred on problem-solving to establish classroom authority. Pupils were expected to apply the logic of mathematics carefully, producing deductive justifications of their answers. In doing so, they developed their use of algebra, whilst connecting problems with mathematical models in order to implement solution routines (which depended upon the logic of the underlying model). However, these values were interwoven with the values of compliant productive participation in the classroom, and understanding the use of the exercise in developing authority, deriving from the value of the qualification they would eventually achieve (Elliott et al., 1999). The successful pupil would orient his actions toward the institutional voice (Williams et al., 2008), focused upon grade achievement. Pupils were expected to produce sufficient quantities of work, applying mathematical operations and actions, efficiently. This efficiency encouraged the recognition of types of problems and the recollection of textual solution forms. Building upon such forms in more elaborate problems required making connections with methods that might have been established within another topic, or accepting as valid the connection made by the contextualisation of a method. Pupils' purposes relating to their focus on friendship, gaining credit and establishing achievement reinforced the exchange value of their capability.

In their relation to classroom mathematics, the pupils in this research had some affinity with “quietly disaffected” pupils (Nardi and Steward, 2003) who experience school mathematics as “TIRED”: having characteristics including tedium, isolation, rule-and-cue following, elitism and depersonalisation. The similarity of the descriptions raised the question as to why my research participants did not come across as disaffected, but were still determined to succeed.

9.1.2 Relation to systemic tensions

Analysis of the teacher interviews revealed awareness of some of the tensions detailed in chapter 8, whilst others remained “latent” (Goodchild and Jaworski, 2005). Here I consider how the choices made in attempted resolutions promoted particular values within the activity. For the purpose of clarity, the resolutions are described here individually.

However, just as the tensions were overlapping, the means taken to deal with them had effect throughout the activity, and could relate to more than one tension.

To help pupils deal with the *subordination* and *orientation tensions*, teachers presented participation as a valued end in itself. M's emphasis on positive personal relations and achievement was intended as a motivation and means of finding enjoyment in the activity, whereas W's acknowledgement of the imposition upon pupils would be accompanied by demonstration that teachers also acted within a system of expectations, and that everybody fared better when meeting their obligations. K noted that that approach worked best with pupils who had enjoyed a positive personal history with mathematics, as they could more easily understand and meet expectations. Teachers also tried to illustrate the use value of mathematical capability within the activity and give this personal meaningfulness, by emphasising the "ego kick" (K, I028) pupils could gain from recognising their own development. However, this measure of self-esteem had currency only in comparison with other pupils' capabilities (Lave and McDermott, 2002): one pupil's comparative success constituted another's failure, and vice versa. My analysis suggested that the motivations introduced by teachers sustained merely as nuances to the institutional voice (Williams et al., 2008).

Different aspects of the *collective ZPD tension* were revealed by teachers' attempted resolutions. In order to personalise the pupils' experience, M had a set of "fun and creative" (I014) tasks which allowed pupils to exercise their subjectivity within certain parts of the syllabus. However, these were topic-specific tasks, and as such highlighted the uniformity of most other tasks. Teachers would also make judgements as to areas of the syllabus in which it would be most enjoyable and productive for the pupils to undertake exploratory tasks. This choice was determined by the relevant scheme of work and the time made available by the pupils' achievement, thus could be constrained to the "top set" pupils, who could work through the necessary syllabus quickly. In relation to the uniform and enduring expectations of development M noted that differentiation of tasks within classes was an ongoing issue that one had to approach anew with each topic area, in relation to the pupils' achievement. This echoed W's judgements as to whether to make

connections between topics, or to start with “a clean slate.” Both approaches retained the primacy of the syllabus, and were directed towards making the experience more enjoyable, even though this might not have been recognised by the pupils. K’s adjustments in question “gradient” represented an unusual relaxation of the collective ZPD to meet individuals’ needs. K saw the purpose of exercises as to “expand and entrench” the mathematics and felt this should originate from individual pupils’ useable mathematics, rather than the requirements of the curriculum. However, this relaxation was still constrained within available resource and time.

Teachers’ sensitivity to individuals when dealing with their enquiries reflected the need to manage the tensions of *subordination* and *pupil fallibility*. M spoke directly of the need to tailor explanations so that a pupil would feel their own needs were being served, whereas K described management of the conversation as giving cues in order for the pupil to make the right connections and feel empowered. These approaches entailed the pupil recognising the diagnostic purpose of conversations, and the expectation that they adopt conventional logic. The tensions could only be resolved through the pupil accepting them, and transforming his own actions in line with the *Gegenstand*, which remained valued above his own logic.

W explained that practice with standard methods was a means of articulating pupils’ ideas, and stressed their effectiveness, presenting them as scaffolds for pupils’ thinking: the notion of *problems-with-solutions* was not seen as tensional. Standard solution forms could be adopted in the first instance, as they enabled pupils to produce some mathematical work of their own on which to reflect for further development. W also saw the use of such methods as a means of aiding the development of conventional algebraic expression, as pupils learned set ways of representing conceptual connections.

As noted above, the teachers were sensitive to *pupil fallibility* and worked to mitigate this tension. M and K expected pupils to adopt their sensibility of the acceptability of making mistakes in the classroom environment. In their use of pupils’ mistakes, they would explicitly recognise that learning mathematics is effortful, and by bringing pupils’

contributions into discussion, the teachers hoped to foster a climate in which pupils would feel confident in mutual support. K focused on creating an “ego-safe environment for the kids to [feel they could] get it wrong” (I028). For K, enabling development in a positive and constructive way was “what we’re about,” in contrast to “harsh” authoritarian and disciplinarian approaches of the past. These approaches created value out of pupils’ mistakes, as they offered opportunities for reflection and contributed to the ongoing activity. Teachers also took opportunities to value pupils’ achievements, insights and queries, although this was subject to constraint. W’s description of having to “pare back” pupils enquiries was a compromise in trying to give them “ownership” of the work they did. Similarly, K tried to highlight individuals’ insights and use their ownership as part of the communal engagement in the lessons. Sharing observations was valued, as was the personal nature of the knowledge obtained.

In relation to the aim of participation, these approaches stressed communal involvement, as they depended upon pupil engagement to sustain this mutually supportive climate. Encouraging reflection on the production enabled pupils to stand aside from their actions and their former selves, whilst also establishing their role in sustaining the activity through participation. Pupils’ work used in this way thus represented the totality of the activity’s production, being a representative of the individual subject, serving the collective action and enabling the subject to stand apart from himself. The teachers saw this use of pupils’ production being inherently connected to an “ego-safe” classroom climate in which making mistakes was permissible. This however led to the emergence of a secondary conflict within activity in which mistake-making was permissible directed towards a future state of the object which should be free of mistakes.

When working with pupils who had had problems with school mathematics in the past, K would give significant attention to dealing with the negative emotions associated with participation. This was described as particularly pertinent with “bottom set” pupils. In these circumstances, K felt it important that pupils were not constrained by normal expectations of order, prioritising instead their emotional experience. This would mean a less strict imposition of order upon the classroom, and reduction in the productive

expectations. In mitigating the tension of *pupil fallibility*, these approaches valued engagement and effort as highly as achievement, but limited transformative opportunity.

The tension of *production versus development* could be seen when pupils worked with a greater degree of engagement on tasks which had a less rigorous productive expectation and would not be assessed. W worked to balance engagement on such tasks with productive practice by issuing wipe-clean boards on which pupils wrote to aid their problem-solving and rehearse the productive skills they needed. Conversely, when full written solutions were required, W's encouragement of working in pencil offered the opportunity to erase or edit workings. The lack of permanent record of their tentative workings made it more comfortable for pupils to note ideas down, and pupils could benefit from the freedom to write without formal expression. These approaches tried to eliminate the tension whilst still valuing the use of text as a deductive tool.

W identified the tension of *tool dependence* in the task form of the exercise, when some pupils could only tackle complex problems after they had had "all the detail in a particular problem" laid out in the preceding easier problems. W had developed a revision mechanism to try to circumvent this, balancing the need for support with the anticipation of meeting varied questions. Pupils would be advised to rehearse methods by working on the first question of three exercises in turn, then the second from the three exercises, and so on. This was described as a middle way between providing the support and "doing something novel and different." The support given by exercises was still valued, although it introduced flexibility to a pupil's rehearsal, promoting some independence from the tool structure.

The use of 'real world' contexts in problems could be considered an attempt to overcome the tension of *bounded practice*, but the process of translation from context to classroom action was undermined through the inclusion of such problems in monothematic exercises. The process of constructing and representing mathematical models of situations did not feature in the visible mathematics of the curriculum: pupils rarely received credit for determining how to model situations mathematically, and discussion of

the validity of contextualisations was actively prohibited in the name of focus on the necessary tasks.

The *production-versus-development* tension of mathematical structures was sustained in conversations over written tasks. Pupils pursued explanations of mathematical structures or information that would enable them to produce complete and correct work. However, justifications given by teachers were constrained by a judgement of what could be communicated in the busy classroom and what the pupils needed to know at that given time. If they felt a pupil would not understand a detailed explanation, then assistance would be given towards completion of the task, reinforcing the exercise as the goal of the work, and the method the tool by which this was achieved.

9.1.3 Producing the pupil: mathematical capability as object and tool

Participation in the activity was identified with working towards learning new skills and developing fluency with them, with pupils' actions framed by notions of development; they had to continually produce work that stretched their previous capabilities, made new connections and developed fluency in required methods. This placed significant value on pupils' facility in applying their mathematical capability, and as such the object of their study then became a tool for use in further development¹⁵. I consider this use here, in order to describe the production of mathematical capability and the pupil in his role. I focus on those elements of practice in which pupils brought the results of their previous learning into action, thereby establishing authority and overcoming the tension of *pupil fallibility*.

Pupils' development in relation to the historical object required not only the adaptation of actions under current focus, but also the re-evaluation of previous actions, as more connections were made between mathematical structures. "Fitting it all together" required the use and adaptation of previously learned mathematics, as more results were

¹⁵ A list of actions/operations that came to the fore in interviews can be found in Appendix 4.

taken as assumed, and expectations of valid and necessary workings changed. Adapting to these changing productive expectations involved looking upon one's former self with a critical eye whilst also making use of one's established capabilities. This use of pupils' non-self-identity (Roth, 2010) was expected to sustain up to the contingent end-point of the IGCSE examination.

In light of the notion of learning in advance of development, the tool/object distinction became weakened, in the dialectical relation between the mathematics used and the new mathematics developed. By co-opting known mathematical results and methods as tools into their work, pupils accessed concrete bases from which new understandings could develop. At the same time they also gave new meanings to those bases, thus further concretising them. As they encountered more complex problems in their work, pupils built upon routines. They were required to manipulate problem scenarios to reveal simpler problems; solution of these smaller problems then led to the construction of a solution of the original. Thus problem-solving could be seen as a process of decomposition and re-composition, in which pupils had opportunities to build chains of reasoning and develop novel methods. However, this feature of tasks was most visible toward the end of an exercise, and might only be reached by a proportion of the pupils. In this case, mathematical authority became self-generating, as only those pupils who had already made greater progress would have the opportunity for such actions.

In all tasks, pupil action depended upon the recall and application of mathematical 'facts'. Mathematical results acquired the status of 'facts' through their use in the classroom: the combination of repeated derivation, ease of justification, frequency of use, consensus, production by reliable tools (the calculator) and use in further problems together contributed to the extent to which results might be treated as facts. Facts could then be recalled and used without justification. This co-opting of previously learned mathematics was seen most frequently in the use of number, depending upon and reinforcing the concrete nature of the number system. During the period of data collection, the use of number as a tool was a more prevalent feature of lessons and tasks than investigation of properties of number. The number system was used as a concrete objective entity whose

properties encouraged precision, accuracy and reasoning. Pupils exercised their capability through fluency with number in dealing with mathematical structures. A pupil's authority lay not only in the quantity of recall of such facts, but in their fluent use in justification of further results.

The tool-nature of the number system was also embodied in the tools of the maths kit. The constraints and affordances of these tools related to the constraints and affordances of the number system, as used in measure and construction. Pupils demonstrated their capability not only through correct use and proficiency with these tools, but also through making judgements as to when to use them. The pupil who depended upon the calculator only to the extent that the teacher approved could lay greater claim to mathematical authority as he could recall and justify more results without that tool.

Pupils spoke of "mastering" topic areas, indicating the reliability with which they could solve the problems of that topic and recall any necessary results. However this notion of proficiency rested upon a perception of a bounded class of problems constituting that topic. This perception was sustained through exercises and teaching materials and revealed when teachers asked pupils to devise their own problems. When offered the opportunity for a demonstration of mathematical authority, invented tasks had to be appropriate to the topic and correctly worked out. However, pupil expectations were constrained by the contents of the tasks they had previously encountered, and they would replicate the types of problems they had seen in exercises, if they were not to risk creating nonsensical or inappropriate problems, or incorporating unforeseen complications.

Pupil authority could be seen in the capacity for managing the interrelations of operations and actions in their work. On the one hand, confidence in the action of applying a routine method would not necessarily be undermined by an operational mistake. Adept pupils could edit out arithmetic mistakes within an otherwise correct solution. Conversely, sensitivity to the construction of methods revealed the relationship between fluency and understanding. The facility with which pupils could explain and discuss underlying structures contributed to their authority and further development: in explicating implicit

details pupil made them available for connection and use in solving problems. Through reference to their production pupils could articulate the connections they had made; determining correct answers to problems indicated that one's method would be worth sharing in conversation. When pupils could manage the multiple purposes of producing written methods (justification, checking argument, communication, doing work) they were able to reflect upon their work to answer queries corresponding to each of these needs. However, opportunities for pupils explicitly to consider their production in relation to those multiple aims were rare.

Building mathematical action upon a basis of capability with specific structures constructed acting mathematically as being about rigour and justification via exploiting the resistant nature of mathematical structures. Aaron's analogy of mathematics as a machine implied the consistent logic of mathematical action and connection between and within structures. This logic was discovered and assimilated through productive problem-solving practice, attaining "correct" answers to problems. Consequently authority was developed upon a basis of progressively unifying the resistant *Gegenstand* of visible mathematics with the structures as objects of thought (*Predmet*). Through such engagement the values associated with work extended to cognition, in support of production:

- DVS: *So when you're doing an exercise or doing some work, as well as getting something on the page... is it important to have something going on in your head as well?*
- Thomas: *Yeh, like to make like how... Like if you just like guess a question and its right... but you have to think to yourself like 'how did I get that?' ... I asked M [who] went over it but I did still didn't get it. M explained it to me, then I looked at the next question, but I kind of got a friend to do it.*
- (I009)

Written work represented exercising skills of deduction and justification: pupils sustained an association of working with pen and paper and having to "think hard" (Thomas and ZR, I002) to attain the right answers. Applying a method to more complex problems required reflection on the reasons behind it, which should aid further development. Pupils would

call upon teachers to justify methods in terms of logical connections in thought and action. These connections enabled pupils to determine which operations were both valid in response to the structure and useful in response to the problem.

Through this exploration I came to see the pupils were produced as having self-generating capacity, as they worked towards acquiring facts and the capability of making further connections with them. Tool use enabled pupils to encounter and exploit the resistant qualities of mathematical structures, thus bringing them into their own capabilities as they were revealed. Pupils were thus produced as non-self-identical (Roth, 2010) in an activity which continually recreated itself, whilst nonetheless sustaining orientation toward the static goal of the IGCSE examination. Their mathematical authority resided in placing their capability within this context of continual self-improvement, guided by the norms of the classroom and the visible mathematics of the curriculum.

This goal relates to, but is distinct from, the development of learner transformability and autonomy (Yackel and Cobb, 1996; Blin, 2004; Lim and Chai, 2004). The development of autonomy was not prominent as a feature of the activity, as the anticipated endpoint did not assess pupils on inquiry, novel argument or transformation of mathematical knowledge to novel circumstances. Pupils had little experience in arranging their own actions in bringing mathematical capability to sense-making or problem solving in the world. As such the activity promoted an I-rationale (Mellin-Olsen, 1981) in developing mathematical authority.

9.2 Shared notions: Characteristics of mathematical action

In this section I consider the emergent characteristics of mathematical action, and the extent to which they were associated with classroom values by pupils. These characteristics emerged from data in all interviews across the research period. Not all participants made the same observations, nor did they necessarily agree with each other. Issues emerged from individuals' own concerns and were not explored exhaustively across all participants. In coming to a picture of the pupils' point of view, these accounts focus on what participants introduced to the interviews, rather than to look for omissions from a

pre-determined notion of the object. The use of texts and tools and the relationships between actions and their constituent operations has been presented within the preceding analysis. The characteristics of action discussed in this section are:

- Applicability
- Mental effort and practice
- Solving problems
- Adhering to methods
- Making connections and managing information
- Fulfilling purpose
- Making sense

9.2.1 Applicability of mathematical action

Participants made repeated connection between their classroom action with number and their action outside the classroom. However, over time the nature of this connection had changed. In early years, they had learned counting and basic arithmetic at home with their families. This had continued in primary school, from which participants reported that their experience in mathematics had largely been about learning arithmetic. In secondary school, number was central as a tool in solving problems. The participants anticipated this use in their future lives, in monetary or financial transactions, or in problems involving measure. These were described as “important” applications of mathematics, and represented areas of life which could become material for school work. In developing fluency, the applicable actions of the classroom became demoted to ‘common sense’ operations: pupils were sufficiently familiar with these aspects of mathematical action that they had lost their effortful character, and hence were described as less mathematical.

Evans (2000a) notes that recall and application of mathematical action depends upon familiarity with the action and the appropriateness to the context. My research suggests that recall was a key feature of mathematics in the classroom: ease of recall promoted positive valorisation of both the subject and object, and recall was a valued operation in

its own right. Recall depended upon and reinforced the concrete nature of mathematical results (as “facts”), and thus concretising the object was a feature of the subject and their own mathematical development. However, this concretisation was valued in the terms of the activity, weakening connection with the world outside, where recall plays a less prominent role than transformation (Noss et al., 2000; FitzSimons, 2002, 2005; Kent et al., 2007).

The participants all explained in later interviews (during year 10) that they anticipated uses for the knowledge and skills of school mathematics in later life. However, the *personal meaning* and *bounded practice tensions* sustained in that, aside from the use of number and measure, they could not locate those uses: action in the classroom had not related meaningfully to practical action in the world outside the classroom or their imagined futures. Jamie had earlier summarised this contradiction by explaining that mathematics was everywhere in everyday life, but that it was difficult to find (I012). Pupils suggested that the mathematics likely to be used in one’s career would be more complicated than that learned in school, and therefore beyond current imagining. However the examples offered were of ‘more, harder’ problems of the type they had frequently encountered (reinforcing the observations of Picker and Berry (2000)). Alex hypothesized that through undertaking further study, the everyday uses of mathematics might become clearer (I022).

In relation to this tension, participants demonstrated discomfort with learning something which seemed to be of minimal use value:

Thomas: um, well when it’s like word problems and stuff like um, I don’t know, you’re working things out, but if you don’t want to have maths in your job, I don’t think at some point they’re that relevant to me because like algebra and triangles and stuff, I don’t really think that will help me.

(I025)

As illustrated by the interview extract on pp. 195-6, pupils held that whilst mathematical activity could be abstracted from common experience, such abstraction would not happen in everyday action. A conflict between the apparent purposefulness of arithmetic action

and the purposelessness of algebraic action came to the fore here. In the classroom, and across the school, the exchange value of mathematical authority was reinforced above the use value of mathematical capability.

9.2.2 Mental effort and Practice

Visible demonstrations of effort were expected of pupils, as teachers continually shifted the ZPD through which they worked. The expectation of fluency with mathematical tools resulted in a focus on practice with methods, through which pupils could observe where their skills were lacking and then address these omissions. These expectations constructed engagement with tasks and reflection upon their outcomes (in attempts to overcome the *pupil fallibility* and *learning in advance of development tensions*) as characteristic features of mathematical action. Participants' descriptions of doing mathematics entailed careful thought and being able to explain one's thinking. Producing an account of one's actions was a central feature of mathematical work; the use of observation or intuition was devalued, as conventional explanations were encouraged. However, producing conventional explanations was in itself effortful for pupils learning to use conventional language and forms. This reflected the *production-versus-development tension*, in which pupils were expected to accept that adopting conventional language would aid their understanding of the mathematical structures represented. The emphasis on rehearsal of methods supported Lim's (1999) association between absolutist views of mathematics and hard work as the root of success. My research also places shape upon the notion of "hard work": it entails pupil compliance, attention to the teacher and the structure of activity, practising textual reproduction and relinquishing subjectivity in the pursuit of mastery.

In relation to the continual need to build upon previous understanding, pupils' attention was drawn to the level of action in their work. The constituent operations within their work passed relatively unnoticed unless mistakes came to light. Actions had become operations through rehearsal and establishing fluency. This consequence of development created a tension within the nature of mathematical activity: to become proficient at mathematical tasks, a pupil practised them until they were fluent, yet in doing so the actions and understanding became demoted to operations and thus lost their character as

effortful mathematical acts. Recall of results and operational fluency became “common sense” operations, receiving less reward having decreasing value in the classroom. As more mathematical actions became useable, their status in terms of visible mathematics was diminished, along with the authority sustained by their use.

9.2.3 Solving Problems

Attaining pre-determined answers to well-designed problems within contextualised tasks was the predominant trajectory of action in the classroom. Pupils worked to become fluent in the deconstruction of problem scenarios, solving sub-problems to construct a solution to a main problem when necessary. The success with which a pupil used the results of previous learning as a tool to solve new problems determined the subjects’ sense of making progress. A focus on solutions thus constructed mathematical action as deductive and accountable. However the recreation of model solutions, and continual reproduction of authority and pupil position, placed emphasis on the written solution as the aim of work, rather than development in the subjects’ deductive capability, preserving the *problem-with-solution tension*. The tension between *use and exchange value* was embedded deep within classroom activity: the results of the problems were of no use to the pupils other than in the credit they earned within the activity. Pupils had had no experience of investigating problems that they had generated themselves, as work was always constrained by the ZPD identified by the teacher.

9.2.4 Adhering to methods

Participants’ descriptions of mathematical action indicated the expectation of fluency in the routine and predictable methods they had to apply. Recognisable problems relating to well-known mathematical structures elicited structured solutions, produced efficiently. Within this work, a pupil’s opinions about problem scenarios were irrelevant and not admissible as conceptual tools in the solution. It was also not permissible to query the purpose or justification of the methods or the problems themselves. Pupils faced the refined task of identifying the relevant mathematical structure and acting upon it. Mathematical objectivity was attained by restriction of the tasks given, the concepts that could be used and the methods with which one tackled a problem. This could be seen as

reinforcing the tension of *pupil fallibility*: problem-solving happened on the terms dictated by mathematical logic and classroom convention, rather than pupils' interests or opinions. This uniformity of experience and outcome contributed to the use of consensus throughout pupils' work. However, accepting the need to adopt methods as given by a teacher or textbook was a means of mitigating the *learning in advance of development* tension: if mathematical capability resided in a discrete class of methods to be acquired in relation to classes of problems, developing this capability became a matter of simple attention and reproduction, without concern for justification or purpose. This also simplified the *roles of text* tension, as texts no longer need to be reflected upon, merely reproduced.

9.2.5 Explaining actions

Classroom activity often involved the exchange of explanations: pupils were expected to attend closely to explanations offered and then work to produce sufficient explanations of their own action (often recreating the teacher's explanation). Explanation was necessary to make sense of the application of a method (in time) which resulted in the production of the written solutions presented in textbooks. As a result, mathematical work was supposed to convey adequate justification of conclusions reached. Pupils' production was then re-produced (through marking) as being fitting or incomplete. Through complying with this obligation, pupils could work to overcome the *pupil fallibility tension*, although the persistence of this expectation would sustain the tension.

Pupils were expected to produce individual accounts of their work, even though they often worked collaboratively on solving problems, sharing ideas and editing each other's production. The working norms of the classroom permitted this collaboration, but at the same time directed the pupils toward occasions when they would have to work without any assistance. The tension of *personal meaning* can be seen reflected here: pupils continually worked on their skill in producing written explanations of answers, with the aim of developing sufficient fluency for examinations. The classroom thus constructed the production of text as the key mathematical action, rather than as a representation of some action which had meaningful qualities outside the activity.

9.2.6 Making connections and managing information

In solving classroom problems, it was understood that one would need to use all the given information in order to make the necessary connections and reach the correct answer. John described the general pattern of problem solving in terms of connecting additional knowledge with the given information: “there’s always something you don’t know, and that you have to find out, using the information you do know, using... some mathematical information you do know” (I021). In the long term, subject development was observable in the extending range of conceptual tools the pupils adopted by subsuming new connections into applicable methods. Whereas in the short term, by working through problems and making sense in action of the method the teacher had illustrated, the pupils’ production converged upon that required or demonstrated as the aim of the lesson. In this process problem-solving was produced as a specific form; using supplied information to recall the fitting method and achieving the correct answer, whilst providing a suitable account. This approach to problem-solving lies in contrast to the process of “laying bare all its details” (Ilyenkov, 1960), highlighting the lack of transformation (on both the subject and object) of the *problem-with-solution tension*.

Reflection upon connections made or wanting was a central feature of the effortful characteristic of mathematical action. Aaron’s description of mathematics as a machine in which everything fits together suggested the potential for pupils to develop a sense of mathematical methods as an integrated whole, connected through common operations and structures. In this sense, the ‘gaps’ in a pupil’s understanding were defined in relation to curricular expectations. Visible curricula, examination syllabi and social expectations determined which tasks the pupils encountered in the classroom, thus development was oriented toward filling only those gaps identifiable within these bounds (and indicated via the collective ZPD). The visible mathematics of the curriculum was reproduced through pupils’ work, as the pupils were produced as more or less successful in relation to its contents.

9.2.7 Fulfilling purpose

Demonstrations of external purpose did not form a prominent part of classroom activity. This resulted from a combination of pupil disinterest in explanations and teacher focus on developing fluency in well-defined methods. Even when supplied by teachers, explanations of purpose or application of methods outside the classroom were easily ignored by pupils. Over time, pupils' aims focused upon achievement of tasks the teacher presented and the development of fluency with mathematical methods: teachers were constrained in the extent to which they could invoke additional purposes to engender interest. The actions of the classroom become encapsulated within examination expectations, with limited relation to behaviours which co-opted additional material tools, conceptual structures or unconventional symbolic reasoning.

This disjunction between the classroom and the world both stemmed from and reinforced the exchange value of mathematical achievement as marked by the public examination; this was also presaged throughout school life in the pupils' need to produce good grades and high test achievement. This inward focus also resulted in a sense of mathematical action being learned in order to act in ways which would only be valued in the classroom: participation was oriented towards learning how to participate more successfully. Pupils were structured to comply with this imposed orientation, wherein practice and fluency were the aims, with the examinations as a point of closure. John explained that if a pupil wanted to master the necessary techniques he shouldn't worry about why they are being learned, but focus on developing fluency:

John: ... generally we know we have to do it we know we have to get on with it, we've sort of gotten over the stage of wondering why and we're more bothered with getting on with it, finishing it, doing the test, getting high scores and then we can be concerned about more trivial things like 'why?'
(I021)

9.2.8 Making sense

Participants spoke of making sense of their mathematical actions in three distinctly different ways: within the activity of the classroom; within the structures of mathematics;

in the context of their lives and futures. These ways related to the development of mathematical capability and authority, and could be seen in light of the certainty with which pupils were able to make judgements in these different contexts.

As they progressed through the school, the activity of the classroom changed little in terms of teacher-pupil interactions, productive expectations or compliance with predetermined aims. In the activity of the classroom, the division of labour saw the teacher determining the order, timing and presentation of topics, with the pupils obliged to interpret the actions and operations of the teacher in terms of their own productive action. The pupils' practice involved deconstructing and reforming the teacher's operations in order to demonstrate their learning. This pattern sustained throughout their school experience. Undertaking tasks which replicated aspects of this labour could be recognised as relating to mathematical activity, and a pupil knew what to do in response. However, work with the structures of mathematics had become continually more challenging, and whilst developing from an increasing (and increasingly) concrete basis, always revealed there was more to learn, up to the point of the examination. Awareness that there was "more mathematics" after the examination was not a concern for the research participants. Identifying an endpoint to study was a means of overcoming the *learning in advance of development tension*.

Thomas articulated the connection between mathematical logic and making sense. He illustrated the nature of developing capability when explaining that "you know when you're right because it makes sense" (I017), and that one could make sense of written examples when they followed the logic one knew. The use of consensus reflected an imperative in that mathematical logic and methods *should* make sense to a pupil; following the deductive logic of mathematics should lead everyone to the same understanding. The use of such logic had received authorisation through the practice of the classroom: in discussions and through feedback on written work a pupil's developing capability for acting mathematically would be produced as fittingly logical. Failure to make sense reinforced the *pupil fallibility tension*, rather than raising questions about mathematical logic.

Connecting mathematical action in class with their lives outside the classroom was a contentious issue for the pupils. They were able to assert that arithmetic and an awareness of shape would be useful in the world. Socially meaningful problems within their family lives were identified: these problems involved elements of negotiation of meaning alongside the use of arithmetic and shape. Alex offered a particularly illustrative example in which he was asked to wash half the windows of his house; the meaning of 'half' was negotiated as a rough calculation of area of a number of windows. Pupils identified examples of working with conversion between units, estimation, division and proportion, and using understanding of area, volume and dimensions in their daily lives. However, other than the use of number and measure, topics retained little efficacy in their imagined use outside the classroom. The use of formal geometry and algebra were rejected. These findings recreated those of Abreu *et al.* (1997). However, participants articulated informal structured action outside the classroom, described as having a 'mathematical' character. For example, the connection between playing chess and solving logic puzzles was sustained in a conjectured use of equations and rules in order to make progress, and the need to understand a specialised notation. Jamie summarised this approach as a "mindset" (I026) to problem-solving.

9.3 The meaning and value of transformability

My literature review (§2.3, §3.3) indicated that everyday use of mathematics predominantly lies in transforming mathematical relations and representations to particular situations and needs, often in response to dilemmas or toward particular goals (Masingila, 1993). Familiarity with situations leads to the development of pragmatic reasoning schemas (Cheng and Holyoak, 1985) in which both flexibility of action and situational features are preserved. In an increasingly technological culture, the need to reveal the models within our technologies becomes imperative, necessitating confidence with tools to "bring the model to life... and express its structure" (Noss, 1997). The increasing presence of technological mediation of our actions elevates the need to identify the role that mathematics plays in the world, as "constructive, concerned and reflective citizen[s]" (OECD, 2003). I understand these imperatives as a call for pupils to be

empowered to make mathematical rules and structures subordinate to the aims of their practice (Martin et al., 2009) as a resource for their activities (McDermott, 2013). This requires the skills to mathematise reality in the service of non-mathematical goals (Nunes et al., 1992), continually transforming both the world and the acting person in dialectic interplay of social needs and resistant reality. In this section I consider the extent to which classroom activity can be seen to anticipate these future transformative needs.

I have shown that classroom activity was oriented toward the production of grade achievement. This can be seen now not only from the point of view of teacher accountability, but also in terms of the pupils' focus and aims. The primary responsibility of teachers and focus of pupils was attaining the highest possible examination grades. This focus has been shown to be both a reaction to and a contributory factor in the bounded nature of the activity. The focus on practice of skills and adoption of forms of expression negated potential dialectic of subject-object relations that would sustain outside the classroom. Working within the stable social arrangement of the classroom, and producing responses to classes of pre-determined and structurally similar problems, pupils' ability to shape the context in which they act was deliberately inhibited. This inhibition stemmed from both the teachers' obligations to the school and the pupils' desire for clear and tangible goals. Rehearsal arose from the need to develop certain specific skills, and the identification and prioritising of those skills lay in the hands of the teacher. Consequently, pupils had no sense of the object being transformed through their developing capacity. The activity structure of the mathematics classroom contained an abnegation of the person/context dialectical relation which would be experienced by the pupils when determining and dealing with problems outside the classroom.

Associated with this limitation was a restriction upon the understanding of transformation. The reformulation of knowledge and skills was a key feature of doing mathematics in school, yet the contingent end-point of this reformulation hindered recognition of the self-generating transformability of the subject. Reflection on the activity of the classroom did not form part of the activity, and thus this potential meaning of learning in mathematics was subsumed under task aims. Subject transformation took

place in mathematics lessons, but an awareness of this was not fostered in the activity of the classroom. The object as created in the classroom was saturated with transformative potential, but harnessing this was inhibited by restrictive classroom practices derived from the need to maintain order and orientation toward examination achievement. The “myth” of mathematics as an incorrigible body of knowledge (Ernest, 1996), was sustained by the lack of dialectical influence between subject and object in the classroom.

The implications of the metaphors of *convergence* and *emergence* became clear: pupils’ convergent development was directed toward the visible mathematics of the curriculum, and their production as successful learners. Observation of the tensions within the activity and expansive action in response to them was suppressed in the persistent stability of the classroom. This suppression was instigated by both teachers and pupils, as they maintained orientation toward the endpoint of public examinations. The resultant object between their respective activity systems (Figure 7, p. 113) was thus outlined by the curriculum and substantiated by social relations that had sustained around the pupils’ productive participation. This object was constructed with generative potential that could promote further transformation in the individual as they act in the world. This transformation would be identified with an *emergent* object. However, participation in a framework in which authority was judged upon conventional representations and sanctioned meanings had the effect of associating such transformation with the vertical mathematisation of the classroom, and thus devoid of value in everyday practice. The compliant pupil could achieve success within the school system by developing a *convergent* object.

9.4 Locating mathematical action

The articulation of the relationship between capability and classroom authority offers a means of describing how pupils might become empowered to locate mathematical action outside the classroom. In this section I consider the two aspects of ‘locating’ mathematical action, finding and placing, in relation to classroom action.

Deriving from curricular statements of capability and content, mathematical action in the classroom was found when pupils were asked to deal with the structures, tools and representations of visible mathematics. The privileged position of a set of structures (algebraic, graphical and geometric) and their representations influenced notions of “proper” mathematics. Action outside the classroom which invoked working with these structures, undertaking algorithmic decision making or recognising the use of symbolic representations was recognised as mathematical. In this respect, finding mathematical action in the world was related to an evocation of the object and tools of the mathematics classroom and the visible mathematics of the curriculum.

In contrast, placing mathematical action would involve transcending classroom activity; preserving values and mathematical structures as one chooses, appropriating tools to establish authority in situation-specific terms whilst developing additional and temporary meanings. In the terms of activity theory and CHAT, the participants’ discussion of the uses of arithmetic and spatial relations suggested that this authority had been established with those structures which had been used at an operational level (i.e. had become tools). The use of these tools could be invoked with little explicit indication of their need. However, it was difficult to represent and use more sophisticated (vertically mathematised) structures in problems whose meaning extended beyond the classroom. Pupils had also had little experience of determining, using and reflecting on derived mathematical models, i.e. instantiating novel structures as tools within problem scenarios, and “bringing these models to life” (Noss, 1997) through their work. Their authority in such action remained limited within the classroom, and always subordinate to the teachers’ authority. At the heart of pupils’ engagement with mathematical structures lay a persistent tension between continual change in the subject and stability of the activity, alongside the incorrigibility of mathematical logic.

The tri-partite structure of norms in my analysis differs from the description of norms as social or sociomathematical offered by Yackel and Cobb (1996), revealing the maintenance of order that overlaid the norms of work and defined the activity in which mathematical development happened. This suggests that the development of autonomy

needs to be seen within a frame of social order and authority in order to understand how pupils can be equipped to transcend the classroom.

10 Findings and reflections

Over the course of the last three chapters I have detailed my exploration of the mathematics classroom as a 3rd-generation cultural-historical activity system. This exploration originated with the question “How is mathematics located in the practice of the mathematics classroom?” which then developed as I attempted to “lay bare all its relevant details” (Ilyenkov, 1960). My shift in focus from attitudes towards mathematics to values ascribed to mathematical activity placed an emphasis on the continual reconstruction of cultural categories of value. Adopting CHAT as my main analytic tool emphasised the aims and productive actions of the pupils within the classroom activity system. Recognising the agency of the pupils within their role as learners enabled me to explore the co-construction of the activity and the object.

With this framework I found that the object appeared as classroom mathematical authority; exhibited in conventionally acceptable production, this authority was sustained in relation to the visible mathematics of the curriculum and developed through compliant productive participation. This process resulted in the production of the pupil in terms of his grades and expectations of further development. This production both depended upon and reinforced the social stability of the classroom as pupils progressed towards IGCSE examinations. Jones (2011, p. 368) notes that CHAT research has repeatedly acknowledged and analysed the “stultifying” restriction of school learning to curricular specifications embedded within classroom activity, isolated from life and action outside school. Engeström (1996) refers to this isolation as the “encapsulation of school learning.” My research suggests that this encapsulation results from the values sustained in activity by both teachers and pupils.

In this chapter I review my exploration and draw conclusions about the instantiation and implications of this encapsulation. I review the evolution of my research project as a cultural-historical activity in its own right, which had as its object the mathematics classroom as a joint activity system centred upon the object of pupils’ developing

mathematical capability. This review draws together the narratives that have run through the thesis.

The structure of this chapter derives from the initial aims I set out for myself, and the three narratives of the thesis. Following a summary of the activity as it emerged in the analysis I offer my key findings, my contribution to research and review the development in my dual identity (§10.1). I review the progress made toward each of my aims, in light of the rationale for this research. In §10.2, I explore the means by which pupils were empowered to locate mathematical action, and consider the methodological contributions of this thesis. In §10.3 I draw observations about the teaching practice the pupils encountered, in relation to a system defined by examination achievement, connecting my observations to the philosophical background of this research. In §10.4 I critically reflect on my use of CHAT in describing the classroom and suggest possible contributions to theory. In §10.5 I discuss the limitations on this research. In §10.6 I explore the implications for further research and in §10.7 offer my concluding remarks.

10.1 Key findings: Classroom activity and teacher-researcher development

In this investigation I found that elements of the classroom did not map discretely onto single nodes of the activity framework. For example the textbook, an ostensible tool of the classroom, instantiated the object and also inhabited the division of labour. Material and conceptual items which initially constituted the object of study shifted to become “transparent” (Noss and Hoyles, 1996) tools in making further progress. Similarly the school rules were tools with which the teachers sustained the community structure. Consequently I have chosen not to re-present the activity framework with the constituents of the nodes ‘filled in’. After presenting my key findings in this section I describe the resultant object of the activity, the pupils’ mathematical authority, and the co-production of the pupils in pursuing this object. I then review my development as a teacher-researcher.

10.1.1 Key findings

My findings relate to two aspects of the study, the location of mathematical action in the classroom and my development as a teacher-researcher, and are summarised here:

- For the pupils in the study, mathematical action was encapsulated within the activity of the classroom, through the values applied by both the teachers and the pupils. Teachers bore a primary responsibility for high grade achievement, whose value was presaged throughout school life. In pursuit of this value, pupils exercised their agency by actively rejecting connections with meaning and application outside the classroom, and suspending questions relating to the premises of the curriculum and classroom activity.
- Pupils' mathematical authority was located within an enduring system of authoritative relations and task forms: the pupils' development had little influence on the continually recreated activity system. This structural stability suppressed the generative potential of systemic tensions, preserving community positions in relation to the high-stakes endpoint of the examination, and positioning the pupils to internalise particular forms of learning and mathematical activity.
- My research pursuit has explicitly placed my teaching practice in a broader socio-historical context, in which I am better equipped to identify the instantiation of educational aims in the action of the classroom and the structures of schooling. The process of engaging with the literature and undertaking a structured investigation revealed the unintended products of mathematics teaching and the extent to which these products became associated with mathematical capability. The production of pupils in a sustained relation to knowledge and knowledge construction, which they would take into the world, was revealed by characterising the classroom as a tool-and-result methodology (Vygotsky, 1978).
- In developing and substantiating the descriptive vocabulary of *capability* and *authority* this research has broadened my understanding of how "knowing" is socially constituted and constructed as authority. Whilst not an explicit focus of this research, different aspects of "knowing" have come to the fore. These aspects

(as a mathematician, as a teacher, as a researcher, as a pupil) co-exist in the classroom, yet appeal to different necessary substantive manoeuvres and tools, and means by which others can hold one accountable for that knowledge.

10.1.2 Contribution to research: norms, tensions and the object of classroom activity

10.1.2.1 The structure of norms in classroom activity

In their research into classroom culture, Angier and Povey (REF) showed that curriculum, pedagogy and epistemology were drawn together in classroom practices and relationships. They showed that students' understandings of the nature of mathematics were linked to classroom relationships and in turn to the curriculum. In this research I have developed a framework of norms with which to articulate those linkages, and which offer a means of describing the relative influences of teachers, institutional order and the pupils themselves. Vygotsky (1962, p. 56) argues that to explain higher mental functions "we must uncover the means by which man learns to organize and direct his behaviour" and that "the child begins to practice with respect to himself the same forms of behaviour that were formerly practiced with respect to him" (Vygotsky, 1966, p. 39-40). In this research I have shown that forms of behaviour practiced upon the pupils, deriving from social and institutional motives, influenced (and could be in tension with) the emergence of forms of behaviour deemed 'mathematical'. I have also shown that in the dialectic relations between teachers, pupils and the object of study, the pupils themselves import motives that shape classroom norms, and the potential relations between classroom activity and transformative action in the world. The inhibitions on the development of mathematical authority related to the persistence of the norms within the activity structure.

Stone *et al* (2013) similarly describe how "practical-moral knowledge structures the regulatory processes" within classroom learning. They show that regulatory processes of social and moral order are sustained and conveyed through learning practices, and conclude that differing patterns of regulation become evident in mathematical actions in the classroom. In this research I have illustrated the connection between mathematical

action and frameworks of social order, and revealed the part played by the teacher in making this connection. Stone *et al* contrast teachers' instructional practices; my research indicates that to adequately describe classroom activity, one should also take into account the teachers' institutional role and responsibilities. How the teacher manages the tensions within classroom activity will relate to the importance they accord to each of the norms. Recognising the emphasis placed on the production of the pupil in relation to each norm offers the means of generalising this framework to other classrooms and disciplines. The contributory actions and operations of those norms may also vary between classrooms. However, the connection between institutional order and developing personal authority in a discipline through participating in classroom work would persist. The production of pupil and disciplinary authority could be explored through this framework.

10.1.2.2 Identifying tensions in classroom activity

At the outset of my investigation, I had rejected the assumption that pupils are necessarily motivated toward the object "mathematics." In this research I have uncovered the implications of this acknowledgement. The community tensions (table IREF) derived from the need and responsibility of the teachers to engage the pupils' interest, whilst maintaining a stable activity in which all pupils understood what was expected of them and could play a fitting part in the social order. In this respect, mathematics is no different from other disciplines. The production of the pupils as "good" members of the classroom and school community played the dominant part in a dialectical relation with their mathematical production. As such, the identification of these tensions offers the means of exploring classrooms in other disciplines, to understand educational achievement in relation to participation and production. Ponitz *et al* (2009) found that higher levels of behavioural regulation predicted stronger levels of achievement across a kindergarten year, particularly in mathematics, and concluded that behavioural regulation aids the learner in adapting to the structured demands of schooling. They suggest that helping "students successfully manage their behavior in school will undoubtedly have benefits beyond mathematics achievement" (ibid, p. 616). My research indicates that compliance

with the social order might lead to 'success' in school mathematics, but remains in tension with the transformative needs of mathematical action outside school.

Tensions uncovered in this research also relate to the subject-object relation in the mathematics classroom. Pupils were expected to identify and work toward an object that would be created and comprehended only as the result of their work. For the pupils in this research, their primary understanding of mathematical concepts came through engagement with their written representations. In undertaking learning in advance of their own development, pupils placed value upon their produced work in terms that they understood: the currency of grade achievement and communal position emphasised the exchange value of mathematical work above its use value. Without means of convincingly reproducing the conditions of work outside the classroom within school mathematics, or of making valid connection with pupils' everyday lives, the pupils had no means of establishing the use value of mathematical action. The object of mathematical authority thus became a tool in the pupils' production of themselves in the school community. Mathematical actions were thus doubly encapsulated (Engeström, 1996); within both the school curriculum and the value system of school achievement.

The framework of tensions I discovered has the potential to become a first step in the development of classroom practice: CHAT posits that identification of tensions should lead to development of the activity. Through understanding the values that come to the fore in resolving these tensions, teachers could become equipped to examine how their own practice relates to the qualities of educational achievement and mathematical authority. Also, by articulating the tensions which impinge upon pupils' actions, they can become active in productive resolution of the tensions, and thus begin to transcend the limitations of classroom practice.

10.1.2.3 The object of activity

To the extent that pupils could exercise their mathematical capability (comprising their facility with actions and operations such as those listed in Appendix 4) in meeting the expectations of the curriculum and the productive requirements of the classroom, they

could claim mathematical authority. This was aided by reflecting upon the characteristics of action and the uses and values ascribed to their production. The relationship between capability and authority was tensional: in establishing authority, pupils made use of the inherent transformability of their capability, but expectations of transformation were limited by a (contingent) endpoint against which their authority would be enduringly valued. The goal of the public examination grade acted as both motivator and inhibitor in that it was a meaningful goal with significant value, but whose achievement precluded exploratory or expansive action.

The value of pupils' mathematical action therefore lay predominantly in the ultimate exchange value of their attainment. This exchange value was anticipated within the activity, and sustained across the pupils' shift in attention from interpersonal relations with teachers to their own production in the school. Value was earned through observing the social and working expectations of the classroom, as the basis of progression in the vertical mathematisation of the curriculum. Consequently the meaningfulness of mathematical actions resided less in the structural connections made than in classroom and curricular permissions and appeals to authority, which came to constitute pupils' own mathematical authority.

In the realisation of this object, the pupil was constructed as inherently fallible, overcoming this construction only by continually producing evidence of his development. In his role as a learner he was accountable to the teacher and the curriculum, whose methods, results and structures he was expected to assimilate as they were revealed. Through such productive participation the pupil should internalise the resistant properties of mathematical structures. Participation entailed the use of material and calculation tools upon which he was expected to depend for certain tasks, in the name of efficacy. This dependence rendered the investigation of select structures unnecessary. In relation to the teacher, the pupil was positioned to develop mathematical authority through imitating and adopting the behaviours demonstrated. The teachers were not only mediators of the curriculum, but also the arbiters of all appropriate action in the classroom. Having established compliant participation, their role in determining the correctness and

suitability of pupils' production came to the fore. This authoritative relation became more prominent as pupils approached the IGCSE examination and relied more upon the teacher's judgement.

10.1.3 Dual identity narrative

This research was initiated by my own predisposition to enquiry, and early professional training which had established a view of teaching as a research profession. Through the research, this predisposition has developed into a dual identity, the two aspects of which have been both complementary and conflicting, whilst sustaining an engagement with the aims of teaching practice and the values sustained in the mathematics classroom. These identities present differing perspectives on the efficacy of the mathematics classroom, whilst both stem from a belief in the benefit of mathematical capability in making sense in and of the world.

Roth *et al.* (2004) illustrate that CHAT can be used to articulate identity as an outcome of participation in historically situated practices involving specific tools. They identify the "struggle of making and remaking" identities and having them understood by others through production and exchange (Brickhouse and Potter, 2001, in; Roth *et al.*, 2004). Reflecting upon my actions in this research in terms of the relations in which I play a part has enabled me to articulate my personal development, in terms of resources, goals and mediated action. Here I first consider my identity as a teacher, then as a researcher, then the interplay between these identities.

Through adopting the framework of dialectic philosophy and activity theory, I have come to view my teacher identity as an achievement in the situated activity of the school. I am also now equipped to view my practice as inhabiting a wider socio-historical and philosophical context, affording me the position of "increasingly informed participation" (Edwards, 2000, p. 198). My preliminary exploration of the literature encompassed issues and perspectives that are rarely made evident in the classroom; the basis from which I draw information seen to be relevant has become much wider, as my understanding of teaching practice has expanded to encompass notions of the production of pupils as

learners and the means by which the classroom empowers them to transcend classroom specifics.

My classroom identity as a teacher “is an aspect of identity that arises anew every day and for each class” (Roth et al., 2004, p. 55). Viewing my teaching identity as an ongoing process in which decisions are informed by sensitivity to the situation, I am now equipped with a set of constructs with which to observe, explain and act in the classroom. The set of tensions identified in this research (Table 10, p. 253) represent one possible ‘toolkit’ with which to work. However, the ongoing nature of activity and my reflection upon it mean that this toolkit is amenable to adaptation in response to further observations. Awareness of the tensions that pervade classroom activity sensitises me to the complexities of introducing additional goals.

Reflection upon the productive effect of the mathematics classroom has revealed the extent to which pupils were mediated by the pursuit of grade achievement, and the relation between social meaning of such achievement and personal empowerment. Discerning the distinction between capability and authority has shed light upon the production of pupils in relation to the curriculum. Remillard (2005) identifies differing relations of teachers to curricula (following or subverting; drawing on; interpreting; participating with): as a teacher I am now equipped to explore these different relations in practice.

As a researcher my identity developed first through orienting myself in the field of mathematics education research: working with the tools of the literature, comparing theoretical frameworks in relation to my goals, articulating those in suitably rigorous terms and (perhaps most importantly) understanding the mediating relationship these actions have shared with the writing of this thesis. In the *Tätigkeit* of my PhD research project, I have come to see these items as resources which both enabled and constrained my development. The tools I developed for this research (the analytical framework and the notions of authority, capability and ‘locating’ mathematical action) have resulted in an empowering description of classroom activity, but have precluded considerations of

power, concept formation and learning transfer, which might have been encompassed within a different theoretical framework.

Understanding identity as dynamic highlights the need for continual reaffirmation through engaging in practices. As a part-time student participation in the research community was sporadic and occupied interstices in the working life of a teacher. Consequently the empirical story told in this thesis derives from my own involvement in the tasks of research and the extent to which I could sustain focus on the goal of answering my questions. The hiatus in data collection represented a period of time in which I had no engagement with the research. Re-assuming a research identity required re-familiarising myself with the literature, my own writings and findings and the participatory structure of academic study.

Considering interplay between identities, Roth *et al.* (2004) note that “culture associated with practices in a field can be enacted in other fields, thereby creating contradictions in those fields” (p. 51): My two identities remain in tension as my engagement with wider social and intellectual concerns questions the practices of the school. Engeström *et al.* (1995) suggest that recognising and working within “polycontextuality” can offer a multi-dimensional view of expertise; I would contend that my experience in this research demonstrates the (use) value of “horizontal expertise and boundary crossing” (ibid, p. 332) in identifying and articulating problems within mathematics education. However, being apprised of the imperviousness of structures of activity is not the same as being empowered to change them. Roth *et al.* identify the “weakness of cultural boundaries” (2004, p. 167) as offering the potential for generative action. However, my research would suggest that the intransigencies of school structures (Jaworski and Goodchild, 2006) will be difficult to overcome, and that the classroom is a particularly enduring form. Seeing the classroom as a tool-and-result methodology (Vygotsky, 1978), both prerequisite and product of the community, that assumes its own validity and effectiveness, revealed its continual self-recreation, admitting no change in community relations. Pupils, teachers and the classroom were produced as a unity, mutually sustaining each other. The

community created the classroom (the “toolmaker’s tool” (Newman and Holzman, 1993)) as a means of developing mathematical ability.

My challenge is now to develop mechanisms by which the knowledge gained from research can be “informed, used, constructed and shared” (Edwards, 2000, p. 199) within the activity of the school. My research has pointed toward the potential effectiveness of examining teaching through cycles of expansive learning (Engeström, 2001), but my understanding of the demands upon teachers’ time and attention suggest this would be prohibitively difficult to implement, if permitted. The introduction of research interests and imperatives could result in professional conflicts, and would require careful management as a boundary-crossing enterprise (Walker and Nocon, 2007).

Determining how to use my analytic framework brought to the fore the difference in claims to knowledge afforded to the different identities. In order to substantiate discoveries within the genre of PhD research, the framework was established as a tool with which to collect and make sense of empirical data. Using CHAT as a tool, with its ontological and epistemological distinction from prevailing models, enabled me to stand apart from teaching practice and offer intellectual rigour to my claims. In contrast, any claims I wish to make within teaching practice would have to be based primarily upon demonstrable effectiveness in terms of the curriculum and attainment. The intellectual and philosophical basis of the framework would be of little concern. I see this comparison as reflecting the distinction between mathematical capability and authority, and the extent to which the acting person is empowered in a given situation to manifest their capability as authority.

My exploration of the literature revealed the widespread depiction of school mathematics as a particular set of contextualised practices which were derived (but distinct) from the practices of academic mathematics and perceived workplace needs (e.g. Ernest, 1998; Burton, 1999; Boaler, 2000; Boylan, 2004). Studies such as these raised the question of schooling as primarily a process of social enculturation in which concerns of individual self-fulfilment are secondary. As a researcher, this raised the troubling question of the

effectiveness of research in informing public policy and curriculum development. As a teacher, my attention was drawn to the goals which can sustain in classroom activity, and the extent to which this depiction might be recognised within schools and used as a lens with which to focus on the production of the subjects of learning. I offer this research as a (personal) first step in tackling issues in which the production of mathematical capability as a characteristic of the person entails issues of meaning acquired and ascribed, creative actions and routine operations, and the continual “dance of agency” (Boaler, 2002b).

10.2 Locating mathematical action; Methodological contribution

Whilst being methodologically “open” (Nardi, 1996b) CHAT analyses share the common feature of application of the EMT to detailed descriptions of events in order to understand patterns of activity. The connection between the data and the EMT can be by researcher’s *fiat* (e.g., Engeström, 1998; Coupland and Crawford, 2002; Groves and Dale, 2004; Ho, 2006) or close attention to the data in order to inhabit the subjects’ point of view (e.g. Wells, 2002a; Hardman, 2007; Roth and Lee, 2007; Jaworski and Potari, 2009). In this section I consider my use of grounded theory to connect CHAT with the empirical data, in order to assess my progress toward understanding how mathematical activity is located in the classroom.

Upon reviewing this research, I came to view the analytical framework I had established as stronger than a set of constructs suggesting a methodology, but rather a *paradigm*: “a basic belief system or worldview that guides the investigator, not only in choices of method but in ontologically and epistemologically fundamental ways” (Guba and Lincoln, 1994). I could now see that the problem with my initial analysis was not only in the lack of insight revealed, but in my application of CHAT. Whilst the EMT was an element of my analytical framework, my application of it as a heuristic device was epistemologically inconsistent with the framework itself. In trying to use it in this way, the possibility of developing intersubjective understanding was limited and the effect of dialectical mediation ignored: I was undermining the Vygotskian principles as they applied to my own learning. Through adopting the grounded approach, my learning remained open to those

principles, and the object of my research came to the fore through a process of negotiating my own ZPD. In this way I could “accept responsibility for [my] interpretive role” (Strauss and Corbin, 1994) not only through providing through description of my method, but by placing my learning in the same evaluative frame as that of the subjects in the research.

Nussbaumer (2012) notes that the complexity of CHAT is revealed by the lack of consistency with which its concepts are used in research: of 1577 papers claiming to use CHAT in education research, only 21 out found to “actually use CHAT theoretical constructs.” In this research, the shift away from a direct application of CHAT concepts to the data was a response to a contradiction within my own activity: resolving the difficulty became a generative act, consistent with the theory, and led to a richer description of the classroom than would have otherwise been possible. The importance for research of this shift lies in the reflexive consistency I sought to establish, the connection of my work with the historical and philosophical background of the theory, and the dialectic relation that was sustained between theoretical constructs and the data. Through maintaining these connections and making them available to the reader, I have sought to achieve consistency and manage the potential complexities of applying the theory to the classroom. I contend that this application could be replicated in other classrooms, to shed further light on the experience and outcome of classroom learning.

10.2.1 Classroom values in locating mathematical action

The importance of production in the activity had been theoretically pre-empted by CHAT. However, through my use of grounded theory, I was able to substantiate the specific nature of pupils’ production in the mathematics classroom. The impasse in data analysis was resolved by using grounded theory to reveal what was relevant to the research participants (Charmaz, 1995). I was then able to return to my analytical framework (Table 2, p. 116) with the coding structure to make sense of the classroom as experienced by them. Extending the “constant comparison” method (Strauss and Corbin, 1990) to my codes and the constructs of CHAT enabled me to articulate the values and tensions in the activity. The grounded analysis revealed the co-constructed constraints upon work

produced and the intimate relation between production and participation. Exploring this relation uncovered the values at play, within the framework of norms. Values were identified in the processes and actions of pupils and teachers; the grounded approach enabled me to appreciate these from the pupils' point of view.

Mathematical action in the classroom was primarily valued through the production of structural connections in problem-solving, exchanged and reproduced as indication of an individual's capability. However, topic focus and rehearsal of problem forms promoted specific connections and results: an emphasis on fluency with specific methods thus directed pupils towards finding anticipated connections and producing anticipated responses. The classroom promoted the erasure of idiosyncrasies in pupils' work (to the extent that reproducing a standardised artefact was considered fitting action), satisfying teachers' expectations and adopting procedural methods without generating or pursuing one's own questions. The value thus ascribed to locating mathematical action lay in identifying the 'correct' tools for action in response to pre-formed problems, and recreating approved workings to achieve the expected answer. Correct tools were those which corresponded to the structure within the problem, and were considered appropriate for current use.

Pupils' responses to contextualisations of mathematical action suggested that the *myths of reference* and *participation* (Dowling, 1996) were sustained as a means of achieving the value of the mathematics qualification (Elliott et al., 1999). Whilst the pupils did not attribute value to the real-world scenarios used to illustrate mathematical structures, nor did they challenge them in the midst of activity. They were treated as devices with which to place mathematical action in the classroom, rather than as indications of where it might be found in the world. Through these mechanisms, the value of locating mathematical action related primarily to the well-achieving pupil, rather than to the use value of mathematical action in making connections in the world (see §9.1.1).

10.2.2 Curricular aims

By retaining connection with the philosophical basis of activity theory, my research has enabled me to offer insights into how the pupils were positioned in relation to the activity and the aims that became apparent in the “enacted” curriculum (Remillard, 2005). Drawing my attention to human relations with the *Gegenstand* and *Predmet* aspects of the object, in light of the Marxian categories of production and distribution, I was able to discern a relation between a pupil and his own mathematical capability. Through producing his capability as classroom authority (respecting the resistance of structures and instantiating the object of thought in a conventional manner) the pupil provides the material for his own distribution.

The authority displayed when participants located the uses of arithmetic and spatial relations suggested that they were becoming equipped to exercise capability with “functional numeracy” (CBI, 2008). However, in the classroom, this capability was devalued within the cycle of production and exchange, as continual judgements of the pupils’ progress were based upon the use of vertically mathematised tools. The meanings which inhered in these tools related more strongly to classroom production, exchange value and the co-production of the pupil, than did the use of numeracy.

Through gaining entry to the school the pupils had encountered the use of mathematics as a gatekeeper discipline, and experienced the resultant distribution of themselves and their peers. Entry to the school was celebrated and seen as beneficial: all pupils anticipated studying A-level courses and proceeding to university. These expectations reinforced the need for high achievement in mathematics at age 16. The IGCSE course they followed was designed to prepare for further study in mathematics although this was pursued by only two of the participants (see §10.7). Becoming complicit in (and trying to benefit from) the processes of distribution required the suppression of subjectivity, particularly for those who did not intend to continue with mathematics. The *personal meaning tension* could be overcome when pupils accepted the meaningfulness of mathematical action in terms of this distribution:

DVS: Do you feel that generally the problems you're working on have any meaning outside of maths lessons?

Jamie: Um well I'm not sure because like obviously if, if you go for a job or if you go to university, like if I go to Cambridge the first thing they look at will be language and maths. So um and other jobs if you've got a good mark in maths then it's better... If things like, if you don't do well in the exam. So obviously it does have that meaning.

(I026)

In this the pupils were competitors, trying to produce attainment and self-esteem (Lave and McDermott, 2002) in relation to others and the curriculum against which they were all assessed. These competitors were simultaneously produced by the school and consumed in its capacity as a business which attracted future trade through their attainment.

In relation to the notion of mathematical capability as transformability, I found the aims of the visible curriculum enacted through the focus placed upon produced work: pupils' and teachers' expectations of transformation were delineated by reproduction of the curriculum. The classroom used the appearance of connection with everyday experience as a motivating factor, but pupils did not anticipate making meaningful connections to scenarios in their daily lives in which they would undertake mathematical action. When using imagined 'real-world' contexts processes of horizontal mathematisation occurred in the service of classroom aims, but the results of classroom action could not meaningfully be translated back to the contexts. Pupils' classroom actions had had no identifiable transformative effect on the world. Transformation was emphasised only as a means of maximising the exchange value of their capability. Pupils had also had no transformative effect on the visible mathematics of the curriculum, influencing its delivery only indirectly through their collective achievement as judged by teachers. Likewise, pupils found the structures within problems were impervious to their observation, objections or alterations. Mathematical authority was developed through internalising the *Gegenstände* of the curriculum, whose resistance derived not only from internal logic, but also from the specifications of the curriculum. The pupils were produced as accruing a range of canonical knowledge and skills.

10.2.3 Connecting practices

This research did not explicitly investigate mathematical action outside the classroom. However, by using the analytical descriptors of everyday mathematical action, I was able to consider the classroom as the pupils' place of work. This revealed how some aspects of everyday practice were prefigured by the pupils' and teachers' intersubjective habits, and how the activity of the classroom became an organising principle in the pupils' actions.

Noss et al. (2000) determine different notions of "efficiency" in academic and workplace mathematics and cast light on the effect these notions have on mathematical reasoning. My findings would suggest that the priorities adopted by pupils in the research more closely resembled those of the workplace than academic mathematics. The difficulty for the pupils to see meaningful connection between their classroom action and the world outside suggested that structures were not internalised for their "generalisability and abstraction away from the workplace" (ibid, p. 32) of the classroom. Rather, the participants engaged with mathematical structures, producing work and new connections in order to accrue the value of compliant productive participation. The emergence of the *pragmatic reasoning schema* of consensus, the function of trust in the teacher and the teachers' use of quantity of work as a shorthand for achievement all derived from the specific nature of the pupils' workplace.

The use of contextualising scenarios and tasks represented the opportunity to give pupils experience with "boundary objects" (Kent et al., 2007), in finding underlying mathematical structures and using them as a prompt to mathematical action. However the response of pupils to these efforts suggested that the social meaningfulness of such scenarios was either absent or actively ignored. Unless the actions in dealing with scenarios corresponded to items of the visible mathematics curriculum, represented within the textbook, the pupils could not identify what they had learned, and thus could place little value on it in the activity. These observations could be seen in the light of the call to incorporate pupils' "knowledge, experience, histories, identities and images of themselves" (Baker, 2005) in classroom activity: without tasks that could appeal to all

pupils' interests and occupy identifiable places in the curriculum (overcoming the *orientation tension*) whilst preserving social meaningfulness, such efforts sustained the *bounded practice tension* in their treatment by both pupils and teachers.

The use of technologies pre-empted the everyday use identified by Noss (1998), reflecting the extent to which the mathematisation of social life resulted in aspects of personal demathematisation. Tools were associated with specific tasks and relied upon, unquestioningly, in the service of problem-solving. The embedded mathematisation was assumed by the curriculum and trusted by pupils. It was not made amenable to exploration, nor were pupils required to explore underlying structures. Engaging with embedded mathematisation in technologies was not valued within the activity.

Viewing the classroom as a workplace in which pupils' authority was constructed in relation to the visible mathematics of the curriculum revealed that the curriculum called upon understanding of the world in order to develop mathematical capability. This approach lies in direct contrast to the call from McDermott (2013) for mathematics to be seen as a resource for pupils' lives. Classroom activity accessed elements of everyday life to develop mathematical authority, without affording transformability of mathematical methods and structures, in response to these elements.

10.3 Observations on teaching practice

In working toward a substantive theory (Strauss and Corbin, 1994) of mathematics classroom activity the tensions that emerged did not submit to the ordinal classification proposed by Engeström (1987) (Table 1, p. 93). These tensions were best understood, not in terms of their relation to the nodes of the EMT, but in terms of the philosophical background of CHAT and the learning principles of Vygotskian theory. Using grounded theory enabled me to penetrate the formal surface of CHAT, and interpret the empirical data in light of that philosophical background. Identifying the tensions from that standpoint revealed the extent to which the pupils' learning could be said to be alienated (Lave and McDermott, 2002) or commodified (Warmington, 2007). In contrast to the research into "quiet disaffection" by Nardi and Steward (2003), I was able to explore how

the pupils themselves contributed to the alienated characteristics of classroom mathematical action. Whilst pupils' focus on the endpoint of their studies contributed to the alienated characteristics, it also enabled them to overcome disaffection. An experienced understanding of the gatekeeper qualities of mathematical achievement contributed to their motivation. Norwich (1999) contends that such "introjected" reasons, originating outside school, are a strong basis for compliant participation. Nardi and Steward include "a belief that school would not improve career prospect" as a contributory factor to disaffection; I suggest that a shared belief that school success was non-negotiable acted as a barrier to disaffection.

Lave and McDermott (2002) present a Marxian analysis of commodified learning, and describe a set of alienated problems in education with their apparent solutions. They argue that the 'solutions' which sustain only reinforce the problems, whilst revealing the underlying principles of the problems. In order to make sense of the teachers' role in the activity as described here, I adopted their description of problem/'solution'/principles (ibid, pp. 29-32). The language is adapted from Marx's essay *Estranged Labour* (1844):

The systematic result of the surplus value of educational achievement run amuck, competition for high grades results in a focus on efficient pupil participation. This 'remedy' fails to question the principle of individual success being defined by compliant participation in the ready-made structures of school and schooling, achieved through a suppression of subjectivity.

I refer to this situation as the *efficiency focus*.

Deriving the tensions in this thesis from a model of the *Individual* human psyche developing in a *Particular* form of social practice highlighted the lack of coherence between the *Universal* cultural products of learning in school and everyday or workplace needs. In this section, through considering the observed teaching practice and the role of teachers in sustaining efficient pupil participation, I address the production of pupils and mathematical action.

10.3.1 Values

Engeström argues that an expansive and historical approach to school learning would address the question of “why is this being taught in the first place?” (1996, p. 164). My research suggests that in the absence of a coherent presentation of the purposes of mathematical action, pupils intuited the answer to this question through the structure and values of activity. The development of pupils’ mathematical authority resulted in embedding values in action which were derived from compliant productive participation. These values largely related to the need to systematise and monitor pupils’ development through production and exchange of written artefacts. The use of pupils’ work in the classroom represented the totality of the activity’s production, being a representative of the individual subject, serving the collective activity and enabling the subject to stand apart from himself. This totality was made explicit when teachers used pupils’ work in expositions of methods. Thus the pupil was seen (and saw himself) through the written artefacts, and his value as a pupil was related to the value placed upon the work.

Teachers’ practice both presumed and produced the unification of pupils’ mathematical development and socialisation. This was achieved through relations, largely constituted by collective interaction, which valued the maintenance of roles in the community as a basis for the recognition of developing mathematical authority. The work undertaken by teachers to maintain order established pupils’ progression in a collective evaluative frame: undertaking the same tasks at the same time as his peers, each pupil’s transformation was constrained along the same route through a collective ZPD, and would be assessed in comparison with others’. Efforts to harness pupils’ motivations, such as grade achievement and setting position, entailed pupils finding value in comparison with others: “academic success is always achieved over others, and credentials are less about what they allow their owners to do than their non-owners not to do” (Lave and McDermott, 2002, p. 25).

Teachers tried to convey their own notions of purpose through their presentation of the content of the syllabus. However, these had little value or transformative effect within the classroom and hence were easy to ignore. Pupils’ rationales for participation (Mellin-

Olsen, 1981; Goodchild, 2001) acted as filters for these explanations. The primary mathematical values which sustained were efficiency of problem-solving, fluency and recall of results, adopting illustrated methods and practising them until they shifted in quality from actions to operations. Other qualities of mathematical action (such as deduction, justification, rigour, reflection) were rarely articulated explicitly in the cycle of production and exchange. The successful production of fitting workings enabled teachers and pupils to treat these qualities as embedded within methods, even though their procedural nature undermined them. Reproducing these methods in social practice constituted development of the individual whilst relation to the structures of mathematics could remain unexamined.

10.3.2 Institutional aims

Teachers bore responsibility for the attainment of groups of pupils, who were to be treated with uniform expectations, and should contribute to the maintenance of the stable activity. Their activity was informed by institutional pressures to maintain a high level of pupil attainment. This responsibility prohibited development of the activity and inhibited engagement with the generative potential of the tensions. In their position they had no option other than to contribute to the production and distribution of pupils within the school, anticipating their distribution in later stages of education and work. Consequently, the teachers' activity was tensional. Whilst submitting to institutional expectations and the need for efficiency, they attempted to convey their own sense of the values of the mathematical action and overcome the efficiency focus. However, the efficiency focus informed all aspects of the teachers' and pupils' activity. Whilst teachers were seen as arbiters of need and appropriate mathematical action in the classroom, pupils' curricular awareness meant there was little capacity for these broader aims to be explicitly and sustainably valued.

10.3.3 Everyday needs

My decision to use the EMT as a vocabulary of description, rather than a structuring device, enabled incorporating multiple voices and inherent difference in the features of the activity. Grounded theory was used not to reveal what was "there" (Charmaz, 1995),

but to reveal the meanings ascribed to the action, in accord with CHAT's shift away from everyday positivist assumptions. Working from the paradigm also enabled me to stand apart from the mathematics classroom and the pupils therein, to give a critical account of the activity. This approach revealed the inhibited subject-object dialectic, but also enabled me to identify why it should be important. Everyday problem-solving involves the dialectic interplay of scenarios, goals, means and meanings (Noss et al., 2000; FitzSimons, 2005), often in explicit negotiation. The pupils in this research had little experience of this negotiation within the activity. Their capacity for transformation was depended upon, but not related to the purpose of the activity. They rarely encountered situations in which the social meaning of their actions was unclear, the tools to be used were not signalled by the teacher or the task itself, or in which they had to devise the nature of the problems that had to be solved. Without practice of these negotiations, it would be difficult to sustain the claim that the pupils were well prepared for the use of mathematics in their daily lives. The value placed upon participation encouraged the subjects to engage in transformative activity, but reflection upon their capacity for this was not a prominent feature of the activity; transformation took place in terms of the visible mathematics of the curriculum.

Describing the relationship between capability and authority cast light on the means by which pupils were constructed as "knowing." Knowing could be sustained only with the tools and within the expectations and social relations of the classroom. In order for the pupils to be able make claims to knowledge in successive and alternative practices, they would need to adapt to new sets of expectations, relations and tools. I experienced this adaptation in my development as a teacher-researcher. However, in spite of the construction of mathematical results as 'facts' to be used, and the shift of methods from actions to operations, the nature of classroom activity endured. The ascent from abstract to concrete placed more tools at the pupils' disposal, but these tools became increasingly situated within the activity. The construction of pupils as "knowing" might thus be similarly situated.

10.4 Implications for theory-building

By working with a grounded approach, I was “building the research as it ensue[d]” (Charmaz, 1995, p. 48) through the refinement of my research questions. Brown and Dowling (1998) argue that the “systematic and explicit organising of the theoretical space” determines the theoretical quality of research. My shift to grounded theory became such a systematic organisation of the abstract concepts of CHAT; an instance of ascending from the abstract to the concrete. This resulted in a dialectical relation between theory, methodology and data: as the object of my research became more concrete through the analysis, so the research questions were refined. The theoretical and empirical fields were articulated jointly.

3rd-generation activity theory represents two interacting activity systems connecting in the focus upon a shared object. These systems can comprise different communities (Engeström, 2001; Flo and Ludvigsen, 2002; Roth et al., 2002; Venkatakrisnan, 2004). However, in my research, the communities were construed as the same group of people with different subjects at their centres. In this description, the subjects of one system inhabited the community of the other, and thus presented different perspectives on the activity. The recognition and functioning of tools, rules and division of labour thus depended upon the point of view of the subject, and the “difference-in-itself” (Roth, 2010) of features of the classroom could be preserved in analysis. This contributed to my rejection of the “buckets” (Barab et al., 2004) approach whilst also offering a diachronic perspective on the production and distribution of pupils within the school. Understanding features of the classroom as social phenomena, open to varying interpretation and use, created the possibility of describing change and historicity in the activity structure.

However, the anticipated and actual uses of pupils’ produced work indicated that the trajectory of change was minimal, constituting a refinement of focus on the visible mathematics of the curriculum, and a reinforcement of the co-constituted roles of teacher and pupil. Formative dialectic relations between the objects and their activities could not be judged to have occurred. The identification of a set of interrelated tensions in the

activity indicated the potential for “expansive learning” (Engeström, 1996), but also reflected the constraining effect of accountability for examination results.

A further aspect in which the description of the activity in this research might contribute to theory-building related to identifying the object as a property of the pupil subject. Engeström (1998) sets up an analysis of “traditional” teaching and schoolgoing with the object as school texts that are transformed through reproduction. However, I articulated the object as “mathematical capability” in relation to claimed purposes and to understand how this object might direct the unfolding of the activity. Placing the object within the subject and their productive participation enabled me to describe that object in terms of the meanings and values that inhered and could potentially transcend classroom boundaries. However, I feel my conception of the object, as worked out in §5.2.1, warrants further exploration in relation to classroom activity. The dialectical relation between pupils’ motivation and the transformation of their capability was not explored here: investigation into the motivational effects of relative success could supplement understanding of knowledge, practice and identity (Boaler, 2002b).

My use of activity theory also differed from that in other studies in that I aimed to represent the activity of the mathematics classroom across an extended period of pupils’ experience, rather than focus on specific examples of classroom action (e.g. Wells, 2002a) or medium-term projects (e.g. Blin, 2004; e.g. Roth and Lee, 2007). My intention to describe those values which sustained resulted in uncovering the resilience of community structure in spite of pupils’ development.

10.5 Limitations of this research

Adopting a “radically local” (Engeström, 1999c) methodology has positioned me to reach conclusions in relation only to those pupils in the school who participated in the research. I make no claim to simple generalizability of my findings, and am aware that the small size of the pupil participant group might have been the source of unintended bias. The centrality of pupil compliance in the construction of the object could be attributed to the nature of the fee-paying school system, in which the commitment made by families

facilitates the “coercion” (Boylan, 2005) of pupils. However, my aim has been to present a picture that will resonate with teaching practice across a system (and within a culture) in which there is increasing pressure towards high examination achievement.

As a full-time teacher in the school there were severe practical limitations upon my capacity to schedule interviews and lesson observations, and the ongoing analysis of data in fulfilling a grounded methodology was often held up by a lack of available time. The departure of two participants from the school presented a further restriction of my sources. Some of the implications of this have been discussed in chapter 6. The scale of the research had been established with these constraints in mind; nevertheless throughout the analysis I strove to establish robust connections between data, codes and claims, as I was sensitive to possible inadequacies in the data collected.

The ethnic make-up of the school is diverse, as it sits in one of the most ethnically mixed boroughs in the country, with a large Asian demographic. Seven of the original pupils who volunteered to take part in the research were of white Anglo-Saxon heritage, one of Middle-Eastern. However, I did not draw any conclusions about whether race influenced the pupils’ desire to participate and cannot trace any connection with the findings of this research. Similarly, having conducted this research with boys, I make no claims relating to gender.

Conducting a macro-analysis of the classroom required simplification of many issues worthy of exploration. I am conscious that this research has touched upon many issues (such as ability, mathematical representation, ICT use, teacher development) but not explored them in any depth.

10.6 Review and implications

The process of research has been challenging, rewarding, enlightening and frustrating in equal measure. At the close of this thesis it appears that far more has resulted than I set out to achieve, whilst much work remains for me in addressing my aims. In this section I consider the implications for further research and for teachers’ practice in mathematics education.

10.6.1 For further research

Through the process of this research, I developed two frameworks of constructs which are worthy of further application and investigation of other disciplines and of school mathematics with other age groups.

As stated above, the hierarchical relationship between the norms was drawn from the importance given to them by the pupils in this research. Enquiry can now be made into the power of this framework to describe teaching and classroom practices in other disciplines. During the course of the research, differing models of the relationship between the norms were considered; stepping aside from the pupils' conceptions, should they be considered, nested, disjoint, overlapping or wholly or partially coincident? Investigating teachers' conceptions and instantiations of the relations between the norms could shed more light on the structure and outcomes of classroom activity.

The framework of tensions identified here did not derive exclusively from the need to develop mathematical logic in socially ordered classrooms, or from the relation between written representations and mathematical structures. Exploring these tensions in other disciplines and other modes of learning mathematics could offer means of how pupils produce themselves in relation to other disciplines, and develop personal meaning in disciplines which might offer a more immediate appreciation of their use value. As suggested above, the means by which resolutions are effected reveal the values that are sustained in a classroom. Identifying and exploring these tensions in other classrooms could illuminate how disciplines nurture pupil engagement whilst valuing transformability as a feature of the emergent objects.

10.6.2 Implications for teachers' practice

I have found that awareness of the tensions has enabled a more acute articulation of the purposes of mathematical study in classroom conversation, and in managing the pressures of institution and curriculum. In relating my understanding to the development of the object that is mathematical authority, I return to the questions set out by McDermott and Webber (1998) (presented in §2.4) specifically: *By what order of persons in relation to*

what organisation of things are moments put aside as mathematical or scientific?

Exploring the co-constitution of mathematical activity in the classroom has shed some light upon this question. Through exploring the attribution of value in pupils' development and relating their capability to social structures and available tools I have begun to articulate the legacy of the mathematics classroom in "putting moments aside as mathematical". I offer the implications of this study as a set of interrelated propositions referring to the current picture of the curriculum and teaching in the UK as laid out in chapters 2 and 3. Each of these is worthy of further investigation:

There is a need for socially meaningful and transformable mathematical activity throughout pupils' school experience.

From the beginning of my data collection period, the pupils had indicated that their motivation for participating in classroom activity stemmed from the exchange value of achievement and the value of participation itself. The early establishment of this rationale might have relevance to the observation that years 7 and 8 are a crucial period in the determination of attitudes to mathematics (Aiken, 1970). Within the context of clearly identified opportunities for placing mathematical action, the exchange value of productive participation had been intuited by the pupils from an early age. The research by Goodchild (2001) which identified the P-rationale had been conducted with year 10 pupils; my research suggests this rationale might develop well before then.

Individual 'success' in school mathematics represents the extent to which a pupil's aims became aligned with those of their school.

In undertaking this research in a selective school, I worked with a particular group of pupils who enabled me to offer an understanding of the constituents and implications of school success. They all attained A or B grades on the IGCSE. In comparison with the GCSE, this places them in roughly the top third of measured attainment and so might be considered 'successful' in school mathematics. However, the pupils in this research were

not distinguished as high achievers within the school¹⁶ and were distributed throughout the sets (Appendix 1). Having earned their place in the school, they encountered expectations of hard work and success was achieved through suppression of their own interests. The reward for their compliant participation was extensive support from the teachers. For some of the pupils, the exchange value of the mathematics qualification was hard won, when they found it difficult to submit their subjectivity to the expectations of the classroom.

Changes to curriculum content alone will not address shortcomings in the mathematical skills required of people in working and everyday life.

I found that the meaningful qualities of pupils' activity derived from anticipated examination achievement, in the absence of any tangible change in productive relations. Greeno (1992) suggests that connection could be made between mathematical thinking and the concepts and methods of the classroom by structuring classroom activities as mathematical discourses in which pupils can learn to participate. He notes that pupils "are not expected to contribute substantively to the conversation" (p. 41) and suggests that allowing pupils to develop stronger capabilities in participating in questioning discourse would lead to stronger shared understanding of "notations, phenomena, concepts and principles" (p. 60). Whilst my research would support Boaler's (1998) assertion that participation in different classroom practices leads to a difference in kind, rather than degree, of mathematical understanding, I would also contend that opening classroom activity to wider meanings and patterns of participation would be beneficial. Research such as (Hoyles et al., 1999; Dawes, 2007) illustrates the sophisticated use in work of the mathematical action that was required for a pass grade in the GCSE. Forms of classroom activity which could reflect (in tasks and social relations) pupils' capacity to make meaning in the world might be the necessary backdrop to encouraging participation in school

¹⁶ In the IGCSE, A*-B grades account for nearly 90% of the cohort. For that year group's performance statistics, see: <http://www.edexcel.com/iwantto/Documents/June-2011-Final-International-GCSE-Specifications-4AC0-4UR0-UK-only.pdf>

mathematics and increasing the extent to which it is valued outside the classroom, contrary to the conceit that pupils should learn “more, harder” mathematics.

Extending compulsory mathematics education to age 18 should be accompanied by forms of learning which look beyond examination practice.

This research suggests that there has been an historical inversion in the roles of schooling and examinations since the introduction of the School Certificate in 1918. This was a compulsory qualification for 16-year old school leavers and was delineated such that “the examination should follow the curriculum and not determine it” (Norwood, 1943). As has been shown here, the activity of the classroom is multiply determined by the expectation of examinations, which results in the isolation of school mathematical action from everyday and workplace needs. If the extension of compulsory mathematics education to age 18 is to have a beneficial effect for learners (Hodgen and Marks, 2013), that determination should be disrupted.

10.7 Concluding remarks

The picture presented of classroom mathematical activity in this thesis could be considered a pessimistic one, in which teachers largely dole out curriculum specifications to cynical recipients who are single-mindedly focusing on the tokens of success, rather than the substance of development. In this, it reflects my start point as a teacher-researcher who had engaged deeply with school mathematics, and identified the object of activity as a characteristic of the pupil. This thesis presents those aspects of classroom activity which challenged many assumptions I had carried. It is important to note that the pupils in the research were not overwhelmingly dissatisfied with school mathematics, but nor were they deeply interested (c.f. Nardi and Steward, 2003). For them it was largely a means to an end, with occasional points of interest.

Both the pupils’ development and my own continued after the data collection in this study. However, only Jake and John went on to study mathematics at AS-level at the school. (They all completed AS and A2 courses in other subjects.) Jake did not continue to the A2 course, but John did and achieved an A* grade. Throughout his studies he was

noted for scrupulous, methodical and repeated reproduction of examples of methods in his work. Apparently sustaining the approach expressed in his interviews, he did not query beyond the specifications of methods and tasks set out for him. The approach to learning mathematics he had established in earlier years served him well at A-level. The object of his activity might be better characterised as the texts which were consumed and reproduced (Engeström, 1998; Wells, 2002a). It is hard to deny the conclusion that success in school mathematics relates in large part to the reproduction of texts in an otherwise meaningless system of exchange. The value earned by John enabled him to take up a place to study Engineering at a high-ranking university. However, the appropriateness of his approach to studying 'academic' mathematics does not translate backwards to the development of mathematical capability for the workplace.

In the same period, a new Head arrived in the school and, with the aim of "increasing academic standards", introduced a school-wide marking policy, under which nearly all pupils' work receives scores or grades, with comments where necessary. In years 10 to 13, focus on past examination papers is encouraged and anticipated examination grades are issued half-termly, based upon each pupil's recent work. Tightly defined expectations, explicit targets and close attention to examination expectations prevails. My research suggests that this extensive summative assessment will further increase pupils' attention to the reproduction of correct and conventional mathematical texts and dependence upon the role of the teacher, reinforcing the stasis of the activity. It would appear unlikely that this is the only school in which such moves are taking place. I am drawn to the conclusion that, in the face of prevailing political and cultural trends it will be increasingly difficult to maintaining the notion of the object of classroom activity as transformable personal capability.

10.7.1 Developing question narrative/Dual identity narrative

The tensions between my two identities as described above could be recounted in CHAT terms: the contradiction arose through my own subject-object relation being developed within two different communities simultaneously. The resolution to the tension lay in finding the common ground shared by those communities. This expansion of my own

practice has taken place on these two fronts: as a teacher I have developed a sense of the power and purpose of theory in explaining the production of pupils and their abilities; as a researcher I have directed readings and insights from these towards the impact they might have on my practice.

My exploration of Vygotskian psychology and Marxian philosophy revealed to me the potential for this research to initiate practical-critical-revolutionary activity (Marx, 1973, p. 121). As described by Newman and Holzman (1993) revolutionary activity takes place in the day-to-day action of humans, but is distinguished from it by a self-conscious engagement with the processes of history. Revolutionary activity places itself in a broader context than the society of which it forms a part, and aims to influence the developing totality of that society. Revolutionary activity not only entails seeing new things, but changes what seeing is (ibid, p. 28). My intention to develop a new understanding of mathematics classroom practice has already involved looking at the classroom anew. My reflection upon the aims of mathematics teaching and its place in our culture has been only the beginning of this process. The production of this thesis can be seen as a *Tätigkeit*, but it has also constituted a substantial movement in my ongoing professional and intellectual development, and provides the basis for my further exploration:

How can a teacher articulate the transformative benefits of mathematical action for pupils, in the face of the efficiency focus and the intransigence of the classroom as a cultural form?

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Appendices

1 Appendix 1: Data Collection

1.1 Allocations of teachers to pupils over the course of the data collection, and eventual achievement


Participant	Teacher (set/no.of sets)				IGCSE Grade	AS Grade	A2 Grade
	Year 7	Year 8	Year 9	Year 10			
Aaron	W	M	K (3/4)	W (4/4)	A	?	?
Alex	W	M	W (2/4)	C (2/4)	B	-	-
Jamie	W	W	J (4/4)	P (5/4)	B	-	-
Jake	W	W	P (1/4)	C (2/4)	A	C	-
John	W	M	P (1/4)	C (2/4)	A	A	A*
Thomas	W	M	W (2/4)	W (4/4)	B	-	-
MC	W	M					
ZR	W	M					

Table 11: Allocation of teachers to pupils, and setting

Aaron transferred to another school for his post-16 education: I was unable to ascertain whether he studied mathematics beyond 16.

1.2 Example observation notes

Observation R007, January, year 8.

Observation Number: R007	Date: 17-1-08	Period: 4	Room: L3
Teacher: M	Pupil(s): John/Alex		
Room notes:			
<p>Usual classroom for this yr.</p>  <p>John noted by ref (accidentally?)</p>			
Location, age, outlook, storage/equipment, exits, specialization, wear and tear, anything unexpected, decoration, furniture, layout, acoustics, environment, lighting			
Class notes:			
<p>SP - still taught as perm. V. dispersed around of the back - Teacher not making much fuss</p>			
No., age, abilities, class, arrival/groupings, spacing, movement, characters, (non) hand-raisers, territory, seating, equipped, division of labour, roles, communication,			

Observation Number: _____ Date: _____ Period: _____ Room: _____
 Teacher: _____ Pupil(s): _____
 Topic notes: _____

Time	Teacher	Pupils	Questions
00.00	Handing sheets out "Don't turn over!"	Slow Ascare	
4			
05.00	Not "100 club" / exam Not test. / selected 6 5 stays with pupils Time distance speed formula? Testing everything we've done. Time + answer.	Annanys keeps belted - a hedge apology, Dressed pupils. The triangle → Semtex stand? Under? → Does it matter outside the classroom? Rules + expectations of teacher laid out.	Authenticity Legitimacy Legitimacy?

Observation Number:		Date:	Period:	Room:
Teacher:		Pupil(s):		
Time	Teacher	Pupils	Questions	
10.00	<p>Exemplifying to answer Q1</p> <p>IBB not noticed by boys?</p> <p>Head nod to mark on error.</p> <p>Not bothered? \leftarrow</p>	<p>Solve working away on sheet, making answers in immediate space.</p> <p>Take for $s = P/T$ problem.</p> <p>Working out not written down.</p> <p>Plenty of talkers</p>		
15.00	<p>What do you think "average speed" covers all (units)?</p> <p>Recall of Δs &</p> <p>LM returns to John's note</p> <p>Working formula for AR</p>	<p>Corrected as previously?</p> <p>Annotating graph</p> <p>Remember</p>		
	<p>'Write answer - wrong units'</p> <p>Clarity in speech admitted</p>		<p>expected with clarity in working</p>	

Time	Teacher	Pupils	Questions
20.00	<p>Discussion reaped with explanation of <u>deg.</u></p> <p><u>2 mins</u>: Eyes up! Explains that <u>deg's</u> were <u>deliberate</u></p>	<p>Listening to M</p> <p>Quietly completing worksheet.</p>	<p>Does this average connect with the mem?</p> <p>[Hand up as a communication device]</p> <p>Lesson about text use.</p>
25.00	<p>Drawing does together.</p> <p>Rejoice "Marko"</p> <p>Qc)</p>	<p>Ticked "10 mins"</p> <p>Ticked "10 mins" before EMH begins - Hemster!</p> <p>Crosses it.</p> <p>Waiting patiently</p> <p>little bit of playtime?</p>	<p>"You would get marks"</p> <p><u>agreement</u> / <u>authorities</u></p>

Observation Number:		Date:	Period:	Room:
Teacher:		Pupil(s):		
Time	Teacher	Pupils	Questions	
20.00	<p>Discussion recap'd with explanation of diag.</p> <p>2 mins: <u>5yrs up</u></p> <p>Explains that 40's issue was deliberate</p>	<p>Listening to M1</p> <p>Q Cate + Fomuldee</p> <p>Qistles completes unhooked.</p>	<p>Does this average' unhook with the men?</p> <p>(Hand up as a unhooking device)</p> <p>Lesson about text use.</p>	
25.00	<p>Drawing done together.</p> <p>Rep to "Merho"</p> <p>Q ()</p>	<p>Q Tides</p> <p>Ticked "10 mins" before unhooking - Nemo!</p> <p>Crosses it. —</p> <p>Waiting patiently</p> <p>little hnd gldaydren?</p>	<p>"You would get merho agreement / authorizing"</p>	

1.3 Interview Questions

Individual interviews, June 2008

Why do you do the tasks you do in lessons?

Why do you have textbooks?

Why do textbooks have exercises in them?

Why are exercises set out the way they are?

Why do teachers pick certain parts for you to do?

Does it matter if you don't get it all done?

Why does the teacher set you these specific tasks?

Why are you studying that particular content?

Why do you have to do those specific questions?

Has the teacher explained why you're doing what you are?

Do you believe them?

Why do you have to write and draw diagrams?

Why do you set things out in certain ways?

Do all pupils have to do it the same way? (What is constant?)

What reasons have you been given by teachers?

Why do you co-operate with the teacher?

What forms does co-operation take?

Why do pupils sometimes not cooperate with the teacher?

In what ways do pupils not co-operate?

What does this achieve?

How do you work with peers?

What forms does working with peers take?

Do you work with peers outside the classroom?

Why do teachers explain and pupils listen?

When do pupils explain and teachers listen?

Why might pupils mark work? On what occasions?

1.4 Final interview schedule

Can you describe what mathematical activity in class involves?

What are you asked to do?

Does it always revolve around problems? Are there any common features to the problems? Do these problems mean anything outside of maths lessons?

What do solutions look like?

What is the teacher for? Could you learn mathematics without them? Why?

How do conversations about mathematics with friends in class differ from conversations with teachers?

Can you take a look at this problem (below)?

Please talk through what you would do to solve it.

Which bits would you say are the mathematical bits?

What is the calculator used for in lessons?

Is using the calculator a mathematical thing to do? e.g. finding an angle in a triangle, multiplying 10×12 or evaluating a formulas by substitution.

What is your motivation for working in mathematics lessons?

What are you trying to achieve?

Is there a reason for presenting tasks in exercises? As groups of similar questions?

Would you say that doing mathematics is difficult?

What makes it so?

When is doing mathematics easy?

Do you think that other people feel this way?

You've talked in previous interviews about equations having meanings, such as telling about other universes. Can you explain what you mean?

Do you ever do things mathematically outside class? Can you think of any instances when you have done some mathematics?

What did you need to do?

What was the solution?

Is there any use for mathematical activity outside the classroom?

Can you give a general description of what doing mathematics (outside class) involves?

Could you say what mathematics is for or about?

Do you feel you know how mathematics will be useful to you when you have left school?

Please take a look at these tasks: is there anything mathematical in doing them?

Please talk through what you would do to solve them.

Which of the actions you've just described would you consider mathematical?

How could you do them in a mathematical way?

If you have an idea as to what career you may go into, do you see mathematical activity happening in that? Can you explain?

Do you think what you do in class will help with that?

A ladder of length 5m is leaning against a wall.
The bottom of the ladder is 2m away from the wall.
How high is the top of the ladder?

Tasks

Packing a suitcase

Baking a cake

Playing chess

Reading a book

Solving the equation $x^2 = 50$

Solving logic puzzles.

Sharing a pizza with people

Deciding how much paint to buy to paint a room

Checking a bank statement is correct

Buying something in a shop

Agreeing a date and time to meet someone

Playing football

Driving a car

2 Appendix 2: Analysis

2.1 Transcription of interviews – example

Taken from I005, with Aaron, Alex, Jake and Thomas.

104 DVS: I'll just give you a moment to think about that,
105 then we can start to share ideas
106 ok so let's see
107 JS can you start us off with an idea
108 what are the things you wrote down?
109 Aaron: well we come into the room
110 well before that we line up outside the door
111 DVS: ok
112 Aaron: to ah to ah we wait until the teacher gets there
113 and then we go in to the room
114 and then we go into our assigned places
115 which were given at the start of the term
116 and then the teacher talks about what the lesson is
117 a- what we're going to do in the lesson
118 that really is dependent on what subject
119 but if the subject–
120 for example the history teacher
121 gives us tells us what to do
122 and writes on the board what we're going to do
123 and how we're going to do it
124 etc etc a
125 and we just um we have to do it
126 and um they go back ()
127 for example the geography teacher explains as the class goes along
128 and uh then when the lesson's over,
129 well after that we're asked to get our homework-
130 if it's a homework day then we're asked to get our homework diaries out,
131 and write in the homework
132 which is always written on the board
133 then we leave, you're dismissed
134 and they usually go about it in a sort of
135 that block are dismissed
136 sort of you you you are dismissed, then you you you are dismissed
137 and then that's about it.
138 DVS: ok
139 that's quite a list does
140 anybody else have any other

141 yeh Thomas
 142 Thomas: well like
 143 every teacher has their own way of doing stuff
 144 like in maths last year W
 145 only for about two minutes explained something
 146 how to do a certain topic
 147 and then for the rest of the lesson
 148 made us do exercises for the rest of the lesson
 149 which nobody really liked
 150 but maths is a lot better now
 151 but like active lessons like PE and drama,
 152 games and drama
 153 we get dressed and get changed and stuff
 154 but what we've got to do like
 155 after the lesson's ended
 156 when we go back to our normal academic subjects
 157 we normally do an exercise or something to like calm ourselves down like
 158 so we don't...
 159 DVS: so you mean if
 160 in what you call an active lesson
 161 Thomas: yeh, no in drama
 162 DVS you go to the lesson following that is an ordinary lesson,
 163 in drama they do something to calm you down
 164 Thomas: yeh
 165 Aaron: yeh our drama teacher gives us sort of like warming up
 166 what's that word
 167 Alex: exercises
 168 Aaron: exercises
 169 like walk around the room in this sort of fashion
 170 Thomas: that's the last lesson of the day
 171 but um yeh um
 172 in English reading it gets quite boring
 173 at the end X to wake us up
 174 um makes us talk about the book for like a minute
 175 Aaron, John, Alex: yeh
 176 Thomas: without hesitating, deviation, or
 177 DVS: ah, ok
 178 Thomas: repetition
 179 Aaron: () repeat it or like stops or pauses
 180 then like you're out
 181 DVS: ok Jake do you have any more ideas
 182 Jake: ah yeh, um most of the time when we go in to a lesson
 183 people muck about

184 Thomas: yeh yeh, two people from our class ()
 185 Jake: yeh I can think of one or two um
 186 like history and RE all our teachers
 187 our teacher just gives us exercises
 188 so we don't really like that either
 189 'cause I don't find that we learn much
 190 except writing
 191 and in maths um W explains the subject on the board
 192 and gives like loads of examples
 193 so then we do exercises which is good
 194 DVS: oh right yep
 195 Alex: M makes us like copy loads of things down
 196 when M does examples on the board
 197 Thomas: exactly
 198 Aaron: < makes us write lots and lots of things
 199 but says like 'don't write it'
 200 Alex: we listen
 201 and then M says like "now take it down"
 202 Thomas: so we can take it in
 203 instead of just like copying down what M writes
 204 so we can actually take it in
 205 DVS: oh is that the explanation that M gives,
 206 Thomas / Alex: yeh
 207 DVS: so you
 208 am I getting this right
 209 you follow it through whilst M's talking about it
 210 Thomas / Alex: yeh
 211 DVS: and then you write it down
 212 Alex: and then we write
 213 Thomas: and once we're finished or we need help
 214 we just go up and start a big queue
 215 Alex: M gets really cross if people start writing down while she's explaining it
 216 DVS: ok
 217 Thomas: yeh M gets really cross if people don't ask for help if they're stuck
 218 DVS: right
 219 Aaron: M also gets cross if we come in late
 220 Thomas: M gets cross all the time
 221 #general giggle#
 222 DVS: right ok
 223 one of the things I ah ()
 224 you talked about things you do,
 225 what about what you use in class #hands raised#
 226 ah go on AR you haven't said anything yet

227 John: well, ah, I need to think about this one
 228 BT: we use pens
 229 DVS: even if it seems really obvious it's worth mentioning
 230 John: ok, well actually,
 231 in maths um EMH prefers us to use pen, not pencil,
 232 DVS: ok
 233 John: last year he preferred us to do it in pencil
 234 because you can rub it out um
 235 Aaron: she likes us to do it in pen
 236 because that's how it's done in the exam
 237 DVS: ok, right, so we had pen
 238 is there anything else that you use in all your lessons
 239 or that you're expecting to use in your lessons?
 240 #hands raised#
 241 Ok let's work our way down the table
 242 what were you going to say AR
 243 John: um we always have like a textbook and exercise book.
 244 DVS: ok, Jake?
 245 Jake: um for maths we use protractors rulers and compasses
 246 most of the time
 247 and we use pens and pencils
 248 DVS: ok, Thomas?
 249 Thomas: and calculators um like
 250 for games we use our games kit and trainers
 251 Aaron: sometimes in games say like timing you have-
 252 we can bring in a stopwatch
 253 or use a stopwatch
 254 Thomas: yeh
 255 Aaron: that happened last year
 256 DVS: ok,
 257 Alex what were you going to say
 258 Alex: no, I was going to say calculators
 259 DVS: you were going to say calculators
 260 what um, how do you um get around the situation
 261 how do you deal with the lesson
 262 if you don't have the right things with you?
 263 Aaron: it's very easy to –
 264 you either borrow something-
 265 Jake: you either borrow something
 266 or you have to do it at home
 267 Thomas: yeh you have to a- ask someone
 268 and then a-ask the teacher
 269 and if M doesn't have any spares just like borrow some

270 but you know you're going to get a big mouthful in maths if you don't have
 271 the right stuff
 272 John: yeh
 273 Aaron: and M once said uh, said,
 274 told us to write in our homework diaries
 275 bring a calculator for today
 276 Thomas: yeh I brought the wrong bag
 277 'cause my other bag had it in it
 278 no but I found out that all along my calculator was actually in the bag
 279 DVS: ah, right
 280 Alex: well say if you like-
 281 well say if you like forget your book
 282 they give you a sheet of paper
 283 and you have to copy it out at home if you haven't brought your books with
 284 you
 285 Thomas: yeh we're not allowed to stick it in we're only allowed to-
 286 Alex: yeh you're not allowed to stick it in
 287 Jake: I stick it in
 288 DVS: what would you say then, was
 289 what would you say is um
 290 of a pupil in general
 291 what is expected of a pupil if they go into a lesson?
 292 Thomas: to do their work to ()
 293 Jake: um to do what teacher says
 294 most of the time it's to be quiet and do your work
 295 and not to distract anybody else
 296 #hands being raised whilst BT speaks#
 297 DVS: ok, what were you going to say AR?
 298 John: always to come quickly
 299 especially for maths and English
 300 Aaron: yeh we can't be late
 301 Thomas: yeh maths

2.2 Open Coding

2.2.1 Example of line-by-line open coding; extract from I004, with Jamie and MC.

Used the methods of the teacher	
Not understanding	
just following instructions	
Not returning to work.	
Learning ICT skills	
Using maths websites	
"specific programs"	
Marking by school year	Using BBC Bitesize
	early on setting.
Teacher setting single books	
Introduces fun and learning	
Referring to specific website	
Learning skills	
Doing 'clays': puzzles	
Improves general knowledge.	
Doing puzzles	
Questions	
Achieving success in virtual games	
Helping general knowledge	
Learning ICT skills	
Doing SATs	
Marking term by school year	

Which was the way the teacher was teaching them but they still didn't understand and so the kids they just did and then didn't go back to it again.

DVS:

ok. Right, so did your, in your school, did your class go to this computer lab?

Jamie:

yeh, yeh, sometimes

I did a lot on some maths websites

DVS: right, did you have any specific programs that you used?

Jamie:

= umm.. maths=

MC:

=we did, in year five= we went on GCSE bitesize, that's all we were going on

DVS:

ok

that's all we had to do the teacher just sat-

MC:

I had um, what I did

although it's a bit of fun it can still help your learning,

Jamie:

it's just called maths fun and games,

www-dot-mathsfunand-games-dot-com,

and just, just doing stuff like Sudoku

and making just general knowledge better

DVS:

=m-hm=

=and= just doing little puzzles to make ()

Jamie:

and you have to do another question

and score another try and stuff and

it just helped our general knowledge really.

DVS:

right, ok.

So you had a computer lab as well in your school

MC:

yeh, we did, we had two

DVS:

two? Right, ok. Right now, what was I thinking...

ah, you mentioned SATs did you do SATs in your school?

MC:

yeh in year six

2.2.2 Open coding becoming focused

I001

Progressing through school	
Studying for	SF
Teacher setting expectation	Teacher leading DP
Development of mathematics	
Knowing the mathematics	KM
Being examined	SF
Teacher setting tasks	TDP
Subject-specific arrangements	
Specialization in mathematics	
Complying with norms	TR WN
Teacher explaining mathematics	TR (contrasted with teacher expectations)
Marking work	
Equipping for task	UE
Writing	W
Doing arithmetic	DA FT
Filling out	W
Making judgements of the teacher	MS
Teacher responding to pupil	TR
Teacher knowing mathematics	Not a
Using IWB	(W) UE
Teacher preparing teaching materials	TDP
Using PCs and software	UE
Being given tasks	TDP
Being rewarded	TDP TR
Teacher doing examples and writing on the board	TDP, W WN
Teacher owning the classroom	TDP
Working on problems	WN
Finding tasks difficult	FE
Group experiencing problems	SE
Working from books	WN
Accessing PCs and internet	UE
Moving to designated computer space	UE
Working in groups	WN
'Doing' topics	FT
Having basic equipment	UE
Changing schools	
Using show-me boards	WN (marking patterns?) UE
Writing problems and notes	
Distinguishing between notes and work	WN
Writing notes for self	DM W
Setting one's own mathematical methods	APK
Using writing to work things out	APK
Erasing writing	W
Replacing lost equipment	EN
Distinguishing complex problems	
Enjoying activity	
Subverting shared norms	JRN
Being well-behaved	
Distinguishing types of behaviour	
Writing down teacher's words	DM

Recognising Change?

1001, linguistic → Focused notes.

Finishing work	FA WN	
Teacher collecting and reviewing work	FA /TR	
Achieving consensus	SE	
Teacher responding to achievement	TR	
Setting pupils		
Being in a high or low set		
Showing off	AS	
Pupil self-definition	MS	
Practising topics	FT	
Reinforcing knowledge		
Preparing for tests	} SF	
Topics arising in tests		
Separating pupils for tasks	TD/TR	Querry discussion?
Working on weak areas	FT	
Class working together on same tasks	FT	
Spending time on topics	FT	
Recalling topics	KM.	
Feeling a routine is stupid	FT MS.	
Repeating tasks	FT	
Knowing a topic well	KM	
Moving up through school		
Topics relating to school age	FT	
Learning topics	FT	
Group achieving a good standard	SE	
Being good at a topic (or not)	KM	
Repeating work on a topic	FT	
Using equipment for a topic	APK	
Overlapping of work between schools		
Majority leading the way	SE	
Repeating tasks	FT	
Distributing topics across the year		
Being tested regularly	UN.	
Dividing time into terms		
Teacher reviewing achievement	TR	
Teacher correcting errors	TR	
Judging easiness of topic		
Using mathematical vocabulary	MA APK	
Comparing mathematics and other subjects		
Comparing secondary and primary schools		
Identifying mathematical objects	APK	
Having right/wrong answers	RW	
Having an opinion	MA WN	
Doing tests		
Not being rewarded for opinions	MN WN	
Being detailed	MN WN	
Selecting information to use	MN WN	
Having to get the right answer	MN WN	
Being precise	MN WN	
Rejecting information	MN WN	
All information being relevant	MN WN	

106.

2.2.3 Example of applying purposive coding

Developing relationship between teacher and pupil

I001

- John: He just gave us books and we would do the work set.
- DVS: I see, so, that's really interesting, Could you tell me a bit more about that. Was this teacher able to talk you through the things in the book?
- John: Some things. Some things, he'd be, If you got stuck, he would try to explain it, if he knew, but usually no-one really asked, they just got on with it.
-
- Aaron: Well, our maths teacher, it was like Alex, it was a specific teacher, so we moved around. And, yes, she was quite good at her job, she knew her things, and was quite helpful
-
- Alex: Yes, ah, it was kind of the same as Aaron, she like, she'd done the job for a long time and was very experienced and she had one of these whiteboards which, umm, she did kind of problems on it, but she used um, Smart Notebook, I think it's called, ...
- DVS: Ok
- Alex: ...quite a bit as well. She'd just write the problems up on the board and then she'd sometimes ask pupils come up to the board and work out the problem and write the answer down on the board, the interactive whiteboard.
-
- John: We didn't have anything like that. If something puzzled the whole class, he might write it on the blackboard, but apart from that it was, he'd explain it to us individually.
-
- Aaron: They were like, while, while, while the teacher's teaching they were doing scribbles, and like doodling and stuff.
- DVS: Ok, If they were doodling did that mean they were, did that mean that they weren't listening to the teacher? Or that they had/
- Aaron: It, well, most the time, unless they were like really, sort of quite, really goody-goodyish, and were writing everything the teacher said, most of the time they were just drawing pictures of stick men and stuff.
-

I002

- Thomas: Well, in years, in middle school, we had like top groups. And, umm, I was always like top group umm,
- DVS: OK
- Thomas: and there was one teacher who like really had it in for me, we had arguments, well not, but serious e-mails to my parents and like complaining and stuff and all the time and umm, it got really strange, so we kind of didn't listen to each other anymore so I kind of dropped down a bit of maths.
- DVS: OK.

Thomas: But I was still in the top group,
and my Mum always says I'm really good at it but I don't feel I'm that clever
at it any more.

Thomas: Yeh, and we also like, you know when you like compare fractions to other
things, we used to like get into circles, boys and girls, and there was like a
board with all these fractions on. They used to like show you a card with
numbers on and you had to like prepare it. That was like another maths
teacher and she was really good.

DVS: Can you tell me why she was really good?

Thomas: Well, she made it more interesting and she, umm, well she, she was the
same really than the other teachers, but she was, she just, I don't know,
she was just better, I can't really explain why.

Thomas: Our ICT teacher once, he had this camera, and he tested it out in different
directions,

ZR: To see the screen.

Thomas: Yeh, to see the screen on one computer and half the time on that one
computer. So basically he spent no time looking at the other computers.

DVS: Ok. What happens when you ask a teacher for an explanation of
something?

Thomas: Well, W normally shouts at us and tells us to ask the person next to us, and
then tells us off for talking.

DVS: Ok, ZR?

ZR: Well, if you find that you want to ask the teacher, you put your hand up,
and, because if you shout out you get in trouble. If you explain that you
don't really understand and you ask them to like, elaborate

Thomas: Yeh, we had a French teacher like that, he'd tell us off for shouting out, but
when we put our hand up, he'd be like fifteen minutes before he actually ().

I003

No comments

I004

and the brightest boy in the year?
=was in the middle set

DVS: =m-hm

MC: that's what I couldn't get about it and the teacher told me that I had
switched with him for some reason but later on in the year he came up?
And this other boy called.. I've forgotten his name, G, G something I don't
know his surname, um he went down and we were doing the CGP-

say every three lessons, K, not even three/ two/ um he would do a, to start
off a lesson we'd come in and there'd be a () and we'd do a maths quiz

DVS: ok

Jamie: or just it the unexpected tests and it makes it much better 'cause you, it just tells you that you always need to be prepared for everything that's in front of you and the teachers at our school just didn't do that really, just didn't help us.

I005

JSch: M gets really cross if people start writing down while she's explaining it
DVS: ok

Thomas: yeh M gets really cross if people don't ask for help if they're stuck
DVS: right

Aaron: M also gets cross if we come in late

Thomas: M gets cross all the time

Thomas: well it starts when
if they once they discover the weak side of
if it's like a new teacher
or a really old teacher like
who really can't control a class
#pretend sotto voce# S

Jake: she's not new

Thomas: I know but she's old like

Jake: oh, #laughs#

Thomas: and people like
laugh behind her back because she's not the most elegant of people

#boys all laugh#

Thomas: yeh like me like last year

Alex: not this (again)

Thomas: I had like ten lunchtime detentions in the whole year,
this year I got twelve in about three weeks
and I don't know what's going on

JSch: it's cause of H

Jake: you got ()

Thomas: yeh and one after-school

Aaron: you got, oh my god I don't believe you

John: yeh chemistry

John: ah but S for some reason
none of us feel really threatened by her

Jake: she would () anyway

John: yeh and if one person like is messing around
the whole class will get an after-school detention.

Thomas: yeh and 'cause like
with the new teachers who like don't know your name
for the first couple of weeks,
they just give whole class detentions

because they don't know who's doing what
but after like a year or so
they'll know who's always the troublemaker and stuff
so they'll give them the detention
so they know more about the class
but as they're new they don't
they just, they just gave us a whole ()

Jake: you the boy at the back

do you think teachers on the whole
respond well to particular people (messing around)
or are they a little bit clumsy in the way they go about it?

Thomas: there's one person,
there's one person that can't.
C is, kind of.

Jake: C #giggles#

Aaron: some people are just trying to be clever
but some people sort of like

Alex: sometime its like really

DVS: ok, right now, um

Aaron: some people just want to impress their mates

Thomas: exactly
they're showing off

Jake: trying to be funny

Aaron: like YYYY

Alex: and the teacher gets really angry

Thomas: yeh YYYY he's like always (), don't tell him I said that

1006

DVS: ok and what about um
what do pupils expect of teachers

Jamie: most of them
because they think they're so up themselves
they expect
"oh I'm the best, the teachers don't need to tell me anything"

DVS: that's the pupils,
you think

Jamie: yeh they just think "oh, we don't have to work 'cause the teacher likes us the most"

how do you know that you have been good?
how do you know that you have achieved what you need to

Jamie: most of the teachers will tell you after the lesson
they'll call you back and say
"Oh, you've done really well this lesson"

blah blah blah.
DVS: ok
Jamie: but really you know, you know that you've been well-behaved and that if if
 ever the other person is mucking about and you're being well-behaved the
 teachers will always spot you.

I007

No comments

I008

ZR: it's as thought, it would work but depends on like, circumstances, and teachers
well
All: yeh
ZR: it has to be a teacher that like the pupils respect.
Jamie: we did work in groups today, didn't we in English.
ZR: yeh we did that
Jake: yeh we did that ()
Thomas: that's what we did with the Palmer's pill in groups.
ZR: It has to be a teacher that like we all respect.
ZR: You were saying, if you take it in or not, I was thinking like it depends on what it,
like who's talking about, like if it was something important say if he was showing you a
map on Google, and say it was like so-and-so country, and you're not really interested in
that country, like you do what he says, but you won't listen to like the unnecessary parts
DVS: hm Ok. yeh cause this is something I'm interested in – how you boys would decide
what are the necessary parts and what are the unnecessary parts.
Jake: some people think that everything's necessary but it all depends on the teacher
and the class 'cause if one person starts mucking about everyone does, and like last year
our old maths teacher couldn't control um, a part of the class.
Thomas: K
Jake: yeh K and he just let everyone do the slip and no-one actually did any work or
learned.
Jamie: and he used to get really really stressed and if you said like, because he couldn't
actually like, there were a group of boys in our class, I think me and BT know who they
are. [to BT] don't look at me. And um, and then I was actually one of them I'm not afraid
to admit it but. Um he like couldn't handle us and we sat like you know there's rows in the
classroom O2 is it or O1?
DVS: O1 is the classroom you're thinking of.
Jamie: O2, is that the one upstairs? And then, we would, you know there's a pipe at the
back a big pipe?
DVS: Oh, down, for the radiators?
Jamie: yeh, and we would have like kicked it, and it would make a like yeh
Jake: a noise

Jamie: but there was like seven of us and we would have like kicked it and then we would ask K like, actually no it was like a really clever person would like actually say to K "I don't understand," and K would go {*expressive exhalation*} and K's face would go really red, and just like K did that every time someone like asked a question because K was getting so stressed.

Thomas: yeh cause we have a, W said at Christmas ok as you have two lessons on a thing and at the end you don't want to do that, I promise you we can go there every week we went there for the next week, and then I think we went like a month later and then after that just once, and like after that-

...

DVS: you forgot? Ok. Now Thomas said about what, um what the teacher said you were going to do about the computer room, but then it changed.

Thomas: yeh yeh that was it broken promises.

DVS: ah, you can call it broken promises if you want to

Jamie: yeh but he didn't make a promise

Thomas: yeh he did he said we could go every week and we went twice in the whole year.

DVS: what do you think is the, if a teacher's got a lesson ahead what do you think is the sort of starting point? When they're planning something, what do you think they start from?

ZR: the ability of the class

Jamie: how the children are gonna respond to this or-

DVS: that's interesting

Thomas: yeh, and he's like seeing if they know it or not

Jamie: they try make it as fun as they can but with S, S tries to make it as boring as they can.

Thomas: yeh, she actually tries to make it boring, and then D, he's just like-

Jake: going off the point!

Jamie: D's really safe though I like D.

Thomas: he's like 'I'm looking forward to having people in detention.' You're not meant to like-

DVS: we have wandered off the point again.

Jake: Again, Thomas.

DVS: so does nobody ever say why you're doing it? Hang on, Jamie then Thomas, go on

Jamie: Well some yeh U does. But we've got, it's only in English, we've got one person in our class, I'm not going to mention any names Jake knows who he is, he thinks he's so cool around U and U likes him and everything and uh he he and then uh he there's a guy called Nikhil, I think and he takes the mick out of his name he keeps on saying it and it gets really annoying and he gets our whole class class detentions at lunchtime and its actually not us but U says-

Jake: U says you have to do it at lunchtime.

DVS: how does this relate to what we're saying about

Jamie: 'cause we're saying like how people product in lessons, so how you're doing and what um does he tell you what um he tries to tell us but Dylan just doesn't

DVS: hang on, hang on. Let's go this way round the table this time. Jamie?

Jamie: S. If you do ask that S'll go "Why are you making silly questions, see me at lunchtime at one-fifteen for fifteen minutes."

DVS: is that-

Jamie: you can't ask S something like "Why are we learning Latin?"

DVS: Is that to do with the question itself or is it the way you ask it?

Jamie: it's got to do with both of them really. But sometimes we can ask a question like "Oh, could you just answer one question? Why are we actually like doing this subject, if Latin is a forgotten language?" And S will interrupt going um "Why are you saying it's a forgotten language?" and someone will like answer back, and then like S'll give us detention. But it's just like weird S doesn't let you ask a question. And Mis-, in Physics, I'm sorry I'm going on a bit but, in Physics D will be like-

BT: "Just get on with your work."

Jamie: yeh like D'll go "There's no need for discussion, there's no need for questions, put your hand down."

DVS: ok

Thomas: yeh

Jake: Also for Latin if you did ask, I asked a load of teachers and they said, "Oh, it's just for an interest," but I'm not interested in it. I don't really care about speaking Latin because that's like a waste of a few periods when we could be doing other subjects that are more interesting ().

1008.1

Alex: take that down. Yeh if M catches anybody writing it down when M's writing it, M gets very angry and tells you to stop writing it and like, no pens, no pens in hands and put them on your book.

DVS: what were you going to say?

John: I was going to say one lesson Sam, for some reason, I don't know whether M was in a bad mood that day but M kept saying, picking on Sam, taking everything out on Sam saying like Sam, what have you done to your tie.

DVS: Who is Sam? [responses] oh yes I know who you mean.

Alex: M was taking it out on Sam, I'm not sure if M was very angry or not, well yes he wasn't concentrating but I think M was very quick to snap on to that

Aaron: yeh

DVS: right, ok

Aaron: yeh because luke, was sitting right next to him had his top button undone and so did Sam

Alex: and M immediately just snaps to Sam

Aaron: M immediately went to Sam, so

DVS: ok I think we've wandered a little bit off the point here

Aaron: well that wasn't actually, I think we were getting away because me and Kavan, we kept asking question after question to R, and eventually the tracks moved from the light to really sort of complicated () and I actually enjoyed that because it sort of taught me about giving different theories, we we're here and who we are.

DVS: ok, so that interesting because, I'm really pleased that you enjoyed it so much that sounds really good, ah, so was the point of that lesson to make you think about why we're here and what for is that something you've thought-

Alex: that's something they sort of went on to explain

Aaron: we weren't actually meant to go on to that but

Aaron: no but in like RSP I remember with J they had a game about something and then eventually it all escalated into this huge battle, well battle of words and everyone was like 'oh no no you're wrong because like this happened,' and then 'no you're wrong because this happened.'

DVS: so was this battle as you called it working out in quite a civilised way-

Aaron: yeh

Alex / John: #laugh#

John: until the chairs started flying

Aaron: and someone started shouting

Alex: Mr R got quite angry

Aaron: he wasn't angry

Alex: well he wasn't angry, he liked the way that we were just like talking about the facts but he thought the level had got a bit too high

Aaron: well sometimes, this happened a long time ago, in my old school, my teacher, my dad taught me something and told me that's the only way to do it and my other teacher called Miss C, she said 'no there's another way to do it' she showed me how my dad was absolutely wrong and () the answer and I told this and he wrote a letter saying() and then um he wrote the letter back to Miss C and Miss C wrote a letter explaining exactly why that method was correct and with a very detailed diagram about () and I carried on

DVS: so-

Aaron: and I had the letter for my dad, ()

DVS: so who was it, was it your teacher who was correct or were they both right?

Aaron: well, no, they actually were both right, because my dad only thought there was one way to do it, and that's his way, and miss C knows there's another way to do it, and so.

I009

No comments

I010

John: well W tends to be more sort of ...
like T, they explain
I don't really know how, they seem friendly
and they make like childlike comments about people and things

you know
it makes the children like more interested by saying like, like
and if someone's making comments

DVS: ok, right right

John: or not paying attention ... like being called names.

DVS: ok – so does that stand out,
is that different from other teachers

John: well like M tends to sort of tell us to be quiet and like if we keep talking
then she says she's going to put us all in a detention but she never really
does it, never gets round to doing it.

John: 'cause it would be pointless
that wouldn't really matter
you know you could just leave everything blank
and write the answers in as you go along

DVS: ok, go on,
so if your teacher took in your work and they saw all the right answers

John: then they'd get the wrong idea about you y'know if someone cheats in a
test and the teacher thinks they're a genius so if you do it honestly you get
the right grade

I011

No comments

I012

Thomas: that's what we said last time,
that he told us how to do it for three minutes,
then made us do work for the rest of the lesson
but M helps you
-interruptions-
but it's not really the teacher
if the guy next to you knows you can ask him

I013

No comments

I014

DVS: so obviously that then leads on to does that then affect the way that you
teach it?

M: absolutely I mean my first port of call is to gain their trust and then if I've
gained their trust they're going to trust what I'm telling them and to make
it fun, to make it interesting, umm to see the relevance in it. She was just
() "you just have to know this for the sake of it" it was just for sitting an
exam so I like to, like the purpose behind maths, why are you doing this it's
not just a formula there is another reason behind it

DVS: ok

M: and that's why I kind of try and get down to the level of the boys and try and make maths then, maths lessons I would say obviously they're very formal to a certain extent but I always like a kind of air of informality, that boys can just walk up to the front of the classroom if they need help, they can just ask whenever or whatever they need to and I hope that they've got that respect that they don't overstep that mark and it's very, very rare that they do.

DVS: it's that's one of the things I've noticed it does seem as if, it does seem as if there's a very, you've put a lot of effort into getting the right social atmosphere

M: absolutely

DVS: ok

M: I think personally that's very important I think if you can get their interest, particularly in year eight, it's one of those years they've got no formal exams the previous year they've got no formal exams this year or next year, they're kind of a bit in limbo, so if you can keep their interest and then when they start their IGCSE next year I think that's really important.

M: Umm, that would depend on the time of the year I think in the first week or so, it's very formal, it's very, it's quite, I think I would stand more and talk at the board rather than talk to the child, I think once you get to know the child, that to me is such an important thing, 'cause you bring it down to their level. Speaking to X I answer questions very very differently than if I went to Y, who's far weaker than X. Because X you can talk to him using mathematical words

DVS: m-hm

M: say for example things like frequency, or you know things like that, but with Y you'd have to say 'what's the total'

DVS: yes

M: and then if you think there's () on top of that it's really dependent on the level of the boy, that's why I think it's important for the teacher to know the children individually not just as a class. It really does come in to it.

DVS: with them. So yeh, ah MC how would you sort of describe him and his maths ()

M: describe him... umm I really like MC I think he's got a real flair for maths

DVS: ah-ok

M: I think he's got a real flair but he really doesn't like writing it down he'd far far much prefer to talk through a problem, give you an answer tell you how he's got it than actually sit down and think "two plus two is four and four is (one)" and you know he doesn't like that he gets really distracted easily but he's one of the boys that you have to give him a problem

DVS: oh, something that's actually going to engage him

M: that he actually has to work through if I just gave him sums that he could do in his head and didn't actually require any thought then he would be an absolute nightmare in class

DVS: ah-ok. Thomas?

M: (intake of breath) he's not very confident, not a very confident boy at all with maths he likes to sit near someone all the time that he knows is good at maths if he doesn't sit next to umm oh gosh Alex

DVS: Alex?

M: or X and if he starts to get questions right I really have to pounce on that during the lesson 'cause that will let his confidence improve, 'the silly behaviour stops chatting stops, turning round stops so if he feels that he's doing the right thing and using the right thought right train of thought he really appreciates that and we just take it from there

DVS: ok, and John

M: he's a strange one thought he was very very weak at the start of the year um had to take him aside and have a good long conversation with him, gave him much more simplified work at the start he worked on it and now I would say he's I wouldn't say he's up at the top end of the scale but he's far far more confident and he tends to think he can't do it or he makes up his own reasons but once he gets an idea in his head it's very difficult to get it out of his head so unless he's one hundred percent clear on each step, he makes up, he makes up silly mistakes and sometimes he's like one of those boys, "where the hell has he got that idea from?" #laughs#

DVS: #laughs#

M: you know so I'll go back to the beginning again, but again that's the way you talk to the child, you say "Oh, John let's go back to the beginning" rather than "that's wrong, you've got to start again".

DVS: uh-huh

M: a bit like that he, he picks up the ()

DVS: uh-hm. Z, ZR

M: He is um very bright, very bright in maths, doesn't like to show it, likes to be the joker

DVS: ah, ok

M: so he's one of these people that actually he's useful when I want answers, he will answer a lot of questions but he wouldn't want the others to know that he's good

DVS: right. Ok, that is interesting ah

M: and what's good was, he was on the ski trip

DVS: oh of course, he broke his

M: yeh and its amazing what that's done for kind of for the relationship between me and him. () you know other things like that talking about all the things they do ()

DVS: ok. Aaron

M: absolute nutter. #laughs#
DVS: #laughs#
M: airhead personified no no at first yeh, he was a really scatty boy, you know equipment books, what are they ma'am. Had to have a stern stern talk with him to get him settled, but said that it's put the onus on him mathematically and behaviourally he has now moved himself away so he's working really hard he's actually doing very very well at the moment and I think he's really improved and I'm hoping we'll get ()
DVS: ah-ok, oh of course you gave them a test yesterday and uh, umm Alex
M: absolutely adore him he wouldn't have any bond at all at first even though he was very good at maths until he knew me he was very very quiet and now him and me have got a bond together he's an absolute star absolute star he's the one not has he got a knack but has he got a bond with the teacher he got on with them ()
DVS: mm it's been interesting well with all of them it's been interesting. Alex has made the most frequent references to exams
M: yep
DVS: does he ()
M: I think that's because he's got the whole serious he is a very serious a very
DVS: he is very serious
M: very serious child and I'm not sure if that's coming if he has older parents maybe whether they're putting pressure on, or maybe older brothers and sisters that might be an idea whereas with Thomas who is as scatty as can be, he's an only child with older parents so kind of have no one who been through it before where probably Alex has
DVS: right yes that could be it
M: that's what i think, yeh.

I015

No comments

I016

No comments

I017

DVS: ok, ok, right, I think, did I see rightly that your questions, the questions that you had were split into different groups?
Thomas: yeh we had to have five rounds. We had four rounds and a bonus round.
DVS: ah ok,
Thomas: yeh and my bonus question was "Who is M's hero?" and there are large pictures of David Beckham on the wall, so someone would say that but it's actually Madonna or M's mum.
DVS: ok, did you know that answer to that one before?
Thomas: yeh because I'd asked M

DVS: that was in the bonus round
Thomas: I'd already asked M

I018

No comments

I019

No comments

I020

No comments

I021

No comments

I022

No comments

I023

No comments

I024

No comments

I025

DVS: Ok, and is it- do you find it necessary to have a good relationship with the teacher, a good working relationship?
Thomas: yeh because if you don't get on with the teacher you're never really going to be able to understand because you may not listen to the teacher or they may not answer questions properly or they'll try and ignore what you say.

I026

No comments

2.3 interrelations of focused codes – example.

FT and ON

As part of the ON, the teacher decides when and in what arrangement pupils will study mathematics. Hence, topicalisation of mathematics exists as a means of managing the curriculum provision. Mathematics is formed into pre-packaged topics by curriculum material such as schemes of work and textbooks, which thus become tools for the teachers in their arrangement of teaching, but also form part of the division of labour in guiding the pupils, and presenting the object to them. Arrangements of mathematics also obtain from teachers' own learning experiences. Hence access to, and the emergence of, the object becomes arranged around certain focal ideas, and it is arranged with a sense of progression amongst these ideas over the course of the subjects' school experience. Topicalisation in itself is a tool for approaching the object, instantiated in the teaching materials the teacher co-opts. Structures initiated by curriculum writers become the learner's experience of mathematics to a certain extent. However practical considerations are such that a teacher does not have complete freedom in choosing their materials: materials are chosen on a school basis, and a teacher's work in offering pupils access to the object is mediated by these materials. Was there evidence of teachers' relationships with mathematics also being mediated?

One key contradiction which emerges here is the boundedness of areas of mathematics: these bounds and labels enable learners to get a handle on mathematics, step-by-step, allowing notions to grow into useable concepts, but at the same time are entirely contingent, dependent on language and teaching habits for their existence, thus do not well represent the connected nature of mathematical activity.

The division of labour in the community is such that the teacher manages these topics areas and the pupils follow the teachers' lead. This can lead to frustration or boredom on the subjects' part if they feel they have had enough exposure or practice in a topic area – this frustration has to be dealt with by the pupils or managed as part of ON by the teacher. The teachers' aims are to provide each pupil with sufficient and proportionate experience of topic areas in order for them to achieve certain standards, but also within a manageable framework of time. Teachers spoke of adjusting the tasks given to pupils in order to remedy weaknesses when necessary.

2.4 CHAT terminology used in categories – additional example

Category: Studying For

Studying For is identifiable by a professed aim to achieve a certain standard in an examination being the guiding focus for the work being undertaken. This has the effect of turning the work into 'revision': rehearsal of techniques and recognizing forms. This short-term aim has many consequences, which will be detailed here. The first is that the rehearsal of those skills takes place with little variation and in predictable contexts, thus inhibiting the learner's capacity to see when and where they would be useful.

Periods of time close to the examinations may be identified with *Studying For*; these periods could be a single lesson or could stretch over terms. The working norms of classrooms may be disrupted for periods of time when *Studying For*: homework may be set to a new or unusual routine, different exercises may be administered and the use of past examination papers comes into play. Practical activities were reported by participants in maths lessons, but it was suggested these had been put aside when practice for exams became a focus of the lessons.

Various tools are co-opted into activity in certain ways by *Studying For*. The contribution from one of the teachers interviewed indicated her use of textbook was determined by how well she felt it contributed to pupils' preparation for the IGCSE exam, regardless of which year they were in. The use of past examination papers is intended to expose pupils to the structure and expectations of the examinations, although it has wider consequences, as will be shown below. Even the choice of pen or pencil as a writing tool might be influenced by a teacher's feelings as to how pupils should be prepared for examinations. Participants reported using texts as a key part of their revision: facts and examples would be rehearsed and compared with the documents they had created or had been handed by teachers. A focus on written examples entailed reproduction and duplication as inherent parts of revision, becoming operational as the pupils' authority extended. In the midst of a test, JSch reported, one could then become confused between a choice of methods; a question in a test set by one teacher that elicits possible options of methods advocated by other teachers raised the question of what to write. He felt he should give the method suited to the teacher who set the test. Classroom convention and the division of labour then have a more powerful effect than the objectivity of mathematical activity.

Community relations may be shifted subtly by *Studying For*. Pupils' desire to know what they need to do for the exam, rather than what mathematical skills they need to learn, inhibits communication in that pupils are keen to focus on answers and therefore less likely to speak to teachers who are interested in solutions. Focus can turn to what they feel they 'need' to know, and to be able to produce.

The most powerful way in which *Studying For* was seen to influence the activity of the classroom was in the expectations which resulted from this focus. Teachers can settle on a trajectory of development which has the examination as its end point, an attitude which is happily adopted by the pupils. Examination structures, such as the right/wrong characteristic which ultimately earns credit or does not, can be adopted in classrooms, to varying degrees of intensity. The object can be 'reduced' to a set of correct answers. The most potent case seen here is the description of 'marks' as a means of judging what is necessary or appropriate in answering a question. One participant had had sufficient exposure to past examination papers that he described solutions and their content in terms of the marks they could earn (or lose). Marks thus become a currency in the education, as a result of over-familiarity with the marking schemes for tests. Marking conventions are taken as inherent characteristics of good mathematics.

The obverse side of *Studying For* is the periods of time in which one does not feel this is the case: teachers reported that in those periods they could undertake activities that broadened pupils' understanding of mathematical activity. However these periods of time were not unproblematic, as examination success was seen as a motivating factor: not having an exam to focus on meant that teachers had to work harder to keep pupils engaged by doing 'fun' activities.

The emergence of *Studying For* suggests that examinations in mathematics, rather than being a mediating aspect of school-mathematical activity, stand as a proxy object.

3 Appendix 3: Exercises and worksheets

All CIMT materials are used with permission and can be accessed at <http://www.cimt.plymouth.ac.uk/projects/mep/default.htm>

3.1 The 100 Club

The 100 Club

Complete 100 correct answers in under 5 minutes to become a member of the 100 club.
On entry to the 100 club you receive 1 commendation and 1 further commendation for each faster time

1	$-4 \times 8 =$	26	$9 \times 11 =$	51	$-9 \times 8 =$	76	$10 \times -8 =$
2	$-5 \times -3 =$	27	$9 \times -8 =$	52	$8 \times -1 =$	77	$8 \times 12 =$
3	$3 \times -8 =$	28	$-7 \times -4 =$	53	$0 \times -12 =$	78	$-12 \times 4 =$
4	$-3 \times 6 =$	29	$-1 \times -11 =$	54	$-12 \times 6 =$	79	$8 \times 4 =$
5	$-5 \times -3 =$	30	$-8 \times 11 =$	55	$0 \times 8 =$	80	$5 \times 5 =$
6	$-2 \times 11 =$	31	$3 \times -7 =$	56	$8 \times -5 =$	81	$-6 \times 4 =$
7	$-7 \times -6 =$	32	$11 \times -4 =$	57	$-6 \times -5 =$	82	$-7 \times 7 =$
8	$8 \times -3 =$	33	$-11 \times 3 =$	58	$-1 \times -12 =$	83	$4 \times 10 =$
9	$12 \times -12 =$	34	$-6 \times -4 =$	59	$2 \times -11 =$	84	$0 \times -3 =$
10	$12 \times 0 =$	35	$12 \times -11 =$	60	$-5 \times 10 =$	85	$7 \times 8 =$
11	$2 \times 12 =$	36	$-11 \times 5 =$	61	$-4 \times 4 =$	86	$-2 \times 4 =$
12	$-9 \times 9 =$	37	$12 \times -7 =$	62	$-6 \times -5 =$	87	$8 \times 2 =$
13	$6 \times 3 =$	38	$-4 \times 0 =$	63	$0 \times -5 =$	88	$7 \times 3 =$
14	$8 \times -10 =$	39	$4 \times 6 =$	64	$4 \times -4 =$	89	$-2 \times 6 =$
15	$-2 \times 4 =$	40	$-5 \times -5 =$	65	$4 \times 9 =$	90	$4 \times -4 =$
16	$12 \times 0 =$	41	$-8 \times 8 =$	66	$-9 \times 8 =$	91	$10 \times -9 =$
17	$-1 \times -12 =$	42	$-9 \times 0 =$	67	$-9 \times 5 =$	92	$-1 \times 6 =$
18	$11 \times -8 =$	43	$-6 \times 6 =$	68	$0 \times -5 =$	93	$12 \times -8 =$
19	$3 \times -6 =$	44	$-1 \times -5 =$	69	$9 \times 5 =$	94	$-3 \times 0 =$
20	$11 \times -5 =$	45	$5 \times 12 =$	70	$3 \times 10 =$	95	$-3 \times -8 =$
21	$-5 \times 3 =$	46	$-10 \times 2 =$	71	$-5 \times -2 =$	96	$-8 \times 6 =$
22	$-7 \times -7 =$	47	$-9 \times 12 =$	72	$11 \times 8 =$	97	$-7 \times -5 =$
23	$8 \times 2 =$	48	$1 \times -1 =$	73	$3 \times 7 =$	98	$3 \times -6 =$
24	$7 \times -3 =$	49	$1 \times -2 =$	74	$1 \times -3 =$	99	$-3 \times 10 =$
25	$5 \times 6 =$	50	$-11 \times 8 =$	75	$8 \times 6 =$	100	$-9 \times -4 =$

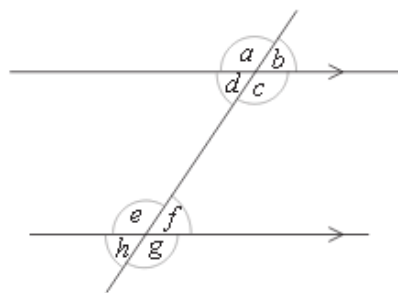
3.2 Angles in parallel lines: exercise



Exercises

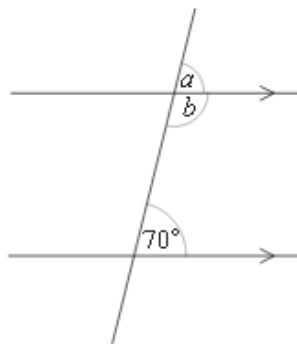
1. Which angles in the diagram are the same size as:

- (a) a ,
(b) b ?

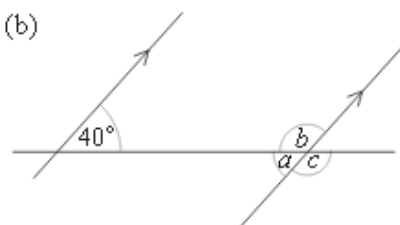


2. Find the size of each of the angles marked with letters in the diagrams below, giving reasons for your answers:

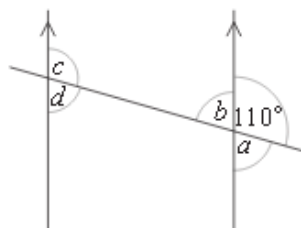
(a)



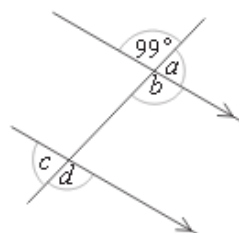
(b)



(c)



(d)

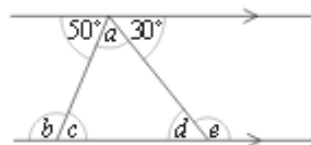


3. Find the size of the three unknown angles in the parallelogram opposite:

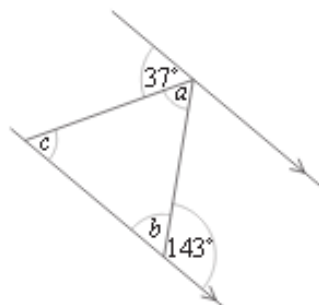


4. One angle in a parallelogram measures 36° . What is the size of each of the other three angles?
5. One angle in a rhombus measures 133° . What is the size of each of the other three angles?

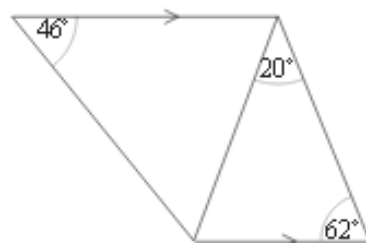
6. Find the sizes of the unknown angles marked with letters in the diagram:



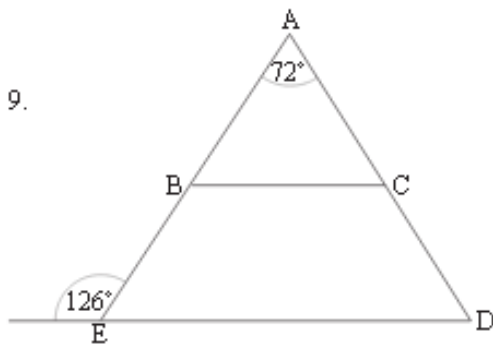
7. (a) In the diagram opposite, find the sizes of the angles marked in the triangle. Give reasons for your answers.
- (b) What special name is given to the triangle in the diagram?



8. The diagram shows a bicycle frame. Find the sizes of the unknown angles a , b and c .



- 9.



BCDE is a trapezium.

- (a) Find the sizes of all the unknown angles, giving reasons for your answers.
- (b) What is the special name given to this type of trapezium?

3.3 Flight paths: Worksheet

ACTIVITY 11.2

Flight Paths

The map of Britain given on Sheet Activity 11.2a is drawn to a scale of 1 : 3 000 000. Use the map to answer the following questions:

1. What are the bearings of:
(a) Newcastle, (b) Edinburgh, (c) Glasgow, (d) Manchester from London? Also find the bearings *from* each of these cities *to* London.
2. (i) A plane leaves London at 1400 hours, travelling on a bearing of 322° at 480 km per hour.
 - (a) On Sheet Activity 11.2a, draw the flight path of the plane.
 - (b) How many kilometres will the plane travel in 30 minutes?
 - (c) On the map, which city might it have reached in that time?(ii) Another plane leaves Bristol at 1400 hours, travelling in the direction of Newcastle at a steady speed of 390 km per hour.
 - (a) Draw its flight path.
 - (b) Where will it be at 1425 hours?
 - (c) Design a new flight path which avoids the Manchester area by flying at least 30 km to the west, and then changing direction towards Newcastle. With the plane travelling at 390 km per hour, give the new flight path in the form

	Bearing	Duration of Flight
1		
2		

Extension

You are the flight director of a new small airline based at Manchester. You have leased 4 identical planes which fly at an average speed of 450 km/h, and need refuelling after a maximum distance of 600 km. You would like to provide at least one daily return flight from Manchester to each of the other cities marked on the map.

Design: (a) *timetables* for the daily use of each plane (starting no earlier than 0700 hours, and finishing no later than 2200 hours; allow at least 40 minutes between arrival and departure at any airport).

(b) *flight paths* for all flight movements, ensuring that they do not conflict with one another.

ACTIVITY 11.2a

Flight Paths - Map

Map of UK drawn to scale 1 : 3 000 000



3.4 Constructing Nets and Triangles: Exercise

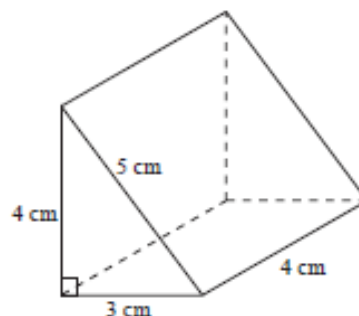
6.5 Nets of Prisms and Pyramids

In order to draw the nets of some prisms and pyramids, you will need to construct triangles as well as squares and rectangles.



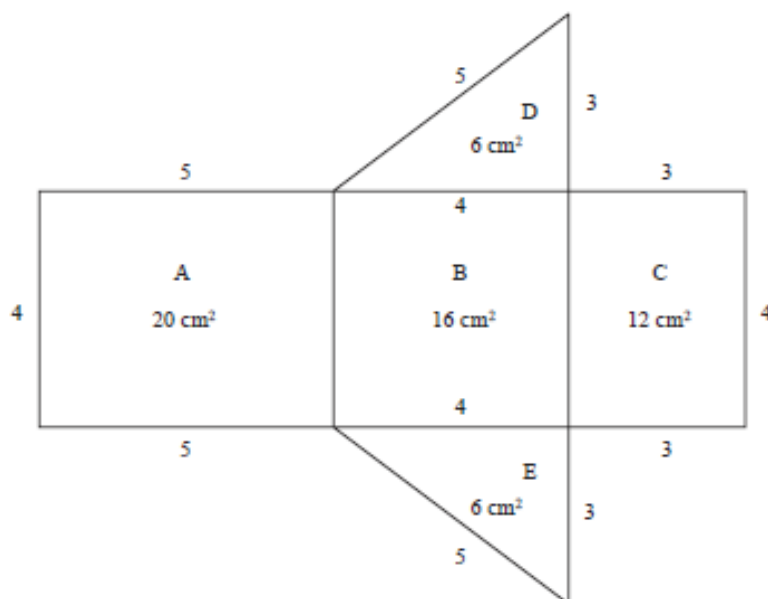
Example 1

- Draw a net for this triangular prism:
- Calculate its surface area.



Solution

- A net is shown below where all lengths marked are in cm.



- The area of each part of the net has been calculated.

$$\begin{aligned}
 \text{Surface area} &= \overset{\text{A}}{(5 \times 4)} + \overset{\text{B}}{(4 \times 4)} + \overset{\text{C}}{(4 \times 3)} + \overset{\text{D}}{\left(\frac{1}{2} \times 4 \times 3\right)} + \overset{\text{E}}{\left(\frac{1}{2} \times 4 \times 3\right)} \\
 &= 20 + 16 + 12 + 6 + 6 \\
 &= 60 \text{ cm}^2
 \end{aligned}$$



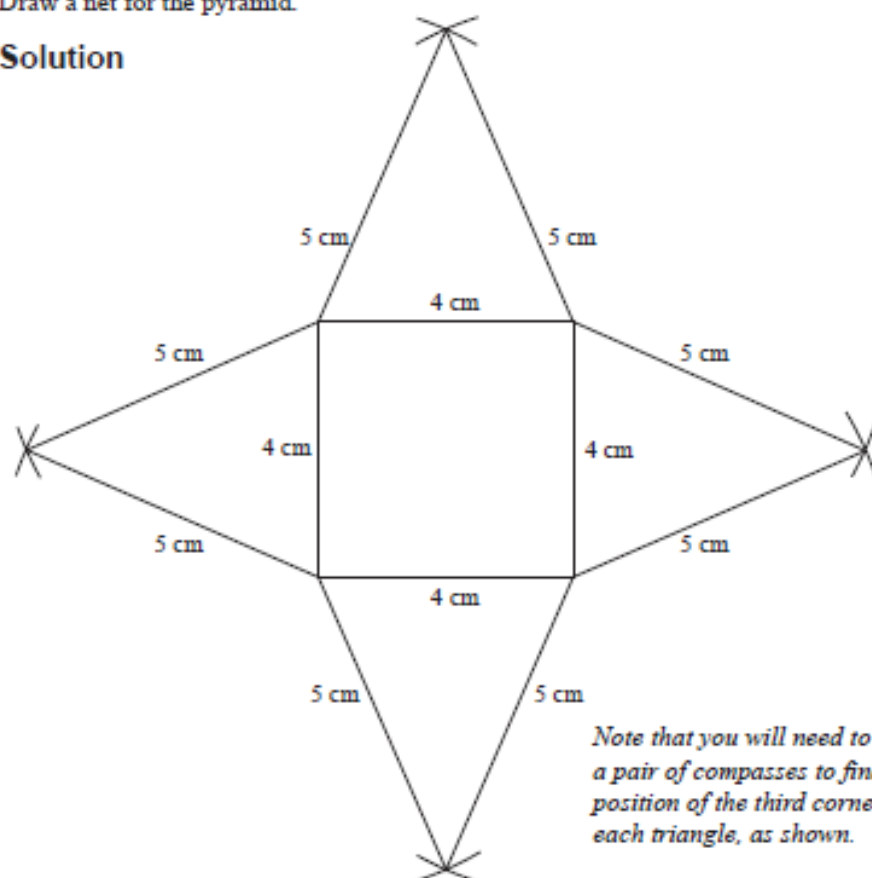
Example 2

The square base of a pyramid has sides of length 4 cm. The triangular faces of the pyramid are all isosceles triangles with two sides of length 5 cm.

Draw a net for the pyramid.

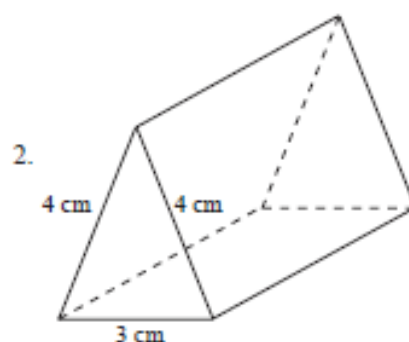


Solution

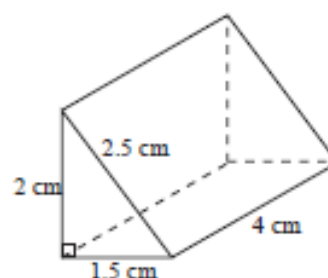


Exercises

1. Draw a net for the triangular prism shown opposite:

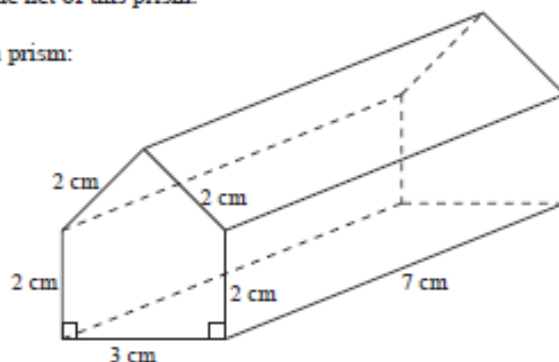


2.

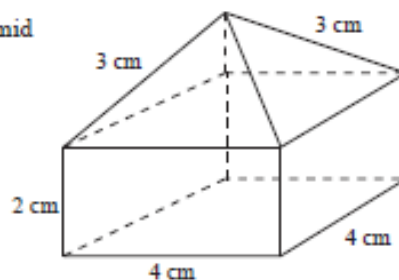


Draw a net for this prism, on card.
Add tabs, cut it out, and then construct the actual prism.

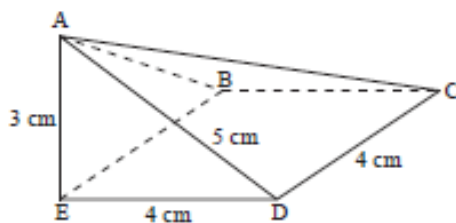
3. A pyramid has a square base with sides of length 6 cm. The other edges of the pyramid have length 6 cm. Draw a net for the pyramid.
4. A pyramid has a rectangular base with sides of lengths 3 cm and 4 cm. The other edges of the pyramid have length 6 cm. Draw a net for this pyramid on card, cut it out and construct the pyramid.
5. A tetrahedron has four faces which are all equilateral triangles. Draw a net for a tetrahedron, which has edges of length 4 cm.
6. A square-based prism has a base with sides of length 5 cm and vertical height 6 cm. Draw the net of this prism.
7. The diagram shows a prism:



- (a) Draw a net for the prism.
 - (b) Find the height of the prism.
8. A container is in the shape of a pyramid on top of a cuboid, as shown in the diagram opposite. Draw a net for the container.



9. The diagram below shows a square-based pyramid; the base is horizontal and AE is vertical. Draw a net for this pyramid.



3.5 24-hr clock: Exercise

Method B



$$38 \text{ mins} + 5 \text{ minutes} = 43 \text{ minutes}$$

$$\text{Total time} = 4 \text{ hours } 43 \text{ minutes}$$



Example 4

The time in the United Arab Emirates is 4 hours ahead of the time in the UK.

- What is the time in the United Arab Emirates when it is 3:00 p.m. in the UK?
- If it is 2:45 p.m. in the United Arab Emirates, what is the time in the UK?



Solution

- The time in the United Arab Emirates is 4 hours ahead, so it is 7:00 p.m.
- Four hours behind 2:45 p.m. is 10:45 a.m.



Exercises

- Convert the following times to 24-hour clock times:
 - 6:45 a.m.
 - 6:45 p.m.
 - 2:20 p.m.
 - 11:40 p.m.
 - 10:30 a.m.
 - 10:15 p.m.
- Write the following 24-hour clock times in 12-hour clock times, using 'a.m.' or 'p.m.':
 - 1642
 - 0832
 - 1042
 - 2236
 - 2318
 - 1520
- Which of the 24-hour clock times below are *not* possible times. Explain why.
 - 1372
 - 1758
 - 2302
 - 2536

4. David gets on a train at 0845 and gets off at 1132. For how long is he on the train?
5. A journey starts at 1532 and ends at 1830. How long does the journey take?
6. Marco boards a ferry at 1842 and gets off at 0633 the next day. For how long is he on the ferry?
7. In Venezuela the time is 4 hours behind the time in the UK.
 - (a) What is the time in Venezuela when it is 3:00 p.m. in the UK?
 - (b) What is the time in the UK when it is 2:30 p.m. in Venezuela?
 - (c) What is the time in the UK when it is 11:15 p.m. in Venezuela?
8. The time in Norway is 1 hour ahead of the UK. It takes $3\frac{1}{2}$ hours to fly from the UK to Norway.
 - (a) A plane leaves the UK at 10:15 a.m. (UK time). What is the time in Norway when it lands there?
 - (b) The plane flies back and lands in the UK at 7:22 p.m. (UK time). At what time did the plane leave Norway?
9. The time in Paraguay is 4 hours behind the UK.
The time in Macao is 8 hours ahead of the UK.
 - (a) What is the time in Macao when it is 6:00 a.m. in Paraguay?
 - (b) What is the time in Paraguay when it is 3:30 p.m. in Macao?
 - (c) What is the time in Macao when it is 8:30 p.m. in Paraguay?
10. A ferry takes $26\frac{1}{2}$ hours to travel from the UK to Spain. The time in Spain is 1 hour ahead of the UK.
When do you arrive in Spain if you leave the UK at:
 - (a) 0830 on Monday (b) 1742 on Friday (c) 2342 on Sunday?

13.3 Time and Money

In this section we consider problems that involve both *time* and *money*.



Example 1

One day, Zoe works from 0930 until 1800.

She is paid £5.20 per hour.

How much does she earn for her day's work?

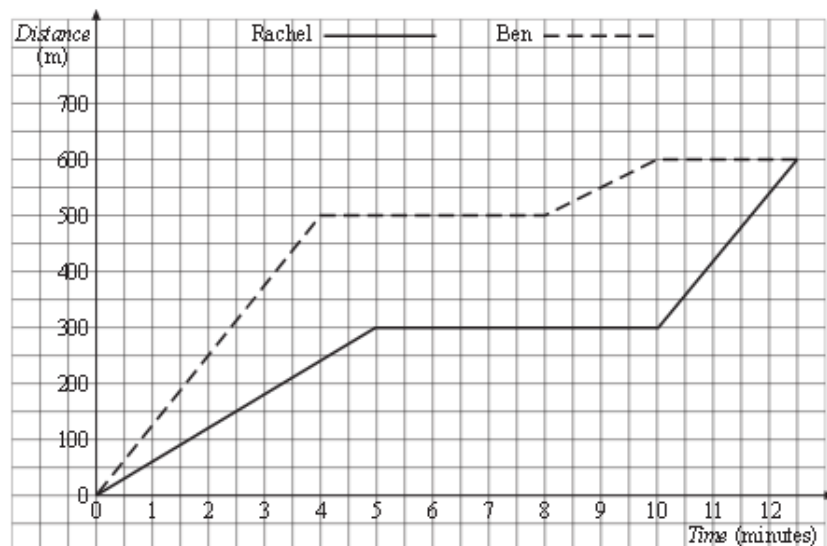
3.6 Speed-Distance-Time: Exercise

MEP: Demonstration Project

Teacher Support Y8B

UNIT 18 *Speed, Distance and Time* **Extra Exercises 18.4**

1. The graph shows how Rachel and her brother, Ben, walk to school.



Answer the following questions, giving all speeds in metres/minute.

- How far do they walk to get to school?
- How long does it take Ben to get to school?
- How long does it take Rachel to get to school?
- For how long does Ben stop on the way to school?
- For how long does Rachel stop on the way to school?
- Calculate Ben's speed on the first part of his journey.
- Calculate his speed on the last part of his journey.
- Calculate Rachel's speed on the first part of her journey.
- Calculate her speed on the last part of her journey.
- Calculate the average speed at which Ben travels on his way to school.
- Calculate the average speed at which Rachel travels on her way to school.
- Convert your answers to parts (h), (i), (j) and (k) to m/s.

3.7 Enlargements: Exercise

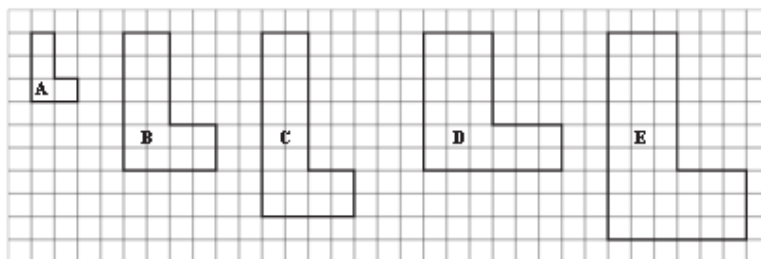
19.1

MEP Y8 Practice Book B

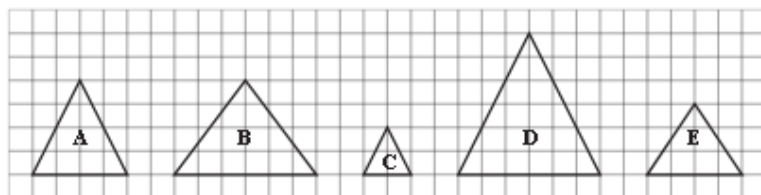


Exercises

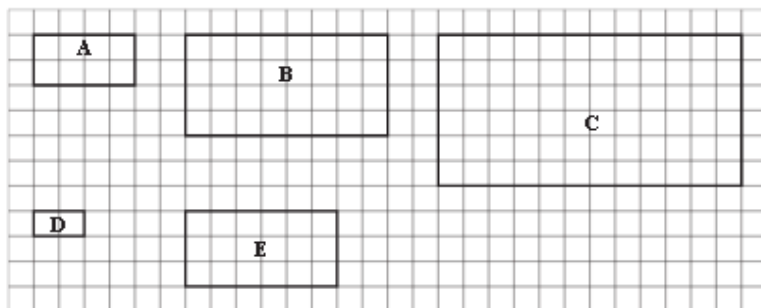
1. Which of the following shapes are enlargements of shape A? State the scale factor of each of these enlargements.



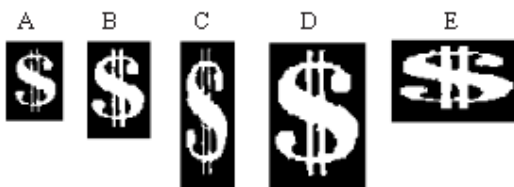
2. Which of the following triangles are *not* enlargements of the triangle marked A?



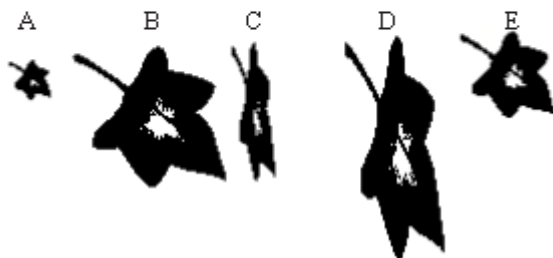
3. The diagram below shows four enlargements of rectangle A. State the scale factor of each enlargement.



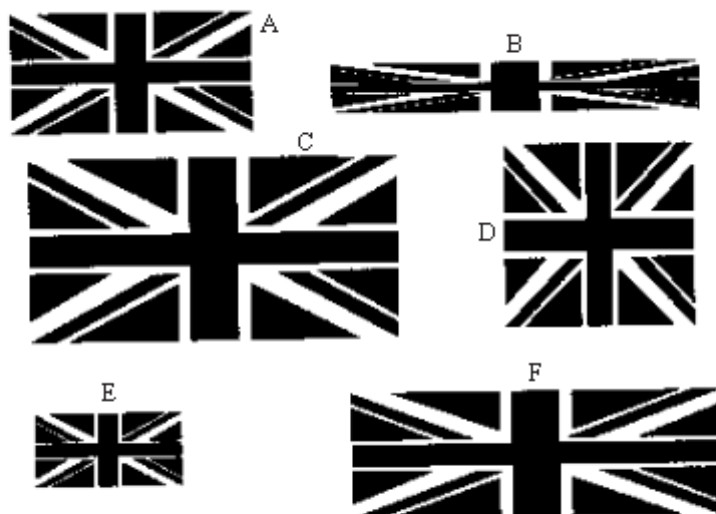
4. Which two signs below are *not* enlargements of sign A?



5. Which two of the leaves shown below are enlargements of leaf A?



6. Which of the flags below are enlargements of flag A?



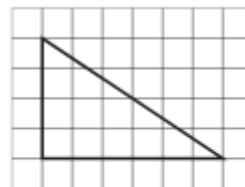
7. Draw enlargements of the rectangle shown with scale factors:

- (a) 2 (b) 4
(c) $\frac{1}{2}$ (d) 3



8. Draw enlargements of the triangle shown with scale factors:

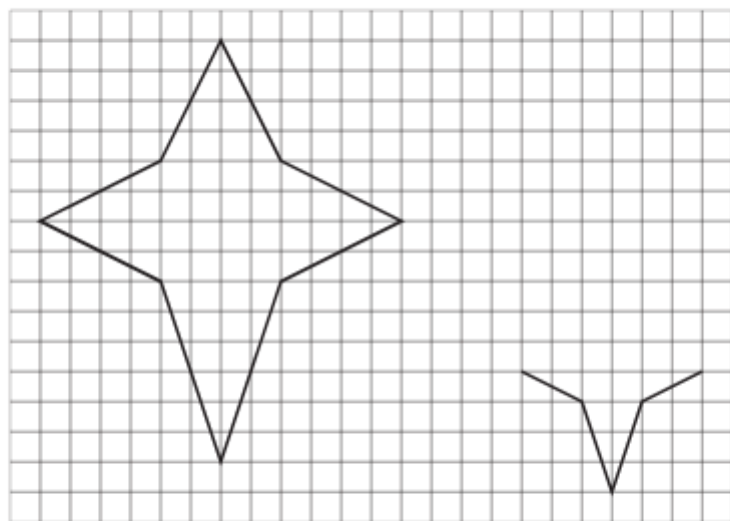
- (a) 2
(b) 3
(c) $1\frac{1}{2}$



9. Denise has started to draw an enlargement of the shape below. Copy and complete her enlargement.

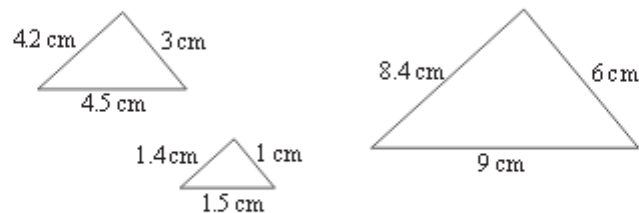


10. Kristian has started to draw an enlargement of the shape below. Copy and complete his enlargement.



19.2 Similar Shapes

Similar shapes are those which are *enlargements of each other*; for example, the three triangles shown below are similar:



3.8 Similar shapes: Exercise



Example 4

A sphere has a volume of 20 cm^3 . A second sphere has 4 times the radius of the first sphere. Calculate the volume of the second sphere.



Solution

The radius is increased by a factor of 4.

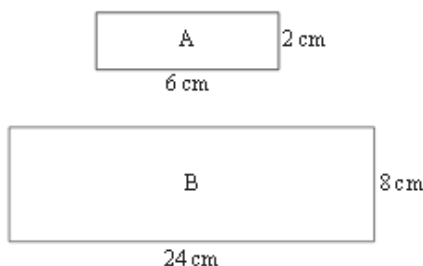
The volume will be increased by a factor of 4^3 .

$$\begin{aligned}\text{Volume} &= 20 \times 4^3 \\ &= 20 \times 64 \\ &= 1280 \text{ cm}^3\end{aligned}$$



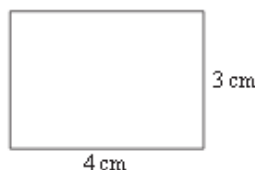
Exercises

1. Two rectangles are shown below:



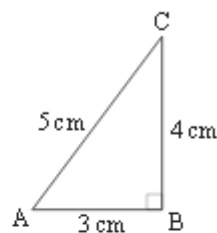
- (a) Calculate the area of each rectangle.
(b) How many times longer are the sides in rectangle B than those in rectangle A?
(c) How many times bigger is the area of rectangle B?
2. Calculate the area of the rectangle shown if it is enlarged with a scale factor of:

- (a) 2 (b) 3
(c) 6 (d) 10



3. The following table gives information about enlargements of the triangle shown, which has an area of 6 cm^2 .

Copy and complete the table.



<i>Length of Sides</i> <i>Base Height</i>		<i>Scale Factor</i>	<i>Area</i>	<i>Area Factor</i>
3 cm	4 cm	1	6 cm^2	1
		2		
	12 cm			
	16 cm			
15 cm				
		6		
30 cm	40 cm		600 cm^2	100
4.5 cm				

4. The parallelogram shown has an area of 42 cm^2 .



The parallelogram is enlarged with a scale factor of 5.

Calculate the area of the enlarged parallelogram.

5. The area of a circle is 50 cm^2 . A second circle has a radius that is 3 times the radius of the first circle. What is the area of this circle?
6. Two similar rectangles have areas of 30 cm^2 and 480 cm^2 . Describe how the length and width of the two rectangles compare.

3.9 Revision tests: Exercises

MEP: Demonstration Project

Teacher Support Y8B

UNIT 12 *Formulae*

Revision Test 12.2 (Academic)

1. If $a = 6$, $b = 7$ and $c = -3$, calculate:

(a) c^2

(b) $3a + 2b$

(c) $4a - 7c$

(d) $b(a - c)$

(7 marks)

2. Calculate:

(a) $(-6) + (-7)$

(b) $6 - (-9)$

(c) $84 \div (-4)$

(6 marks)

3. Solve the following equations:

(a) $x + 7 = 5$

(b) $2x + 5 = 19$

(c) $6x - 3 = 15$

(d) $\frac{x}{7} + 2 = 4$

(e) $3(x + 4) = 6$

(9 marks)

4. Make x the subject of each of the following formulae:

(a) $y = 2x - 1$

(b) $y = \frac{x}{2} + 3$

(4 marks)

5. Solve the equation $x^3 = 20$, correct to 1 decimal place. Show all your working.

(4 marks)

UNIT 13 *Money and Time***Revision Test 13.2**
(Academic)

1. Write the following times as 24-hour clock times:
(a) 3:15 p.m. (b) 7:45 a.m.
(2 marks)
 2. Write the following 24-hour clock times as 12-hour clock times, using 'a.m.' or 'p.m'.
(a) 1742 (b) 1108
(2 marks)
 3. Jamil goes shopping and spends £2.82 in one shop and £6.49 in another. He starts out with exactly £15.
(a) How much does he spend?
(b) How much money does he have left?
(4 marks)
 4. A bus journey takes $3\frac{1}{4}$ hours.
(a) At what time does John complete the journey if he starts at 0732 ?
(b) If he arrives at 1408, at what time did he start his journey?
(4 marks)
 5. Ali gets on a train at 1620 and gets off again at 1847. For how long was he on the train?
(2 marks)
 6. Julie is paid £5.20 per hour. How much will she earn if she works for:
(a) 6 hours,
(b) 40 hours ?
(5 marks)
 7. Joseph works for 30 hours and is paid £135. How much is he paid per hour?
(3 marks)
 8. Anna arrives at her friend's house at 1847 and leaves at 2053. How long does she spend at her friend's house?
(3 marks)
-

Revision Test 13.2

9. Malcolm is paid £7 per hour. The table shows the times he started and finished work one weekend.

<i>Day</i>	<i>Start Time</i>	<i>Finish Time</i>
Saturday	1130	1500
Sunday	0800	1430

How much did he earn for this weekend's work?

(5 marks)

UNIT 14 *Straight Line Graphs***Revision Test 14.2**
(Academic)

1. (a) Plot the points with coordinates,
 $(-2, -3)$, $(3, -3)$, and $(3, 2)$
- (b) When you join up the points, what type of triangle have you drawn?

(4 marks)

2. Calculate the gradient of each of the following lines:



(6 marks)

3. The coordinates of three of the corners of a rectangle are listed below:
 $(-2, 4)$, $(-3, 7)$ and $(4, 6)$
- (a) Draw the rectangle.
- (b) Write down the coordinates of the other corner.

(5 marks)

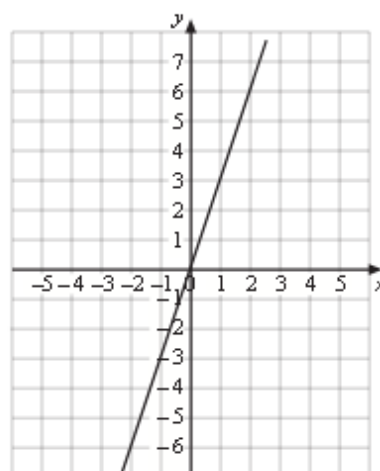
4. (a) Draw a line through the points with coordinates,
 $(3, 4)$, $(4, 6)$ and $(5, 8)$
- (b) What is the *gradient* of the line?
- (c) What is the *intercept* of the line?
- (c) What is the *equation* of the line?

(7 marks)

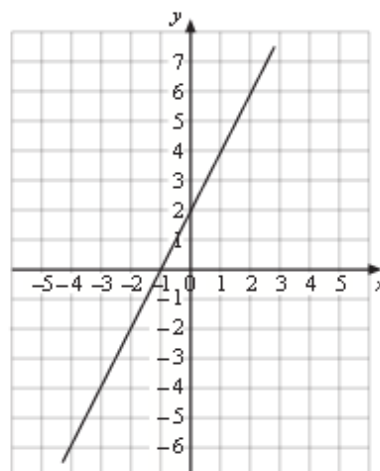
Revision Test 14.2

5. Determine the equation of each of the following lines:

(a)



(b)

*(8 marks)*

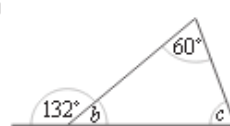
UNIT 15 *Polygons***Revision Test 15.2**
(Academic)

1. Calculate the size of each of the angles marked with a letter in the following diagrams

(a)



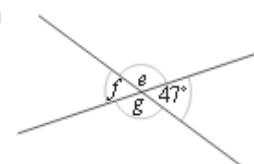
(b)



(c)



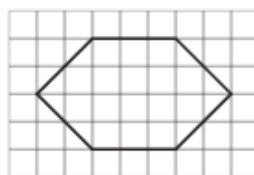
(d)



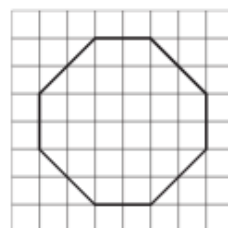
(12 marks)

2. Copy the following shapes and draw in
- all*
- their lines of symmetry.

(a)



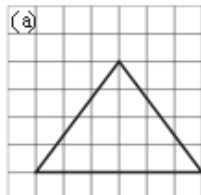
(b)



(4 marks)

3. State the order of rotational symmetry for each of the following shapes:

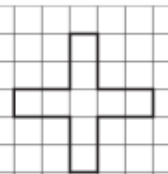
(a)



(b)



(c)



(3 marks)

Revision Test 15.2

4. Calculate the sizes of the interior and exterior angles of a regular octagon.

(5 marks)

5. A regular polygon has an exterior angle of 30° . How many sides does the polygon have?

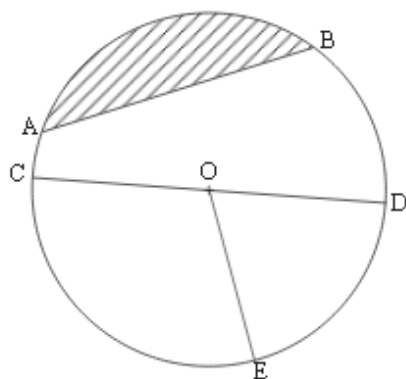
(3 marks)

6. Draw a shape with 6 lines of symmetry, and show clearly all 6 lines.

(3 marks)

Revision Test 16.1

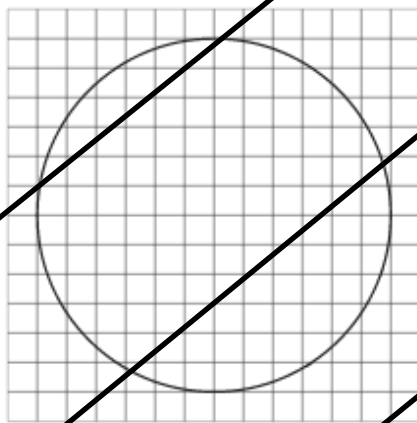
6. The following diagram shows a circle which has its centre at O:



- (a) What is the name given to the line AB?
- (b) What is the name given to the line CD?
- (c) What is the name given to the line OE?
- (d) What is the name given to the shaded area?

(4 marks)

7. Estimate the area of the following circle:



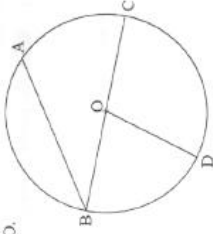
(4 marks)

3.10 Revision: Circles and cylinders

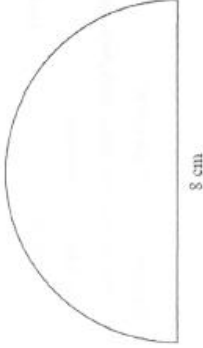
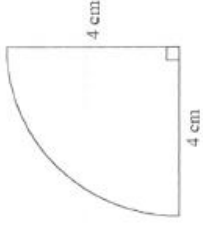
Mr Vickery HWK
Mathematics 8R

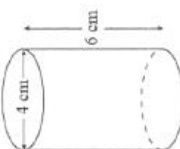

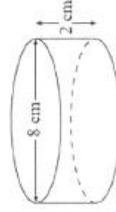

Circles and Cylinders work Sheet
Set Thur 06/03/08 Due Fri 07/03/08

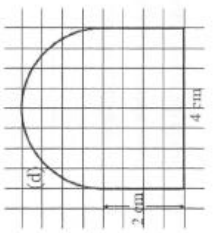
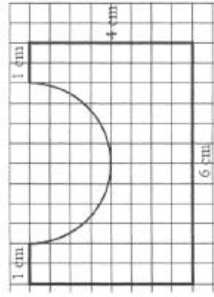
	<p>(a) Draw a circle of radius 4 cm.</p> <p>(b) Inside the circle, draw an isosceles triangle which has 2 sides of length 6 cm.</p> <p>(c) What is the length of the third side of the triangle?</p> <p>(d) Measure the angles in the triangle.</p>
1	<p>(a) Draw a circle of radius 3 cm.</p> <p>(b) Draw a chord of length 4 cm.</p> <p>(c) Shade the minor segment.</p>

2	<p>5</p> <p>(a) Draw a circle of radius 5 cm.</p> <p>(b) Draw a diameter.</p> <p>(c) Mark a point anywhere on the circle.</p> <p>(d) Join this point to each end of the diameter.</p> <p>(e) Measure the angle between the two chords you have drawn.</p> <p>(f) Repeat for some different points on the circle.</p> <p>(g) What do you notice?</p>
3	<p>The diagram shows a circle which has its centre at O.</p> <p>Copy and complete the following sentences:</p> <p>(a) The line AB is a</p> <p>(b) The line BC is a</p> <p>(c) The line OD is a</p> 

5	<p>A circle has radius 20 cm. Calculate:</p> <p>(a) its <i>diameter</i>, (b) its <i>circumference</i>, (c) its <i>area</i>.</p>	
6	<p>Calculate the <i>area</i> and <i>circumference</i> of a circle with radius 6.2 cm.</p>	
7	<p>Calculate the <i>area</i> and <i>circumference</i> of a circle with diameter 9 cm.</p>	

7	<p>Calculate the <i>area</i> and <i>perimeter</i> of the semicircle shown.</p> 	
8	<p>Calculate the <i>area</i> of the following shape:</p> 	

12	<p>A hole of radius 6 cm is cut in a rectangle with sides of length 20 cm and 15 cm. What is the <i>area</i> of the shape that is left?</p>	
13	<p>Work out the Volume and Surface Area of the following shapes.</p> 	
14		

9	<p>A circle has a circumference of 30 cm. What is the <i>radius</i> of this circle?</p>	
10	<p>Calculate the area of each of the shapes shown, which have been formed by adding or removing semicircles and quarter-circles.</p> 	
11		

3.11 Conversion of units: Exercise



Solution

Note 1 gallon \approx 4.5 litres

$$\begin{aligned}\text{Quantity of petrol} &\approx \frac{27}{4.5} \\ &\approx 6 \text{ gallons}\end{aligned}$$

Vera buys approximately 6 gallons of petrol.



Exercises

1. Change the following lengths into inches:

- | | | |
|---------------------|--------------|---------------------|
| (a) 4 feet | (b) 7 feet | (c) 4 feet 2 inches |
| (d) 8 feet 7 inches | (e) 5.5 feet | (f) 2 yards |
| (g) 5 yards 2 feet | (h) 1 mile | |

Change the following lengths into feet or feet and inches:

- | | | |
|----------------|----------------|---------------|
| (i) 60 inches | (j) 48 inches | (k) 17 inches |
| (l) 29 inches | (m) 108 inches | (n) 95 inches |
| (o) 240 inches | (p) 6 inches | |

2. Change the following masses into ounces:

- | | | |
|----------------|-----------------------|----------------|
| (a) 7 pounds | (b) 11 pounds | (c) 36 pounds |
| (d) 904 pounds | (e) 42 pounds | (f) 5.5 pounds |
| (g) 2 stone | (h) 9 stone 12 pounds | |

Change the following masses into pounds or pounds and ounces:

- | | | |
|----------------|----------------|---------------|
| (i) 80 ounces | (j) 128 ounces | (k) 56 ounces |
| (l) 720 ounces | (m) 36 ounces | (n) 77 ounces |
| (o) 8 ounces | (p) 4 ounces | |

3. Change the following volumes into pints:

- | | | |
|----------------------------|-------------------|--------------------------|
| (a) 5 gallons | (b) 11 gallons | (c) 63 gallons |
| (d) 412 gallons | (e) 7.5 gallons | (f) $\frac{1}{2}$ gallon |
| (g) $3\frac{1}{4}$ gallons | (h) 6.875 gallons | |

3.12 Laws of indices: Exercise



Exercises

1. Copy each of the following statements and fill in the missing numbers:

(a) $2^3 \times 2^7 = 2^{\square}$

(b) $3^6 \times 3^5 = 3^{\square}$

(c) $3^7 \div 3^4 = 3^{\square}$

(d) $8^3 \times 8^4 = 8^{\square}$

(e) $(3^2)^5 = 3^{\square}$

(f) $(2^3)^6 = 2^{\square}$

(g) $\frac{3^6}{3^2} = 3^{\square}$

(h) $\frac{4^7}{4^2} = 4^{\square}$

2. Copy each of the following statements and fill in the missing numbers:

(a) $a^3 \times a^2 = a^{\square}$

(b) $b^7 \div b^2 = b^{\square}$

(c) $(b^2)^5 = b^{\square}$

(d) $b^6 \times b^4 = b^{\square}$

(e) $(z^3)^9 = z^{\square}$

(f) $\frac{q^{16}}{q^7} = q^{\square}$

3. Explain why $9^4 = 3^8$.

4. Calculate:

(a) $3^0 + 4^0$

(b) $6^0 \times 7^0$

(c) $8^0 - 3^0$

(d) $6^0 + 2^0 - 4^0$

5. Copy each of the following statements and fill in the missing numbers:

(a) $3^6 \times 3^{\square} = 3^{17}$

(b) $4^6 \times 4^{\square} = 4^{11}$

(c) $\frac{a^6}{a^{\square}} = a^4$

(d) $(z^{\square})^6 = z^{18}$

(e) $(a^{10})^{\square} = a^{95}$

(f) $p^{16} \div p^{\square} = p^7$

(g) $(p^{\square})^8 = p^{40}$

(h) $q^{13} \div q^{\square} = q$

6. Calculate:

$$(a) \frac{2^3}{2^2} + 3^0$$

$$(b) \frac{3^4}{3^3} - 3^0$$

$$(c) \frac{5^4}{5^2} + \frac{6^2}{6}$$

$$(d) \frac{7^7}{7^5} - \frac{5^9}{5^7}$$

$$(e) \frac{10^8}{10^5} - \frac{5^6}{5^3}$$

$$(f) \frac{4^{17}}{4^{14}} - \frac{4^{13}}{4^{11}}$$

7. Fill in the missing numbers in each of the following expressions:

$$(a) 8^2 = 2^{\square}$$

$$(b) 81^3 = 9^{\square} = 3^{\square}$$

$$(c) 25^6 = 5^{\square}$$

$$(d) 4^7 = 2^{\square}$$

$$(e) 125^4 = 5^{\square}$$

$$(f) 1000^6 = 10^{\square}$$

$$(g) 81 = \square^4$$

$$(h) 256 = \square^4 = \square^8$$

8. Fill in the missing numbers in each of the following expressions:

$$(a) 8 \times 4 = 2^{\square} \times 2^{\square} \\ = 2^{\square}$$

$$(b) 25 \times 625 = 5^{\square} \times 5^{\square} \\ = 5^{\square}$$

$$(c) \frac{243}{9} = \frac{3^{\square}}{3^{\square}} \\ = 3^{\square}$$

$$(d) \frac{128}{16} = \frac{2^{\square}}{2^{\square}} \\ = 2^{\square}$$

9. Is each of the following statements true or false?

$$(a) 3^2 \times 2^2 = 6^4$$

$$(b) 5^4 \times 2^3 = 10^7$$

$$(c) \frac{6^8}{2^8} = 3^8$$

$$(d) \frac{10^8}{5^6} = 2^2$$

4 Appendix 4: Productive mathematical operations and actions, identified from pupil interviews

Using specific terminology to refer to items and a specific grammar for their use in expressions

Using rough writing, to express ideas in sketched algebra or words

Producing specific textual forms

Reflecting on ideas expressed in writing

Using written algorithms for accuracy and for justification

Referring to written methods to review reasoning

Producing solutions and answers

Receiving and reading problems (predetermined, pre-packaged and already solved by someone else)

Paying attention to detail

Recalling routines

Recalling facts, definitions and figures

Using known results and techniques to generate new understanding

Progressing from simpler to harder problems

Solving problems that have numerical/algebraic answers

Recalling calculator functions and purpose

Relating results from calculation to the problem situation

Applying mathematical logic and making deductive connections

Accepting right/wrong characteristic of answers

Memorising methods through practice

Learning mathematical logic

Identifying important features of a problem situation

Choosing and applying relevant methods

Identifying problems by type

Explaining ideas in English and mathematical terms

Working with number (counting and arithmetic)

Working with statistical information

Identifying relevant formulae

Using formulae and solving equations

Structuring arithmetic required to solve a problem

Dealing with problems of increasing complexity

Understanding what you know and what you need to find out

Solving sequences of similar problems, of increasing difficulty

Identifying key steps in a method

Sharing ideas with others, agreeing on answers

Adapting previous knowledge and methods

Connecting to previously learned mathematics

Making connections between known areas of mathematics